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BAYESIAN MULTIPLE DEFERRED STATE (BMDS – 1) SAMPLING PLAN WITH WEIGHTED POISSON DISTRIBUTION

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ABSTRACT

Since the first acceptance sampling plan have been developed 80 years ago, a number of selection principles have emerged. This paper presents a new procedure for the construction and selection of multiple deferred state sampling (MDS – 1) plan through the weighted Poisson model with gamma prior.

Keywords

Bayesian Multiple Deferred State – 1(c_1, c_2) sampling plan, Weighted Poisson distribution with Gamma prior.

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Introduction

The practical performance of a sampling plan is revealed by its operating characteristics (OC) curve. Soundarajan (1981) has extended his approach to single sampling plan under the condition of Poisson model for the OC curve. Soundarajan and Vijaraghavan (1989) extended this approach to multiple deferred sampling plan of type MDS-1(0,2) limiting to the acceptance number at 0 and 2. Subramani and Govindaraju (1990) have presented tables of the selection of multiple deferred state MDS – 1 sampling plan for given acceptable and limiting Quality using Poisson distribution. This paper gives tables and procedure for selecting the multiple deferred state MDS -1 sampling plan of Varest (1982) involving operating characteristic curve (OC) using weighted Poisson distribution with Gamma prior.

In order to control the quality of purchased lot, two major alternatives are opened to a buyer. One, complete inspection: single item in the lot is inspected and tested. Two, Partial inspection: a sample of items is taken, the sampled item are inspected and tested, and the lot as a whole is accepted or rejected depending on whether few or many defective items which are found in the sample. This type of sampling, one of many used to control the quality of manufacturing process or lots, is known as acceptance sampling.

Multiple Deferred State Sampling Plan

Multiple deferred state sampling procedures are a natural extension of fixed deferred state sampling plans. These procedures have been found to be appealing to most management personnel in addition to possessing desirable properties with respect to their OC curves.

The operating procedures are similar to those for fixed deferred state sampling plan with the extension that conditional decisions are based on the disposition of a multiple group of future lots. This type of procedure places more emphasis on the early detection of quality degradation than does fixed deferred state sampling procedures. That is, the conditional acceptance criterion for marginal quality is more stringent. The underlying logic behind this type of decision criterion over dependent stage and chain sampling concepts is in the indicator concept. This indicator concept is based on the supposition that the number of defectives observed in a sample provides some insight into the state of the process quality. That is, the greater the number of defectives observed, the more likely it is that the process quality has degraded or is degrading to an unacceptable level. Conditional decision based on future lots is more likely to detect this degradation than those conditional decisions based on past lots.

MDS – 1 Plan

The MDS – 1 plan is applicable to the case of Type B situations where lots expected to be of the same quality are submitted for inspection seriously in the lot production. MDS – 1 plans are extensions of chain sampling plans of Dodge's (1955) type ChSP – 1. Both the MDS – 1 and chain sampling plans achieve a similar reduction in sample size when compared to the unconditional plans, such as single and double sampling plans. The Operating procedure of the MDS – 1 plan as given by.

(1) From each submitted lot, select a sample of n units and test each unit for conformance to the specified requirements.

(2) Accept the lot if x, the observed number of nonconformities, is less than or equal to c_1; reject the lot if x is greater than c_2.

(3) If $c_1 < x < c_2$ accept the lot, provided in each of the sample taken from the preceding or succeeding m lots, the number of nonconformities found is less than or equal to c_1. The lot otherwise rejected.

Weighted Poisson distribution

Rao (1965) introduced the concept of weighted distribution when the samples are recorded without a sampling frame that enables random samples to be drawn. The weight function that usually appears in the scientific and statistical literature is ω(X) =X, which provide the size – biased version of the random variable. The size – biased version of order k, which corresponds to the weight ω (X) =X^k, for k any real positive number has also been widely used. Joan Del Castillo and Perez- Casany(1998) applied the weighted Poisson distribution that results from the modification of the Poisson distribution with the weight ω(X) =X^k can also considered as a mixture of the size – biased version of the Poisson distribution. They fit the weighted Poisson distribution for over dispersion (aggregation) and under dispersion (repulsion) situation. Patil, Rao and Ratnaparki. (1986) have proved that given a random variable X, the weighted version X^k is stochastically greater or smaller than the original random variable X according as the weight function ω(X) is monotonically increasing or decreasing to X. Patil and Rao (1978) pointed out that the importance of the size-biased version of a random variable X. They show that many classical discrete distributions have a size-biased version of the same form with the variable reduced by unity.

In the construction of acceptance sampling plan, size- biased version of random variable about defectives play an important role. The weighted distributions are more suitable distributions than the classical distributions like Binomial, Poisson and Negative Binomial. The weighted Poisson distribution plays an important role in acceptance sampling, mainly in the construction of sampling plans. Each outcome (number of defectives) is specific but can be assigned different weights based on its importance or usage. The probability mass function of weighted Poisson distribution is given by:

$$p(x,n,p,k) = \frac{x^k p(x)}{\Sigma x^k p(x)} \quad x = 1,2,3 \ldots$$

Where

$$p(x) = \frac{e^{-np}(np)^x}{x!} \quad x = 1,2,3 \ldots$$

Here X^k is the corresponding weight for each outcome and 'k' is a constant. The Poisson distribution can be seen as the particular case of the weighted Poisson distribution when k = 0. The probability mass function of the weighted Poisson distribution for k = 1 is

$$p(x) = \frac{e^{-np}(np)^{x-1}}{(x-1)!} \quad x = 1,2,3 \ldots$$

Bayesian Acceptance Sampling

Bayesian acceptance sampling approach is associated with utilization of prior process history for the selection to describe the random fluctuations involved in Acceptance sampling. Bayesian sampling plan requires the user to specify explicit the distribution of defectives from lot to lot. The prior distribution is the expected distribution of a lot quality on which the sampling plan is going to operate. The distribution is called prior because it is formulated prior to the taking of samples. The combination of prior knowledge, represented with the prior distribution, and the empirical knowledge based on the sample leads to the decision on the lot.

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Bayesian MDS-1 plan

Based on Hald (1981) and Suresh, Latha (2001), APA functions for MDS-1 Plan with Gamma Poisson distribution is obtained as

$$\overline{P} = \overline{P}c_1+[\overline{P}c_2-\overline{P}c1][\overline{P}c_1]^m$$

Where

$$\overline{P}c_{1=}\ \sum_{x=1}^{c1} \frac{1}{\beta(x,s-1)} \frac{s^s}{(s-1)} \frac{\mu^{x-1}}{(\mu+s)^{x+s-1}}\ ; x = 1,2,3, \ldots$$

$$\overline{P}c_2 = \sum_{x=1}^{c2} \frac{1}{\beta(x,s-1)} \frac{s^s}{(s-1)} \frac{\mu^{x-1}}{(\mu+s)^{x+s-1}}\ ; x = 1,2,3, \ldots$$

Comparison with Conventional MDS-1 Plan

Tables 1-4 gives the average probability of acceptance values for given nμ,m,$c_1$andc_2.

The probability of acceptance values of Bayesian MDS-1 plan can be compared with conventional MDS-1 plan. Table.5 gives the value of probability of acceptance for conventional MDS-1 plan. It is observed that the probability of acceptance for Bayesian plan is less than that for the conventional plan for given (np, m, c).Therefore, the Bayesian sampling plan is more favourable to the consumer than the conventional plan.

Table 1. Average probability of acceptances values for BMDS-1 plan for given values of nμ, s, and c_1=1, c_2=2, m=0.

nμ \ s	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
0.02	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997
0.03	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995
0.04	0.9988	0.9989	0.9990	0.9990	0.9990	0.9991	0.9991	0.9991
0.5	0.8960	0.8996	0.9017	0.9031	0.9041	0.9048	0.9054	0.9058
0.6	0.8648	0.8680	0.8700	0.8714	0.8723	0.8730	0.8736	0.8741
0.7	0.8332	0.8355	0.8371	0.8382	0.8390	0.8397	0.8402	0.8405
0.8	0.8017	0.8028	0.8037	0.8044	0.8050	0.8054	0.8057	0.8060
0.9	0.7708	0.7702	0.7703	0.7704	0.7706	0.7708	0.7709	0.7710
1	0.7407	0.7382	0.7372	0.7367	0.7364	0.7363	0.7361	0.7360
2	0.5000	0.4752	0.4609	0.4515	0.4449	0.4400	0.4362	0.4331
3	0.3520	0.3125	0.2894	0.2741	0.2633	0.2552	0.2490	0.2440
4	0.2592	0.2136	0.1875	0.1705	0.1586	0.1498	0.1430	0.1377
5	0.1982	0.1516	0.1257	0.1093	0.0981	0.0900	0.0838	0.0790
8	0.1040	0.0645	0.0452	0.0343	0.0274	0.0228	0.0195	0.0171
10	0.0740	0.0406	0.0257	0.0178	0.0132	0.0102	0.0082	0.0068
15	0.0382	0.0162	0.0081	0.0046	0.0028	0.0019	0.0013	0.0009
25	0.0156	0.0045	0.0016	0.0006	0.0003	0.0001	8.42E-05	4.86E-05

Table 2. Average probability of average value for BMDS-1 plan given values nµ, s, c_1=1, c_2=2, m=1.

nµ \ s	2	3	4	5	6	7	8	9
0.02	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997
0.03	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995
0.04	0.9988	0.9989	0.9990	0.9990	0.9990	0.9991	0.9991	0.9991
0.5	0.8960	0.8996	0.9017	0.9031	0.9041	0.9048	0.9054	0.9058
0.6	0.8648	0.8680	0.8700	0.8714	0.8723	0.8730	0.8736	0.8741
0.7	0.8332	0.8355	0.8371	0.8382	0.8390	0.8397	0.8402	0.8405
0.8	0.8017	0.8028	0.8037	0.8044	0.8050	0.8054	0.8057	0.8060
0.9	0.7708	0.7702	0.7703	0.7704	0.7706	0.7708	0.7709	0.7710
1	0.7407	0.7382	0.7372	0.7367	0.7364	0.7363	0.7361	0.7360
2	0.5000	0.4752	0.4609	0.4515	0.4449	0.4400	0.4362	0.4331
3	0.3520	0.3125	0.2894	0.2741	0.2633	0.2552	0.2490	0.2440
4	0.2592	0.2136	0.1875	0.1705	0.1586	0.1498	0.1430	0.1377
5	0.1982	0.1516	0.1257	0.1093	0.0981	0.0900	0.0838	0.0790
8	0.1040	0.0645	0.0452	0.0343	0.0274	0.0228	0.0195	0.0171
10	0.0740	0.0406	0.0257	0.0178	0.0132	0.0102	0.0082	0.0068
15	0.0382	0.0162	0.0081	0.0046	0.0028	0.0019	0.0013	0.0009
25	0.0156	0.0045	0.0016	0.0006	0.0003	0.0001	8.42E-05	4.86E-05

Table 3. Average probability of average value for BMDS-1 plan given values nµ, s, c_1=1, c_2=2, m=2.

nµ \ s	2	3	4	5	6	7	8	9
0.02	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997
0.03	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995
0.04	0.9988	0.9989	0.9990	0.9990	0.9990	0.9991	0.9991	0.9991
0.5	0.8960	0.8996	0.9017	0.9031	0.9041	0.9048	0.9054	0.9058
0.6	0.8648	0.8680	0.8700	0.8714	0.8723	0.8730	0.8736	0.8741
0.7	0.8332	0.8355	0.8371	0.8382	0.8390	0.8397	0.8402	0.84059
0.8	0.8017	0.8028	0.8037	0.8044	0.8050	0.8054	0.8057	0.8060
0.9	0.7708	0.7702	0.7703	0.7704	0.7706	0.7708	0.7709	0.7710
1	0.7407	0.7382	0.7372	0.7367	0.7364	0.7363	0.7361	0.7360
2	0.5000	0.4752	0.4609	0.4515	0.4449	0.4400	0.4362	0.4331
3	0.3520	0.3125	0.2894	0.2741	0.2633	0.2552	0.2490	0.2440
4	0.2592	0.2136	0.1875	0.1705	0.1586	0.1498	0.1430	0.1377
5	0.1982	0.1516	0.1257	0.1093	0.0981	0.0900	0.0838	0.0790
8	0.1040	0.0645	0.0452	0.0343	0.0274	0.0228	0.0195	0.0171
10	0.0740	0.0406	0.0257	0.0178	0.0132	0.0102	0.0082	0.0068
15	0.0382	0.0162	0.0081	0.0046	0.0028	0.0019	0.0013	0.0009
25	0.0156	0.0045	0.0016	0.0006	0.0003	0.0001	8.42E-05	4.86E-05

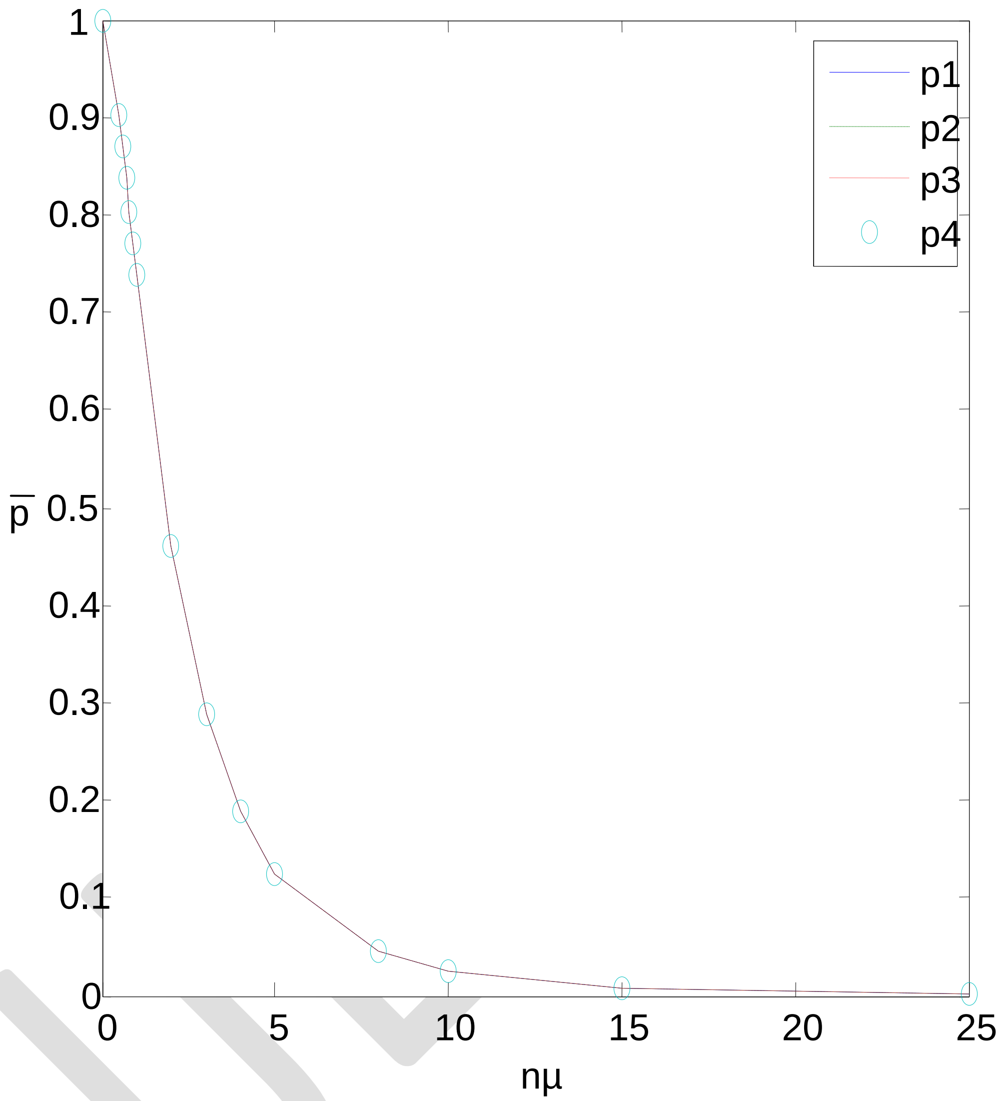
Table 4. Average probability of average value for BMDS-1 plan given values nμ, s, c_1=1, c_2=2, m=3.

nμ \ s	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
0.02	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997
0.03	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995
0.04	0.9988	0.9989	0.9990	0.9990	0.9990	0.9991	0.9991	0.9991
0.5	0.8960	0.8996	0.9017	0.9031	0.9041	0.9048	0.9054	0.9058
0.6	0.8648	0.8680	0.8700	0.8714	0.8723	0.8730	0.8736	0.8741
0.7	0.8332	0.8355	0.8371	0.8382	0.8390	0.8397	0.8402	0.8405
0.8	0.8017	0.8028	0.8037	0.8044	0.8050	0.8054	0.8057	0.8060
0.9	0.7708	0.7702	0.7703	0.7704	0.7706	0.7708	0.7709	0.7710
1	0.7407	0.7382	0.7372	0.7367	0.7364	0.7363	0.7361	0.7360
2	0.5000	0.4752	0.4609	0.4515	0.4449	0.4400	0.4362	0.4331
3	0.3520	0.3125	0.2894	0.2741	0.2633	0.2552	0.2490	0.2440
4	0.2592	0.2136	0.1875	0.1705	0.1586	0.1498	0.1430	0.1377
5	0.1982	0.1516	0.1257	0.1093	0.0981	0.0900	0.0838	0.0790
8	0.1040	0.0645	0.0452	0.0343	0.0274	0.0228	0.0195	0.0171
10	0.0740	0.0406	0.0257	0.0178	0.0132	0.0102	0.0082	0.0068
15	0.0382	0.0162	0.0081	0.0046	0.0028	0.0019	0.0013	0.0009
25	0.0156	0.0045	0.0016	0.0006	0.0003	0.0001	8.42E-05	4.86E-05

Table 5. Probability of Acceptance values for MDS-1 Plan for c_1=1, c_2=2, m=0, 1, 2, 3.

np\m	0	1	2	3
0.02	0.9998	0.9994	0.9990	0.9987
0.03	0.9995	0.9986	0.9978	0.9970
0.04	0.9992	0.9977	0.9962	0.9948
0.5	0.9097	0.7904	0.7180	0.6741
0.6	0.8780	0.7295	0.6479	0.6032
0.7	0.8441	0.6692	0.5823	0.5391
0.8	0.8087	0.6108	0.5219	0.4819
0.9	0.7724	0.5553	0.4670	0.4311
1	0.7357	0.5032	0.4176	0.3861
2	0.4060	0.1719	0.1402	0.1360
3	0.1991	0.0572	0.0501	0.0498
4	0.0915	0.0196	0.0183	0.0183
5	0.0404	0.0069	0.0067	0.0067
8	0.0030	0.0003	0.0003	0.0003
10	0.0004	4.54E-05	4.54E-05	4.54E-05
15	4.89E-06	3.06E-07	3.06E-07	3.06E-07
25	3.61E-10	1.39E-11	1.39E-11	1.39E-11

Figure 1. **Average Probability of Acceptance Curve**



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