Fixed – charge Bi-criterion Transportation Problem

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ABSTRACT

The fixed-charge problem was originally studied by G.B.Dantzig and W.Hirsch in1954. Since then many others including Basu, Pal and Kundu have tried to develop an algorithm for Trade-off in fixed-charge Bi-criterion Transportation Problem. In fact, it is an extension of classical transportation problem, where the cost of transportation is directly proportional to the number of units transported. When a commodity is transported, a fixed cost is incurred in the objective function.

Most of the practical transportation problems have two objectives minimizing of cost and minimizing of time. Thus, the methods of solving such Bi-criterion transportation problems are significant. Most of the methods developed so far have given importance to minimize cost than time or to minimize time than cost. We pro[pose to attempt Bi-criterion transportation problem in which minimization of both time and cost may be achieved.

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In this paper an algorithm has been developed for minimizing cost and time simultaneously. For this we apply the following algorithm.

Algorithm

Step-1 Find the initial basic feasible solution.

Step-2 Find the total time corresponding to initial basic feasible solution.

Step-3 Optimality test for solution obtained in step-1.

Step-4 If not optimal then proceed to find the modified solution.

Step-5 Repeat the process until the infeasible solution is obtained.

Step-6 Choose the cost minimizing value & time minimizing value, this ordered paired will be ideal solution.

Illustrative Example

Let us consider the following transportation problem with m=4 sources i.e. i ϵ I={1,2,3,4} and n=5 destinations B_j, j ϵ J = {1,2, ……..5}. The initial data are presented in Table 1. Each row corresponding to a supply point and each column to a demand point. The total supply 65 is equal to the total demand. In each cell (i,j) top left corner represents the cost C_{ij} required for transporting x_{ij} units from source A_i to destination B_j.

The basic variables x_{ij} are presented in the middle of corresponding cells.

Table-1

i/j	**B$_1$**	**B$_2$**	**B$_3$**	**B$_4$**	**B$_5$**	**Supplies a_i**
A$_1$	4	7	10	9	5	14
A$_2$	6	8	16	9	8	13
A$_3$	7	4	6	10	7	22
A$_4$	5	8	9	10	6	16
Demands b_j	15	10	15	10	15	**Total 65**

Next we proceed to find basic feasible solution by North – West Corner rule.

Table-2

i/j	**B$_1$**	**B$_2$**	**B$_3$**	**B$_4$**	**B$_5$**	**Supplies a_i**
A$_1$	4 14	7	10	9	5	14
A$_2$	6 1	8 10	16 2	9	8	13
A$_3$	7	4	6 13	10 9	7	22
A$_4$	5	8	9	10 1	6 15	16
Demands b_j	15	10	15	10	15	**Total 65**

Total Cost 4x14 + 6x1 + 8x10 + 16x2 + 6x13 + 10x9 + 10x1 + 6x15 = 442

Here time plays an important role, so we now prefer to construct a table of time.

Table-3

i/j	**B$_1$**	**B$_2$**	**B$_3$**	**B$_4$**	**B$_5$**	**Supplies a_i**
A$_1$	11 14	3	10	2	5	14
A$_2$	2 1	7 10	3 2	8	1	13
A$_3$	12	2	4 13	5 9	7	22
A$_4$	9	4	6	3 1	5 15	16
Demands b_j	15	10	15	10	15	**Total 65**

For time calculation, we use the indicator h_{ij} of active transportation route $x_{ij}>0$

$$h_{ij} = \begin{cases} 1 & if\ x_{ij} > 0 \\ 0 & if\ x_{ij} > 0 \end{cases}$$

$$\text{Total Time } T = \sum_{i \in I} \sum_{j \in J} t_{ij} h_{ij}$$

According to above table

Total Time T = 11x1 + 2x1 + 7x1 +3x1 + 4x1 + 5x1 + 3x1 + 5x1
= 11 + 2 + 7 + 3 + 4 + 5 + 3 + 5 = 40

Now we have to check whether the initial basic feasible solution obtained above is optimal. For this we proceed as follows:

Table-4

v_j	$v_1(6)$	$v_2(8)$	$v_3(16)$	$v_4(20)$	$v_5(16)$		
i/j	**B_1**	**B_2**	**B_3**	**B_4**	**B_5**	**Supplies a_i**	**u_i**
A_1	4 14	7	10	9	5	14	**$u_1(-2)$**
A_2	6 1	8 10	16 2	9 -1	8	13	**$u_2(0)$**
A_3	7	4	6 13	10 9	7	22	**$u_3(-10)$**
A_4	5	8	9	10 1	6 15	16	**$u_4(-10)$**
Demands b_j	15	10	15	10	15		

Obviously, the solution is not optimal. So we proceed to find another modified table using most negative d_{ij} (= c_{ij}-u_i-v_j)

Table-5

i/j	**B_1**	**B_2**	**B_3**	**B_4**	**B_5**	**Supplies a_i**
A_1	4 14	7	10	9	5	14
A_2	6 1	8 10	16	9 2	8	13
A_3	7	4	6 15	10 7	7	22
A_4	5	8	9	10 1	6 15	16
Demands b_j	15	10	15	10	15	**Total 65**

Total Cost = 420

Corresponding to this modified value of cost, Total time T = 45.

Proceeding as above after a few steps the solution of the Bi-criterion transportation problem can be achieved in the ordered pair of cost and time. Here it is important to note that the process is repeated until infeasible solution obtained. If we denote the ordered pairs as:

$(Z_1*T_1*), (Z_2*T_2*), \ldots\ldots\ldots\ldots, (Z_n*T_n*)$

Where $Z_1*>Z_2* \ldots\ldots\ldots\ldots\ldots\ldots > Z_n*$

and $T_1*>T_2* \ldots\ldots\ldots\ldots\ldots\ldots > T_n*$

Then we identify the minimum cost Z_n* and minimum time T_1* among the above trade-off pairs. The term (Z_n*T_n*) with minimum cost and minimum time is termed the total solution.

Conclusion

Though minimizing cost and time of a Fixed – charge Transportation problem is of very much Importance particularly in industrial situations, Economics, Inventory control, less attention has been paid to solve this problem. However, the solution of this problem seems to be an ideal solution.

References:

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