

A SUPPORT INVESTIGATION FOR SPR_SODE MODEL FOR DENGUE

¹Mr.S.Dhevarajan, ² Dr.A.Iyemperumal, ³ Dr.S.P.Rajagopalan, ⁴Mrs.D.Kalpana

^{1,2,3}Dr.MGR Educational and Research Institute, University, Chennai-95.

⁴PSB Polytechnic college, Chennai-103.

Abstract

A stochastic ordinary differential equation called SPR_SODE model for the spread of dengue fever is considered to analyze further. In this paper, the existence of endemic equilibrium points of the above said Model is discussed. The new ratio parameter $\diamond = \phi_h / \phi_m$ is defined and the approximation is also verified mathematically. It is also proved that near the bifurcation point the initial direction of the branch of equilibrium points is equal to the right eigenvector of L corresponding to the characteristic value for the above said model.

Key words:

SPR_SODE model, existence, scaled variables, stability and support investigation.

Introduction

Dengue is one of the diseases which is in worldwide and comes under infectious diseases. The work of a carrier (i.e) the medium for transmitting is performed by the mosquito, “Aedes Aegypti” [7]. Four distinct but closely related viruses cause this disease [7]. There is no drug available for the specific treatment of dengue. The symptoms of dengue are evident within 2-7 days of infected mosquito bite [2]. This disease kills about 3 millions of people in a year [8]. Though there are plenty of models for infectious diseases. Though, there is a need for a separate good mathematical model to such a special disease like, dengue fever for better results. In this work, the stochastic ordinary differential equation called SPR_SODE model [3], [4], [5] is considered to analyze further.

SPR_SODE Model

The total population is divided into two major parts, one is the human and another one is the mosquitoes. The human is divided into four, susceptible, exposed, infectious and recovered. The mosquitoes are divided into three, susceptible, exposed and infectious. All the notions and their respective explanations of SPR_SODE model [3], [4], [5] are taken for further analysis without any change. SPR_SODE model is given below [3],[4],[5].

$$\begin{aligned}
 \frac{d}{dt}[SS_h] &= \wp_h + b_h[TP]_h + L_h[RC]_h - \tau_h(t)[SS]_h - \Omega_h[TP]_h[SS]_h \\
 \frac{d}{dt}[EX_h] &= \tau_h(t)[SS]_h - \xi_h[EX]_h - \Omega_h[TP]_h[EX]_h \\
 \frac{d}{dt}[IF_h] &= \xi_h[EX]_h - [RC]_h[IF]_h - \Omega_h[TP]_h[RC]_h \\
 \frac{d}{dt}[RC_h] &= \theta_h[IF]_h - L_h[RC]_h - \Omega_h[TP]_h[IF]_h - \eta[IF]_h \\
 \frac{d}{dt}[SS_m] &= [BIR]_m[TP]_m - \tau_m(t)[SS]_m - \Omega_m[TP]_m[SS]_m \\
 \frac{d}{dt}[EX_m] &= \tau_m(t)[SS]_m - \xi_m[EX]_m - \Omega_m[TP]_m[EX]_m \\
 \frac{d}{dt}[IF_m] &= \xi_m[EX]_m - \Omega_m[TP]_m[IF]_m
 \end{aligned}
 \tag{A}$$

Conversion to fractional Quantities

By dividing each state population by the total population one can scale the state population. Denote each scaled population by small letters. Hence with a minimal amount of workings, one can get,

$$\begin{aligned}
 \frac{d}{dt}[ex]_h &= \frac{\phi_h \phi_m P_{mh} [if]_m}{\phi_m [TP]_m + \phi_h [TP]_h} \cdot [TP]_m \cdot [1 - [ex]_h - [if]_h - [re]_h] - \left[\xi_h + [BIR]_h + \frac{\wp_h}{[TP]_h} \right] [ex]_h + \eta_h [if]_h [ex]_h \\
 \frac{d}{dt}[if]_h &= \xi_h [ex]_h - \left(\theta_h + [BIR]_h + \frac{\wp_h}{[TP]_h} \right) [if]_h + \eta_h [if]_h^2 \\
 \frac{d}{dt}[rc]_h &= \theta_h [if]_h - \left(L_h + [BIR]_h \frac{\wp_h}{[TP]_h} \right) [rc]_h + \eta_h [if]_h [TP]_h \\
 \frac{d}{dt}[TP]_h &= \wp_h + \theta_h [TP]_h - ([DID]_h + [DDD]_h [TP]_h) \cdot [TP]_h - \eta_h [if]_h [TP]_h \\
 \frac{d}{dt}[ex]_m &= \frac{\phi_h \phi_m}{\phi_m [TP]_m + \phi_h [TP]_h} \cdot [TP]_h \cdot [P_{mh} [if]_h + P_{mh} [rc]_h] \cdot [1 - [ex]_h - [if]_h] - [\xi_h + [BIR]_m] [ex]_m \\
 \frac{d}{dt}[if]_m &= \xi_m [ex]_m - [BIR]_m [if]_m \\
 \frac{d}{dt}[TP]_m &= [BIR]_m [TP]_m - ([DID]_m + [DDD]_m [TP]_m) [TP]_m
 \end{aligned}
 \tag{B}$$

Reproductive Number

Now, R_0 can be defined mathematically as, $R_0 = \sqrt{\Re_{hm} \Re_{mh}}$ [1],[2],[3](C)

where \mathfrak{R}_{hm} and \mathfrak{R}_{mh} can be written in notation as,

$$\mathfrak{R}_{hm} = \frac{\xi_m}{\xi_m + [DID]_m + [DDD]_m [TP]_m^*} \frac{\phi_h \phi_m P_{mh} [TP]_h^*}{\phi_m [TP]_m^* + \phi_h [TP]_h^*} P_{mh} \left[[DID]_m + [DDD]_m [TP]_m^* \right]^{-1}$$

$$\mathfrak{R}_{mh} = \frac{\xi_h}{\xi_h + [DID]_h + [DDD]_h [TP]_h^*} \frac{\phi_h \phi_m P_{mh} [TP]_m^*}{\phi_h [TP]_h^* + \phi_m [TP]_m^*} \left(\theta_h + \eta_h + [DID]_h + [DDD]_h [TP]_h^* \right)^{-1}$$

$$\cdot \left[P_{mh} + \overline{P_{mh}} \cdot \theta_h \left(\eta_h + [DID]_m + [DDD]_m [TP]_h^* \right)^{-1} \right] \quad \dots (D)$$

In [3,4,5, S.Dhevarajan et.al, 2013], it was proved that the above said model is asymptotically stable. It is also proved that the domain, existence and uniqueness of the solution of the above said model. The disease free equilibrium of the model is also proved [3]. The solution of the above said model is asymptotically stable. The disease free equilibrium solution is also exists. Asymptotically equilibrium points are near to equilibrium points that are near to them move toward the equilibrium solution [1].

The existence of endemic equilibrium points of SPR-SODE Model

The equilibrium equations for (B) are shown below in (E). In this analysis, Here, the terms $[ex]_h, [if]_h, [rc]_h, [TP]_h, [ex]_m, [if]_m$ and $[TP]_m$ are representing their respective equilibrium values and not their actual values at a given time t.

$$0 = \frac{\phi_h \phi_m P_{mh} [if]_m}{\phi_m [TP]_m + \phi_h [TP]_h} \cdot [TP]_m \cdot [1 - [ex]_h - [if]_h - [re]_h] - \left[\xi_h + [BIR]_h + \frac{\rho_h}{[TP]_h} \right] [ex]_h + \eta_h [if]_h [ex]_h \quad \dots (E1)$$

$$0 = \xi_h [ex]_h - \left(\theta_h + [BIR]_h + \frac{\rho_h}{[TP]_h} \right) [if]_h + \eta_h [if]_h^2 \quad \dots (E2)$$

$$0 = \theta_h [if]_h - \left(L_h + [BIR]_h + \frac{\rho_h}{[TP]_h} \right) [rc]_h + \eta_h [if]_h [TP]_h \quad \dots (E3)$$

$$0 = \rho_h + \theta_h [TP]_h - ([DID]_h + [DDD]_h [TP]_h) \cdot [TP]_h - \eta_h [if]_h [TP]_h \quad \dots (E4)$$

$$0 = \frac{\phi_h \phi_m}{\phi_m [TP]_m + \phi_h [TP]_h} \cdot [TP]_h \cdot [P_{mh} [if]_h + P_{mh} [rc]_h] \cdot [1 - [ex]_h - [if]_h] - [\xi_h + [BIR]_m] [ex]_m \quad \dots (E5)$$

$$0 = \xi_m [ex]_m - [BIR]_m [if]_m \quad \dots (E6)$$

$$0 = [BIR]_m [TP]_m - ([DID]_m + [DDD]_m [TP]_m) [TP]_m \quad \dots (E7)$$

Define new parameter, $\diamond = \phi_h / \phi_m$, to obtain

$$\Gamma \left[\frac{[TP]_m^* \theta [TP]_h^*}{[TP]_m \theta [TP]_h} \right] [TP]_m P_{mh} [if]_m [1 - [ex]_h - [if]_h - [re]_h] - \left[\xi_h + [BIR]_h + \frac{\phi_h}{[TP]_h} \right] [ex]_h + \eta_h [if]_h [ex]_h = 0 \dots\dots (E8)$$

$$\Gamma \left[\frac{[TP]_m^* \theta [TP]_h^*}{[TP]_m \theta [TP]_h} \right] [TP]_h [P_{mh} [if]_h + P_{mh} [rc]_h] [1 - [ex]_h - [if]_h] - [\xi_h + [BIR]_m] [ex]_m = 0 \dots\dots (E9)$$

Bifurcation parameter Γ can be varied, while keeping all other parameters fixed. In terms of the original variables, this corresponds to changing ϕ_h and ϕ_m while keeping the ratio between them fixed. Consider $\diamond = \phi_h / \phi_m$. One can choose the ratio \diamond to sweep out the entire parameter space. By solving for the other variables, either explicitly as functions of the parameters, or in terms of $[ex]_h$ and $[ex]_m$ the equilibrium equations can be reduced as a two-dimensional system for $[ex]_h$ and $[ex]_m$. Now, it is to solve (E7) for $[TP]_m$ explicitly that expressing the positive equilibrium for the total mosquito population in terms of parameters, exactly as in the disease-free (no disease) case [3]: $[TP]_m = \frac{[BIR]_m - [DID]_m}{[DDD]_m} \dots\dots (E10)$

$$\text{Solving for } [if]_m \text{ in (E6) in terms of } [ex]_m, \text{ one can find } [if]_m = \frac{\xi_m [ex]_m}{[BIR]_m} \dots\dots (E11)$$

By rewriting the positive equilibrium for $[TP]_h$ in terms of $[if]_h$ from (E9) as

$$[TP]_h = \frac{[BIR]_h - [DID]_h - \eta_h [if]_h + \sqrt{[BIR]_h - [DID]_h - \eta_h [if]_h]^2 - 4[DDD]_h \phi_h}}{2[DDD]_h} \dots\dots (E12)$$

Using (E12) in (E3), and solving for $[rc]_h$ in terms of $[if]_h$:

$$[rc]_h = \frac{2\theta_h [if]_h}{\left(2L_h + [BIR]_h + [DID]_h - \eta_h [if]_h \right) + \sqrt{[BIR]_h - [DID]_h - \eta_h [if]_h]^2 + 4[DDD]_h \phi_h}} \dots\dots (E13)$$

Given the nonlinear nature of (E2), which is not possible to solve for $[if]_h$ in terms of $[ex]_h$ explicitly.

Now, by using (E12) rewrite (E2), and define the function $[ex]_h = g([if]_h)$ as

$$g([if]_h) = \frac{\theta_h + \eta_h + \frac{1}{2} [[BIR]_h + [DID]_h - \eta_h [if]_h] + \sqrt{[BIR]_h - [DID]_h - \eta_h [if]_h]^2 + 4[DDD]_h \phi_h}}{\xi_h} [if]_h$$

It is clear that $g(0) = 0$. Label the positive constant $g(1)$ as $[ex_{\max}]_h$. As $g([if]_h)$ is a nice smooth and continuous function of $[if]_h$ with $g'([if]_h) > 0$ for $[if]_h \in [0, 1]$ and $[ex]_h \in [0, [ex_{\max}]_h]$, there

exists a smooth function $[if]_h = y([ex]_h)$ with domain $[0, [ex_{\max}]_h]$ and range $[0, 1]$. As $g(0) > 0$, the smooth function $y([ex]_h)$ would extend to some small $[ex]_h < 0$. $[TP]_h$ and $[rc]_h$ can also be expressed as functions of $[ex]_h$ by substituting $[if]_h = y([ex]_h)$ into (E12) and (E13). Now introduce D_1 , the bounded open subset of R^2 defined by

$$D_1 = \left\{ \begin{pmatrix} [ex]_h \\ [ex]_m \end{pmatrix} \in R^2 \mid \begin{matrix} -[ex]_h < [ex]_h < [ex_{\max}]_h \\ -[ex]_m < [ex]_m < 1 \end{matrix} \right\} \dots\dots(E14)$$

for some $[ex]_h > 0$ and some $[ex]_m > 0$. By substituting (E10), (E11), (E12), (E13), and $[if]_h = y([ex]_h)$ into (E1) and (E5) the seven equilibrium equations (E1-E7) equivalently as two equations for the components $\left([ex]_h, [ex]_m \right) \in Y$ can be reformulated. To place these two equations into the Rabinowitz form [6], $u = G(\Gamma, u) = \Gamma Lu + h(\Gamma, u)$, where $u \in Y \subset R^2$, with Euclidean norm $\| \cdot \|$; $\Gamma \in Z \subset R$ is the bifurcation parameter; L is a compact linear map on Y ; and $h(\Gamma, u)$ is $O(\|u\|^2)$ uniformly on bounded Γ intervals. It requires that both Y and Z be open and bounded sets, and that Y contains the point 0. Define Z as the open and bounded set $Z = \{ \Gamma \in R \mid -M_Z < \Gamma < M_Z \}$. This set is defined to include the characteristic values of L , so there is minimum value that M_Z can have, but M_Z may be arbitrarily large with $\Gamma = \frac{\phi_m \phi_h}{\phi_m N_m^* + \phi_h N_h^*}$.

Determination of lower order terms

Now it is to determine lower order terms. (E2) can be written as $f([ex]_h, [if]_h) = 0$, where

$$f([ex]_h, [if]_h) = \xi_h [ex]_h - \left[\theta_h + \eta_h + \frac{1}{2} [[BIR]_h + [DID]_h - \eta_h [if]_h] + \sqrt{[BIR]_h - [DID]_h - \eta_h [if]_h}^2 + 4[DDD]_h \phi_h} \right] [if]_h$$

and use implicit differentiation to write $[if]_h = y([ex]_h)$ as a Taylor polynomial of the form

$$[if]_h = \mathfrak{I}_1 [ex]_h + O([ex]_h)^2 \dots\dots (E15)$$

$$\text{Where } \mathfrak{I}_1 = \left[\frac{\frac{\partial f}{\partial [ex]_h}}{\frac{\partial f}{\partial [if]_h}} \right]_{[ex]_h=[if]_h=0} = \frac{\xi_h}{\left[\theta_h + \eta_h + \frac{1}{2} [BIR]_h + [DID]_h + \sqrt{[BIR]_h - [DID]_h]^2 + 4[DDD]_h \wp_h} \right]}$$

The first order approximations to the equilibrium equations can be obtained by substituting (E15) into (E13) and (E12), and then all three, along with (E11) and (E10) into the equilibrium equations (E8) and (E9). Hence,

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} f_{1_10} & f_{1_01} \\ f_{2_10} & f_{2_01} \end{pmatrix} \begin{pmatrix} [ex]_h \\ [ex]_m \end{pmatrix} + O \left(\begin{pmatrix} [ex]_h \\ [ex]_m \end{pmatrix} \right)^2 \dots\dots\dots (E16)$$

$$\text{Where, } f_{1_10} = - \left[\xi_h + \eta_h + \frac{1}{2} [BIR]_h + [DID]_h + \sqrt{[BIR]_h - [DID]_h]^2 + 4[DDD]_h \wp_h} \right] \dots\dots\dots (E17a)$$

$$f_{1_01} = \Gamma \frac{\xi_m P_{hm} \cdot [BIR]_m - [DID]_m}{[BIR]_m [DDD]_m} \dots\dots\dots (E17b)$$

$$f_{2_10} = \Gamma \cdot \frac{\xi_h \left[[BIR]_h - [DID]_h + \sqrt{[BIR]_h - [DID]_h]^2 + 4[DDD]_h \wp_h} \right]}{2[DDD]_h \left(\theta_h + \eta_h + \frac{1}{2} [BIR]_h + [DID]_h \right) + \sqrt{[BIR]_h - [DID]_h]^2 + 4[DDD]_h \wp_h}} \dots\dots\dots (E17c)$$

$$\left[P_{mh} + \frac{\theta_h \cdot \bar{P}_{mh}}{\left(L_h + \frac{1}{2} [BIR]_h + [DID]_h \right) + \sqrt{[BIR]_h - [DID]_h]^2 + 4[DDD]_h \wp_h}} \right] \dots\dots\dots (E17d)$$

$$f_{2_10} = -([BIR]_m + \xi_m) \dots\dots\dots (E17d)$$

To apply Corollary 1.12 of Rabinowitz [6], we algebraically manipulate (E16) to produce

$$u = \Gamma L u + h(\Gamma, u) \dots\dots\dots (E18)$$

$$\text{Where } u = \begin{pmatrix} [ex]_h \\ [ex]_m \end{pmatrix} \text{ and } L = \begin{pmatrix} 0 & A \\ B & 0 \end{pmatrix} \text{ With}$$

$$A = \frac{\xi_m P_{hm} \cdot [BIR]_m - [DID]_m}{[BIR]_m [DDD]_m \left[\xi_h + \frac{1}{2} [BIR]_h + [DID]_h + \sqrt{[BIR]_h - [DID]_h]^2 + 4[DDD]_h \wp_h} \right]} \dots\dots\dots (E19a)$$

$$B = \frac{\xi_h \left[[BIR]_h - [DID]_h + \sqrt{[BIR]_h - [DID]_h]^2 + 4[DDD]_h \phi_h} \right]}{2[DDD]_h ([BIR]_m + \xi_h) \left[\left(\theta_h + \eta_h + \frac{1}{2} [BIR]_h + [DID]_h \right) + \sqrt{[BIR]_h - [DID]_h]^2 + 4[DDD]_h \phi_h} \right]} \dots (E.19b)$$

$$\left[\frac{P_{mh} + \frac{\theta_h \cdot \bar{P}_{mh}}{\left(L_h + \frac{1}{2} [BIR]_h + [DID]_h \right) + \sqrt{[BIR]_h - [DID]_h]^2 + 4[DDD]_h \phi_h}} \right]$$

and $h(\Gamma, u)$ is $O(u^2)$. The matrix, L , has two distinct eigen values: $+\sqrt{AB}$ and $-\sqrt{AB}$. Characteristic values of a matrix are the reciprocals of its eigen values. The two characteristic values of L by $\lambda_1 = 1/\sqrt{AB}$ and $\lambda_2 = -1/\sqrt{AB}$. As both A and B are always positive, due to our assumption that $[BIR]_m > [DID]_m$, λ_1 is real and corresponds to the dominant eigen value of L . The right and left eigen vectors corresponding to λ_1 are, respectively,

$$v = \begin{pmatrix} \sqrt{A} & \sqrt{B} \end{pmatrix}^T \text{ and } w = \begin{pmatrix} \sqrt{B} & \sqrt{A} \end{pmatrix} \dots \dots \dots (E20)$$

For $M_Z > \lambda_1$, as $0 \in Y$, $(\lambda_1, 0) \in \tilde{\lambda}$. Corollary 1.12 of Rabinowitz [6] states that there is a continuum of solution-pairs $(\Gamma, u) \in \tilde{\lambda}$, whose closure contains the point $(\lambda_1, 0)$, that either meets the boundary of $\tilde{\lambda}$, $\partial \tilde{\lambda}$, or the point $(\lambda_2, 0)$. We denote the continuum of solution-pairs emanating from $(\lambda_1, 0)$ by Π_1 , where $\Pi_1 \subset \tilde{\lambda}$, and from $(\lambda_2, 0)$ by Π_2 , where $\Pi_2 \subset \tilde{\lambda}$. Let Z_1, Z_2, U_1 and U_2 are the sets defined by,

$$Z_1 = \{ \Gamma \in Z \mid u \text{ such that } (\Gamma, u) \in \Pi_1 \} \dots \dots \dots (E21a)$$

$$U_1 = \{ u \in Y \mid \Gamma \text{ such that } (\Gamma, u) \in \Pi_1 \} \dots \dots \dots (E21b)$$

$$Z_2 = \{ \Gamma \in Z \mid u \text{ such that } (\Gamma, u) \in \Pi_2 \} \dots \dots \dots (E21c)$$

$$U_2 = \{ u \in Y \mid \Gamma \text{ such that } (\Gamma, u) \in \Pi_2 \} \dots \dots \dots (E21d) \text{ [Change the variable, } Y \text{ and } Z]$$

The part of Y in the positive quadrant of R^2 can be denoted by Y^+ and defined by $Y^+ = \left\{ \left([ex]_h, [ex]_m \right) \in Y \mid [ex]_h > 0 \text{ and } [ex]_m > 0 \right\}$. The internal boundary of Y^+ can be defined

$$\text{by } \partial Y^+ = \left\{ \left([ex]_h, [ex]_m \right) \in Y \mid \left(\left[[ex]_h > 0 \text{ and } [ex]_m = 0 \right] \text{ or } \left[[ex]_h = 0 \text{ and } [ex]_m > 0 \right] \text{ or } \left[[ex]_h = 0 \text{ and } [ex]_m = 0 \right] \right) \right\}$$

Using the Lyapunov-Schmidt expansion, the initial direction of the continuation of solution-pairs, Π_1 and Π_2 can be determined, as described by Cushing [2]. Here, the proofs only for the expansion of Π_1

around the bifurcation point at $\Gamma = \lambda_1$, the results for Π_2 around $\Gamma = \lambda_2$ are similar. Now, it is to expand the terms of the nonlinear eigen value equation (E18) about the bifurcation point, $(\lambda_1, 0)$. The expanded variables are

$$u = 0 + \varepsilon u^{(1)} + \varepsilon^2 u^{(2)} + \dots \quad \text{..... (E22a)}$$

$$\Gamma = \lambda_1 + \varepsilon \Gamma_1 + \varepsilon^2 \Gamma_2 + \dots \quad \text{..... (E22b)}$$

$$h(\Gamma, u) = h(\lambda_1 + \varepsilon \Gamma_1 + \varepsilon^2 \Gamma_2 + \dots, \varepsilon u^{(1)} + \varepsilon^2 u^{(2)} + \dots) \quad \text{..... (E22c)}$$

$$= \varepsilon^2 h_2(\lambda_1, u^{(1)}) + \dots$$

Where ε is very small, negligible. Now, substitute the expansions (E22) into the eigen value equation (E18) and evaluate at different orders of ε . Evaluating the substitution of the expansions (E22) into the eigen value equation (E18) at $O(\varepsilon^0)$ produces $0 = 0$, which gives us no information.

Relation with Eigen Values

Evaluating the substitution of the expansions (E22) into the eigen value equation (E18) at $O(\varepsilon^1)$, we obtain $u(1) = \lambda_1 L u(1)$. This implies that $u(1)$ is the right eigenvector of L corresponding to the eigen value $1/\lambda_1$. Thus, close to the bifurcation point, the equilibrium point can be approximated by $[ex]_h = \varepsilon \sqrt{A}$ and $[ex]_m = \varepsilon \sqrt{B}$, where ε is arbitrarily small and close to bifurcation point. Hence the initial direction of the branch of equilibrium points, $u(1)$, near the bifurcation point, $(\lambda_1, 0)$, is equal to the right eigenvector of L corresponding to the characteristic value λ_1 .

Conclusion:

In this work, a stochastic ordinary differential equation called SPR_SODE model for the spread of dengue fever is analyzed. The existence of endemic equilibrium points of above said Model is discussed. The new ratio parameter $\diamond = \phi_h / \phi_m$ is defined and the approximation is also verified mathematically. The initial direction of the branch of equilibrium points, near the bifurcation point is equal to the right eigenvector of L corresponding to the characteristic value.

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