Transient Convective Heat and Mass Transfer Flow of a Chemically Reacting Viscous Fluid in a Vertical Wavy Channel with Oscillatory Flux with Heat Sources

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ABSTRACT

In this paper, we discuss the effect of chemical reaction on unsteady free connective heat and mass transfer flow through a porous medium in a vertical wavy channel The unsteadiness in the flow is due to the oscillatory flux in the flow region. The coupled equations governing the flow, heat and mass transfer have been solved by using a perturbation technique with the slope δ as the perturbation parameter. The expression for the velocity, the temperature, the concentration, the shear stress and the rate of heat and mass transfer are derived and are analysed for different variations of the governing parameters β, α, γ, k.

Keywords :   Heat and Mass Transfer, Chemical Reaction, Viscous Fluid, Wavy Channel, Oscillatory Flux.

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INTRODUCTION

Coupled heat and mass transfer phenomenon in porous media is gaining attention due to its interesting applications. The flow phenomenon is relatively complex rather than that of the pure thermal convection process. Underground spreading chemical wastes and other pollutants, grain storage ,evaporation cooling and solidification are the few other application areas where the combined thermo-solutal natural convection in porous media are observed .Combined heat and mass transfer by free convection under boundary layer approximations has been studied by Bejan and Khair [2], Lai and Kulacki [9] and Murthy and Singh [11].Coupled heat and mass transfer by mixed convection in Darcian fluid-saturated porous media has been analysed by Lai [8].The free convection heat and mass transfer in a porous enclosure has been studied recently by Angirasa et al [1]. The combined effects of thermal and mass diffusion in channel flows has been studied in recent times by a few authors, notably Nelson and Wood [14,15], Lee et al [10] and others [23,25].

In recent years, energy and material saving considerations have prompted an expansion of the efforts at producing efficient heat exchanger equipment through augmentation of heat transfer. It has been established [5a] that channels with diverging – converging geometries augment the transportation of heat transfer and momentum. As the fluid flows through a tortuous path viz., the dilated – constricted geometry, there will be more intimate contact between them. The flow takes place both axially (primary) and transversely (secondary) with the secondary velocity being towards the axis in the fluid bulk rather than confining within a thin layer as in straight channels. Hence it is advantageous to go for converging – diverging geometries for improving the design of heat transfer equipment. Vajravelu and Nayfeh [23a] have investigated the influence of the wall waviness on friction and pressure drop of the generated coquette flow. Vajravelu and Sastry [24] have analysed the free convection heat transfer in a viscous, incompressible fluid confined between long
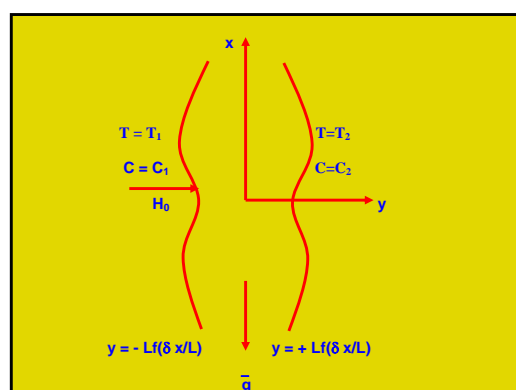
vertical wavy walls in the presence of constant heat source. Later Vajravelu and Debnath [25] have extended this study to convective flow in a vertical wavy channel in four different geometrical configurations. This problem has been extended to the case of wavy walls by McMichael and Deutsch [10a], Deshikachar et al [4a], Rao et al [17a] and Sree Ramachandra Murthy [20a]. Hyan Goo Kwon et al [7a] have analyzed that the Flow and heat/mass transfer in a wavy duct with various corrugation angles in two dimensional flow regimes. Comini et al [3a] have analyzed the Convective heat and mass transfer in wavy finned-tube exchangers.

In many chemical engineering processes, there does occur the chemical reaction between a foreign mass and the fluid in which the plate is moving. These processes take place in numerous industrial applications viz., polymer production, manufacturing of ceramics or glassware and food processing. Das et al [5] have studied the effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction. Muthukumara-swamy [13] has studied the effects of reaction on a long surface with suction. Recently Gnaneswar [6] has studied radiation and mass transfer on an unsteady two-dimensional laminar convective boundary layer flow of a viscous incompressible chemically reacting fluid along a semi-infinite vertical plate with suction by taking into account the effects of viscous dissipation.

The present trend in the field of chemical reaction analysis is to give a mathematical model for the system to predict the reactor performance. A large amount of research work has been reported in this field. In particular the study of heat and mass transfer with chemical reaction is of considerable importance in chemical and hydrometallurgical industries. Chemical reaction can be codified as either heterogeneous or homogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. Frequently the transformations proceed in a moving fluid, a situation encountered in a number of technological fields. A common area of interest in the field of aerodynamics is the analysis of thermal boundary layer problems for two dimensional steady and incompressible laminar flow passing a wedge. Simultaneous heat and mass transfer from different geometrics embedded in a porous media has many engineering and geophysical application such as geothermal reservoirs, drying of porous solids thermal insulation, enhanced oil recovery, packed-bed catalytic reactors, cooling of nuclear reactors, and under ground energy transport. A very significant area of research in radiative heat transfer, at the present time is the numerical simulation of combined radiation and convection/conduction transport processes. The effort has arisen largely due to the need to optimize industrial system such as furnaces, ovens and boilers and the interest in our environment and in no conventional energy sources, such as the use of salt-gradient solar ponds for energy collection and storage. In particular, natural convection induced by the simultaneous action of buoyancy forces resulting from thermal and mass diffusion is of considerable interest in nature and in many industrial application such as geophysics, oceanography, drying process, solidification of binary alloy and chemical engineering. Kandaswamy et al [9a] have discussed the Effects of chemical reaction, heat and mass transfer on boundary layer flow over a porous wedge with heat radiation in the presence of suction or injection. Recently Madhusudan Reddy [10b] has analysed the effect of chemical reaction on double diffusive heat transfer flow of a viscous fluid in a wavy channel.

Configuration of the Problem

FORMULATION OF THE PROBLEM

We consider the effect of chemical reaction on the unsteady motion of viscous, incompressible fluid through a porous medium in a vertical   channel

bounded by wavy walls . The thermal buoyancy in the flow field is created by an oscillatory flux in the fluid region. The walls are maintained at constant temperature and concentration. The Boussinesq approximation is used so that the density variation will be considered only in the buoyancy force. The viscous and Darcy dissipations are neglected in comparison with heat by conduction and convection in the energy equation. Also the Kinematic viscosity ν, the thermal conducting k are treated as constants. We choose a rectangular Cartesian system 0 (x ,y) with x-axis in the vertical direction and y-axis normal to the walls. The walls of the channel are at $y = \pm L f(\frac{\delta x}{L})$

The equations governing the unsteady flow ,heat and mass transfer are

Equation of continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

Equation of linear momentum

$$\rho_e(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}) = -\frac{\partial p}{\partial x} + \mu(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}) - \rho g - (\sigma\mu_e^2 H_o^2)u - (\frac{\mu}{k})u \tag{2}$$

$$\rho_e(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}) = -\frac{\partial p}{\partial y} + \mu(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}) - (\frac{\mu}{k})v \tag{3}$$

Equation of Energy:

$$\rho_e C_p(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}) = \lambda(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}) - Q(T - T_e) \tag{4}$$

Equation of diffusion

$$(\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y}) = D_1(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}) - k_1(C - C_e) \tag{5}$$

Equation of state

$$\rho - \rho_e = -\beta\rho_e(T - T_e) - \beta^*\rho_e(C - C_e) \tag{6}$$

where ρ_e is the density of the fluid in the equilibrium state, Te, Ce are the temperature and concentration in the equilibrium state (u, v) are the velocity components along O(x, y) directions, p is the pressure, T, C are the temperature and Concentration in the flow region,ρis the density of the fluid,μ is the constant coefficient of viscosity ,Cp is the specific heat at constant pressure, λ is the coefficient of thermal conductivity , k is the permeability of the porous medium , β is the coefficient of thermal expansion, Q is the strength of the constant internal heat source ,σ is the electrical conductivity , μe is the magnetic permeability, β^* is the volumetric expansion with mass fraction coefficient, D_1 is the molecular diffusivity and k_1 is the chemical reaction coefficient.

In the equilibrium state

$$0 = -\frac{\partial p_e}{\partial x} - \rho_e g \tag{7}$$

where $p = p_e + p_D, p_D$ being the hydrodynamic pressure.

The flow is maintained by an oscillatory volume flux for which a characteristic velocity is defined as

$$q(1 + k e^{i\omega t}) = \frac{1}{L}\int_{-Lf}^{Lf} u\, dy. \tag{8}$$

The boundary conditions for the velocity and temperature fields are

$$u = 0 \;, v = 0 \;,T = T_1 \;,C = C_1 \qquad \text{on } y = -Lf\left(\frac{\delta x}{L}\right)$$

$$u = 0, \; v = 0, \; T = T_2 \qquad C = C_2 \qquad \text{on } y = +Lf\left(\frac{\delta x}{L}\right) \qquad (9)$$

In view of the continuity equation we define the stream function ψ as

$$u = -\psi_y \,, \; v = \psi_x \qquad (10)$$

Eliminating pressure p from equations (2)&(3)and using the equations governing the flow in terms of ψ are

$$[(\nabla^2\psi)_t + \psi_x(\nabla^2\psi)_y - \psi_y(\nabla^2\psi)_x] = \nu\nabla^4\psi - \beta\, g(T-T_0)_y -$$

$$-\left(\frac{\mu}{k}\right)\nabla^2\psi - \beta^*\, g(C-C_0)_y - (\sigma\mu_e^2 H_o^2)\frac{\partial^2\psi}{\partial y^2} \qquad (11)$$

$$\rho_e C_p\left(\frac{\partial T}{\partial t} + \frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y}\right) = \lambda\nabla^2 T - Q(T-T_o) \qquad (12)$$

$$\left(\frac{\partial C}{\partial t} + \frac{\partial\psi}{\partial y}\frac{\partial C}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial C}{\partial y}\right) = D\nabla^2 C - k_1(C-C_o) \qquad (13)$$

Introducing the non-dimensional variables in (2 .9)& (2.10) as

$$x' = x/L, \; y' = y/L, \; t' = t\omega, \; \Psi' = \Psi/\nu, \; \theta = \frac{T-T_2}{T_1-T_2}, \; C' = \frac{C-C_2}{C_1-c_2} \qquad (14)$$

the governing equations in the non-dimensional form (after dropping the dashes) are

$$R\left(\gamma^2(\nabla^2\psi)_t + \frac{\partial(\psi,\nabla^2\psi)}{\partial(x,y)}\right) = \nabla^4\psi + \left(\frac{G}{R}\right)(\theta_y + NC_y) - M^2\frac{\partial^2\psi}{\partial y^2} \qquad (15)$$

$$P\left(\gamma^2\frac{\partial\theta}{\partial t} + \frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}\right) = \nabla^2\theta - \alpha\theta \qquad (16)$$

$$Sc\left(\gamma^2\frac{\partial C}{\partial t} + \frac{\partial\psi}{\partial y}\frac{\partial C}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial C}{\partial y}\right) = \nabla^2 C - KC \qquad (17)$$

where

$$R = \frac{UL}{\nu} \qquad \text{(Reynolds number)}, \qquad G = \frac{\beta g\Delta T_e L^3}{\nu^2} \text{ (Grashof number)}$$

$$P = \frac{\mu c_p}{k_1} \text{ (Prandtl number)}, \qquad M^2 = \frac{\sigma\mu_e^2 H_o^2 L^2}{\nu^2} \text{ (Hartmann number)},$$

$$Sc = \frac{\nu}{D_1} \text{ (Schmidt number)}, \qquad \alpha = \frac{QL^2}{\lambda} \text{ (Heat source parameter)},$$

$$D^{-1} = \frac{L^2}{k} \text{ (Darcy number)}, \qquad K = \frac{K_1 L^2}{D_1} \qquad \text{(Chemical reaction parameter)},$$

$$\gamma^2 = \frac{\omega L^2}{\nu} \qquad \text{(Wormsely Number)}$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

The corresponding boundary conditions are

$$\psi(+f) - \psi(-f) = 1$$

$$\frac{\partial \psi}{\partial x} = 0, \quad \frac{\partial \psi}{\partial y} = 0 \quad at\, y\eta = \pm f \tag{18}$$

$$\theta(x, y) = 1, C = 1 \quad on \ y = -f$$

$$\theta(x, y) = 0, C = 0 \quad on \ y = f$$

$$\frac{\partial \theta}{\partial y} = 0, \frac{\partial C}{\partial y} = 0 \quad at \quad y = 0 \tag{19}$$

The value of ψ on the boundary assumes the constant volumetric flow in consistant with the hyphothesis (7) .Also the wall temperature varies in the axial direction in accordance with the prescribed arbitrary function t.

METHOD OF SOLUTION

The main aim of the analysis is to discuss the perturbations created over a combined free and forced convection flow due to traveling thermal wave imposed on the boundaries. The perturbation analysis is carried out by assuming that the aspect ratio δ to be small.

Introduce the transformation such that

$$\bar{x} = \delta x, \frac{\partial}{\partial x} = \delta \frac{\partial}{\partial \bar{x}}$$

Then $\quad \frac{\partial}{\partial x} \approx O(\delta) \rightarrow \frac{\partial}{\partial \bar{x}} \approx O(1)$

For small values of δ<<1 ,the flow develops slowly with axial gradient of order δ

And hence we take $\frac{\partial}{\partial \bar{x}} \approx O(1)$

Using the above transformation the equations(15-17) reduces to

$$\delta R(\gamma^2 (\nabla_1^2 \psi)_t + \frac{\partial(\psi, \nabla_1^2 \psi)}{\partial(x, y)}) = \nabla_1^4 \psi + (\frac{G}{R})(\theta_y + NC_y) - M^2 \frac{\partial^2 \psi}{\partial y^2} - D^{-1}F^2\psi \tag{20}$$

$$\delta P(\gamma^2 \frac{\partial \theta}{\partial t} + \frac{\partial \psi}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \theta}{\partial y}) = \nabla_1^2 \theta - \alpha\theta \tag{21}$$

$$\delta Sc(\gamma^2 \frac{\partial C}{\partial t} + \frac{\partial \psi}{\partial y}\frac{\partial C}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial C}{\partial y}) = \nabla_1^2 C - KC \tag{22}$$

Where $\nabla_1^2 = \delta^2 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$

Introducing the transformation

$$\eta = \frac{y}{f(\bar{x})}$$

the equations(20-22)reduces to

$$\delta Rf(\gamma^2 (F^2\psi)_t + \frac{\partial(\psi, F^2\psi)}{\partial(\bar{x}, \eta)}) = F^4\psi + (\frac{Gf^3}{R})(\theta_\eta + NC_\eta) -$$

$$- (M^2 f^2)\frac{\partial^2 \psi}{\partial \eta^2} - D^{-1}F^2\psi \tag{23}$$

$$\delta P(\gamma^2 \frac{\partial \theta}{\partial t} + f(\frac{\partial \psi}{\partial \eta}\frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \theta}{\partial \eta}) = F^2\theta - \alpha f^2\theta \tag{24}$$

$$\delta Sc(\gamma^2 \frac{\partial C}{\partial t} + f(\frac{\partial \psi}{\partial \eta}\frac{\partial C}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial C}{\partial \eta})) = F^2 C - KC \tag{25}$$

Where $F^2 = \delta^2 \frac{\partial^2}{\partial \bar{x}^2} + \frac{\partial^2}{\partial \eta^2}$

We adopt the perturbation scheme and write

$$\psi(x,\eta,t) = \psi_0(x,\eta,t) + ke^{it}\bar{\psi}_0(x,\eta,t) + \delta(\psi_1(x,\eta,t) + ke^{it}.\bar{\psi}_1(x,\eta,t)) + \ldots\ldots$$

$$\theta(x,\eta,t) = \theta_0(x,\eta,t) + ke^{it}\bar{\theta}_0(x,\eta,t) + \delta(\theta_1(x,\eta,t) + ke^{it}.\bar{\theta}_1(x,\eta,t)) + \ldots\ldots$$

$$C(x,\eta,t) = C_0(x,\eta,t) + ke^{it}C\bar{\phi}_0(x,\eta,t) + \delta(C_1(x,\eta,t) + ke^{it}.\bar{C}_1(x,\eta,t)) + \ldots\ldots \tag{26}$$

On substituting (20) in (23) - (25) and separating the like powers of δ the equations and respective conditions to the zeroth order are

$$\psi_{0,\eta\eta\eta\eta} - (M_1^2 f^2)\psi_{0,\eta\eta} = -(\frac{Gf^3}{R})(\theta_{0,\eta} + NC_{0,\eta}) \tag{27}$$

$$\theta_{o,\eta\eta} - (\alpha f^2)\theta_0 = 0 \tag{28}$$

$$C_{o,\eta\eta} - (KScf^2)C_0 = 0 \tag{29}$$

with

$$\psi_0(+1) - \Psi_0(-1) = 1,$$
$$\psi_{0,\eta} = 0, \psi_{0,x} = 0 \qquad at \ \eta = \pm1 \tag{30}$$
$$\theta_o = 1, \qquad C_0 = 1 \qquad on \quad \eta = -1$$
$$\theta_o = 0, \qquad C_0 = 0 \qquad on \quad \eta = 1 \tag{31}$$

$$\bar{\theta}_{0,\mu\eta} - (iP\gamma^2 f^2)\bar{\theta}_0 = 0 \tag{32}$$

$$\bar{C}_{0,\eta\eta} - (KSc\gamma^2 f^2)\bar{C}_o = 0 \tag{33}$$

$$\bar{\psi}_{0,\eta\eta\eta\eta} - ((M_1^2 + i\gamma^2)f^2)\bar{\psi}_{0,\eta\eta} = -(\frac{Gf^3}{R})(\bar{\theta}_{0,\eta} + N\bar{C}_{0,\eta}) \tag{34}$$

$$\bar{\theta}_o(\pm1) = 0 \qquad \bar{C}_o(\pm1) = 0$$
$$\bar{\psi}_o(+1) - \bar{\psi}_o(-1) = 1 \quad \bar{\psi}_{o,\eta}(\pm1) = 0, \bar{\psi}_{o,x}(\pm1) = 0 \tag{35}$$

The first order equations are

$$\psi_{1,\eta\eta\eta\eta} - (M_1^2 f^2)\psi_{1,\eta\eta} = -(\frac{Gf^3}{R})(\theta_{1,\eta} + NC_{1,\eta}) + (Rf)(\psi_{0,\eta}\psi_{0,x\eta\eta} - \psi_{0,x}\psi_{0,\eta\eta\eta}) \tag{36}$$

$$\theta_{1,\eta\eta} - (\alpha f^2)\theta_1 = (PRf)(\psi_{0,x}\theta_{o,\eta} - \psi_{0,\eta}\theta_{ox}) \tag{37}$$

$$C_{1,\eta\eta} - (KScf^2)C_1 = (Scf)(\psi_{0,x}C_{o,\eta} - \psi_{0,\eta}C_{ox}) \tag{38}$$

$$\bar{\psi}_{1,\eta\eta\eta\eta} - ((M_1^2 + i\gamma^2)f^2)\bar{\psi}_{1,\eta\eta} = -\frac{Gf^3}{R}(\bar{\theta}_{1,\eta} + N\bar{C}_{1,\eta}) + (Rf)(\bar{\psi}_{0,\eta}\psi_{0,x\eta\eta} +$$
$$+ \psi_{0,\eta}\bar{\psi}_{0,x\eta\eta} - \psi_{0,x}\bar{\psi}_{0,\eta\eta\eta} - \bar{\psi}_{0,x}\bar{\psi}_{0,\eta\eta\eta}) \tag{39}$$

$$\bar{\theta}_{1,\eta\eta} - ((iP\gamma^2 + \alpha)f^2)\bar{\theta}_1 = (PRf)(\psi_{0,\eta}\bar{\theta}_{o,x} + \bar{\psi}_{0,\eta}\theta_{ox} -$$
$$- \bar{\psi}_{0,x}\theta_{o,\eta} - \psi_{0,x}\bar{\theta}_{o\eta}) \tag{40}$$

$$\bar{C}_{1,\eta\eta} - ((K + i\gamma^2)Scf^2)\bar{C}_1 = (Scf)(\psi_{0,\eta}\bar{C}_{o,x} + \bar{\psi}_{0,\eta}C_{ox} -$$
$$- \bar{\psi}_{0,x}C_{o,\eta} - \psi_{0,x}\bar{C}_{o\eta}) \tag{41}$$

with

$$\psi_{1(+1)} - \psi_{1(-1)} = 0$$

$$\psi_{1,\eta} = 0, \ \psi_{1,x} = 0 \ \text{at} \ \eta = \pm 1 \tag{42}$$

$$\theta_1(\pm 1) = 0 \qquad C_1(\pm 1) = 0$$

$$\overline{\theta_1}(\pm 1) = 0 \qquad \overline{C_1}(\pm 1) = 0$$

$$\overline{\psi_1}(+1) - \overline{\psi_1}(-1) = 1 \quad \overline{\psi_{1,\eta}}(\pm 1) = 0, \overline{\psi_{1,x}}(\pm 1) = 0 \tag{43}$$

NUSSELT NUMBER and SHERWOOD NUMBER

$$(\tau)_{y=-1} = d_6 + Ec\,d_7 + \delta d_8 + O(\delta^2)$$

The local rate of heat transfer coefficient (Nusselt number Nu) on the walls has been calculated using the formula

$$Nu = \frac{1}{\theta_m - \theta_w}\left(\frac{\partial \theta}{\partial y}\right)_{\eta=\pm 1} \qquad \text{where} \quad \theta_m = 0.5\int_{-1}^{1}\theta\,d\eta$$

and the corresponding expressions are

$$(Nu)_{\eta=+1} = \frac{(d_9 + \delta d_{11})}{(\theta_m - Sin(x+\gamma t))}, \qquad (Nu)_{\eta=-1} = \frac{(d_8 + \delta d_{10})}{(\theta_m - 1)}$$

where $\theta_m = d_{14} + \delta d_{15}$

The local rate of mass transfer coefficient(Sherwood Number Sh) on the walls has been calculated using the formula

$$Sh = \frac{1}{C_m - C_w}\left(\frac{\partial C}{\partial y}\right)_{y=\pm 1} \qquad \text{where} \qquad C_m = 0.5\int_{-1}^{1}C\,dy$$

and the corresponding expressions are

$$(Sh)_{\eta=+1} = \frac{(d_4 + \delta d_6)}{(C_m)}, \qquad (Sh)_{\eta=-1} = \frac{(d_5 + \delta d_7)}{(C_m - 1)}$$

where $C_m = d_{12} + \delta d_{13}$

where $d_1, d_2, \ldots\ldots\ldots\ldots, d_{14}$ are constants.

6. DISCUSSION OF THE RESULTS:

We discuss the effect of chemical reaction on unsteady convective heat and mass transfer flow of a viscous, electrically conducting fluid through a porous medium in a wavy channel. The unsteadiness in the flow is due to the oscillatory flux imposed on the flow. The non-linear, coupled equations governing the flow, heat and mass transfer are solved by using a regular perturbation technique with δ the scope of the wavy wall as a perturbation parameter.

The axial velocity (u) is exhibited in fig 1-3 for different values of β, α, γ, k. Fig. 1 represents u with heat source parameter α and chemical reaction parameter k. An increase in k≤1.5 leads to a depreciation in |u| and enhances with higher k≥2.5. |u| experiences an enhancement with α≤4 and reduces with higher α≥6. The effect of wall waviness on u is shown in fig.2. It is found that higher the constriction of the channel walls larger |u| in the flow region. An increase in the Wormsely number γ results in a depreciation in |u| everywhere in the flow region (fig. 3).

The secondary velocity (v) which arises due to the non-uniform boundary is shown in figs. 4-6 for different parametric values. An increase in the chemical reaction parameter k results in an enhancement in |v| in the entire flow region (fig. 4). |v| enhances with heat source

parameter α≤4 and reduces with higher α≥6. The variation of v with β shows that higher the constriction of the channel walls larger the magnitude of the secondary velocity (fig. 5). An increase in the Wormsely number γ results in a depreciation in |v| in the entire flow region (fig. 6).
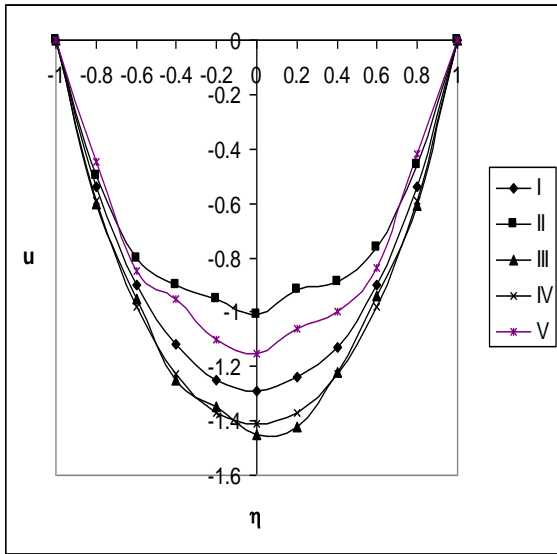

Fig. 1 : Variation of u with K & α

	I	II	III	IV	V
K	0.5	1	1.5	0.5	0.5
α	2	2	2	4	6
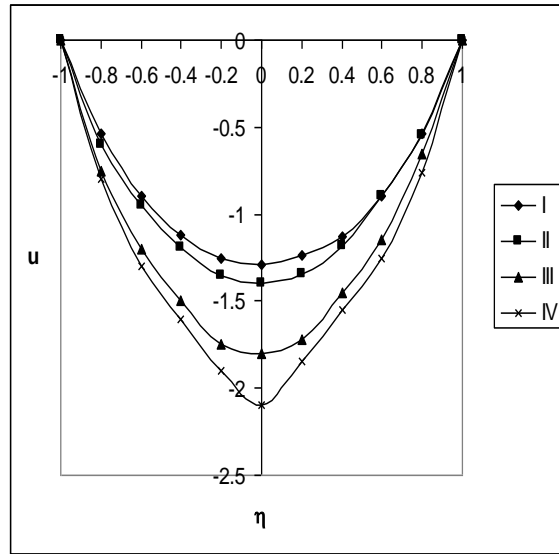
Fig. 2 : Variation of u with β

	I	II	III	IV
β	-0.3	-0.5	-0.7	-0.9
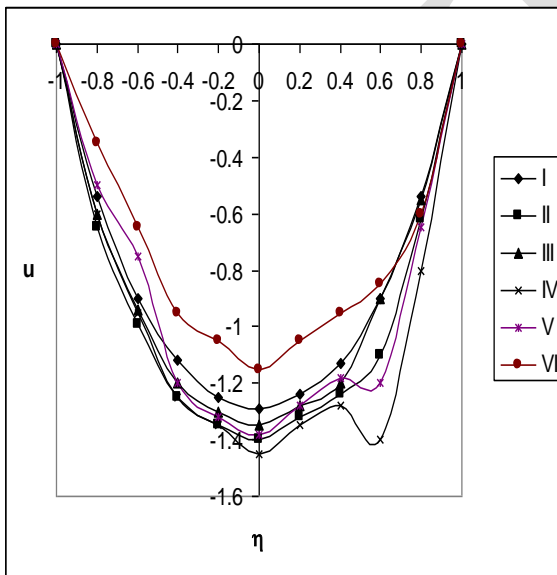
Fig. 3 : Variation of u with R & γ

	I	II	III	IV	V	VI
R	35	70	140	35	35	35
γ	0.5	0.5	0.5	0.7	0.9	1.2
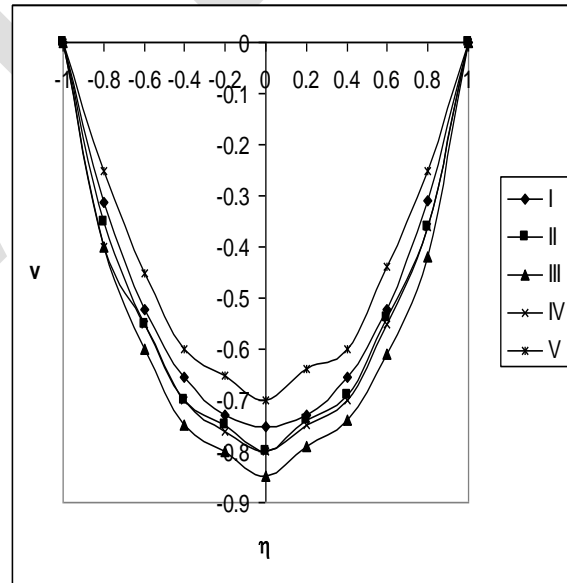
Fig. 4 : Variation of v with K & R

	I	II	III	IV	V
K	0.5	1	1.5	0.5	0.5
R	35	35	35	70	140
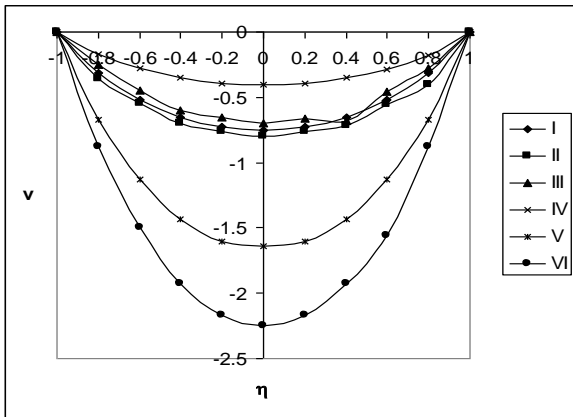
Fig. 5 : Variation of v with α & β

	I	II	III	IV	V	VI
α	2	4	6	2	2	2
β	-0.5	-0.5	-0.5	-0.3	-0.7	-0.9
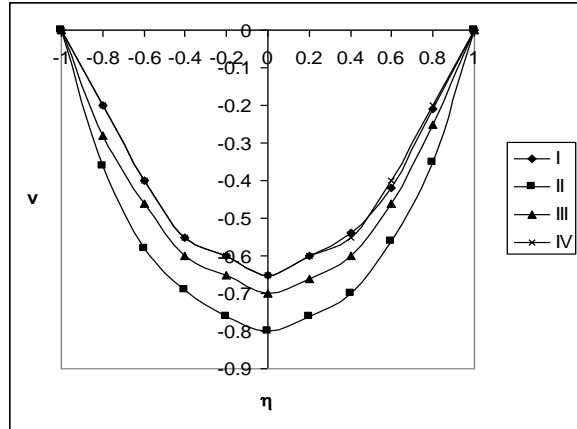
Fig. 6 : Variation of v with γ

	I	II	III	IV
γ	0.5	0.7	0.9	1.2
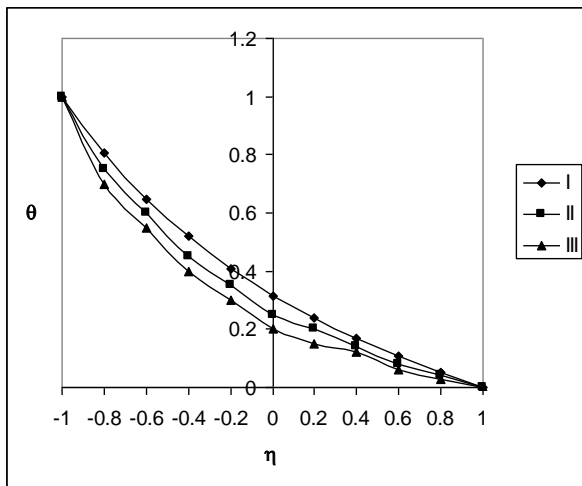
Fig. 7 : Variation of θ with K
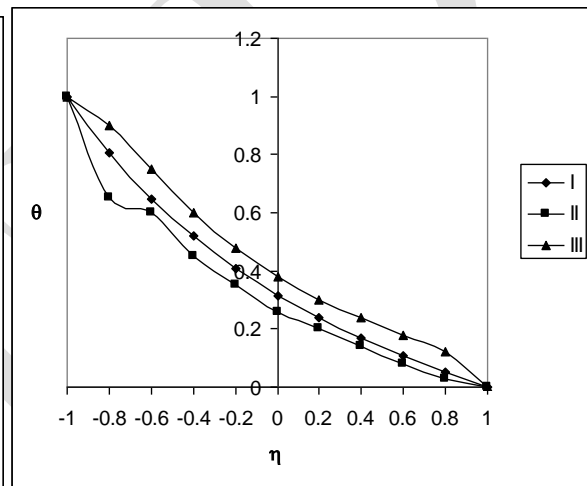
	I	II	III
K	0.5	1	1.5

Fig. 8 : Variation of θ with α
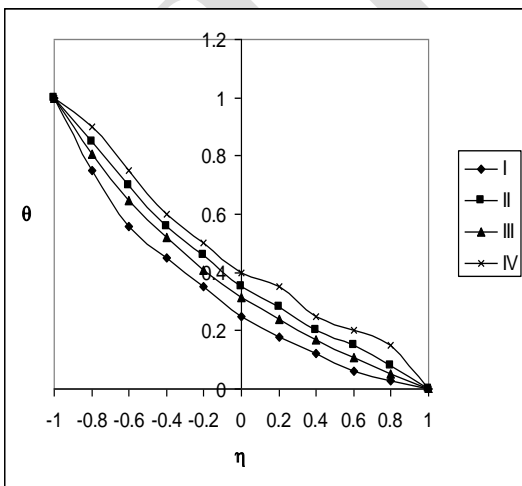
	I	II	III
α	2	4	6

Fig. 9 : Variation of θ with β

	I	II	III	IV
β	-0.3	-0.5	-0.7	-0.9
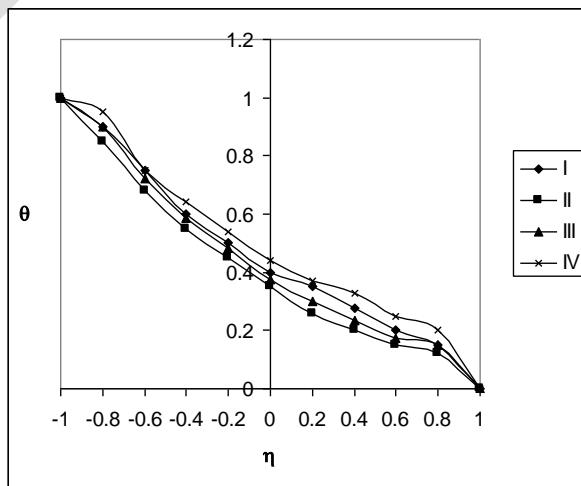
Fig. 10 : Variation of θ with γ

	I	II	III	IV
γ	0.5	0.7	0.9	1.2

The non-dimensional temperature (θ) is exhibited in figs 7-10 for different parametric values. The temperature reduces with increase in the chemical reaction parameter k(fig.7). θ reduces with α≤4 and enhances with higher α≥6 (fig. 8). The variation of θ with β shows

that higher the constriction of the channel walls larger the temperature in the entire flow region (fig. 9). From fig. 10 we find that the actual temperature experiences an enhancement with increase in the Wormsely number γ.

The Concentration distribution (C)is exhibited in figs. 11-13 for different parametric values. An increase in the chemical reaction parameter k results in an enhancement in C (fig.11). Higher the strength of the heat source smaller the concentration (fig. 31). The effect of wall waviness on C is shown in fig. 33. Higher the constriction of the channel walls larger the concentration.
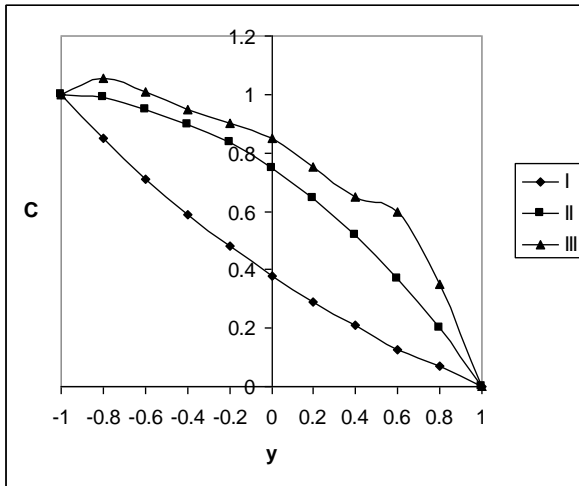
Fig. 11 : Variation of C with K

	I	II	III
K	0.5	1	1.5
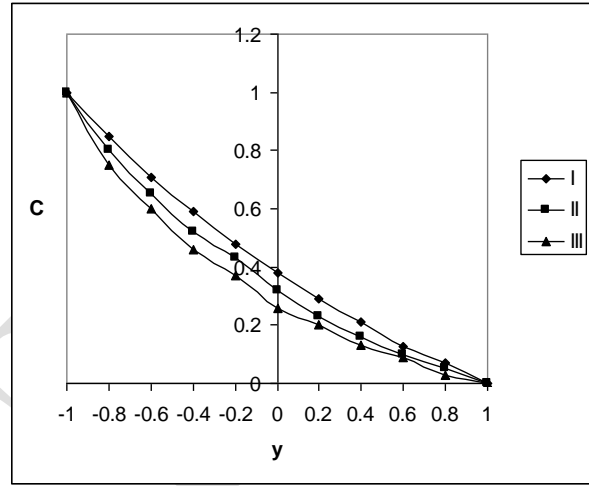
Fig. 12 : Variation of C with α
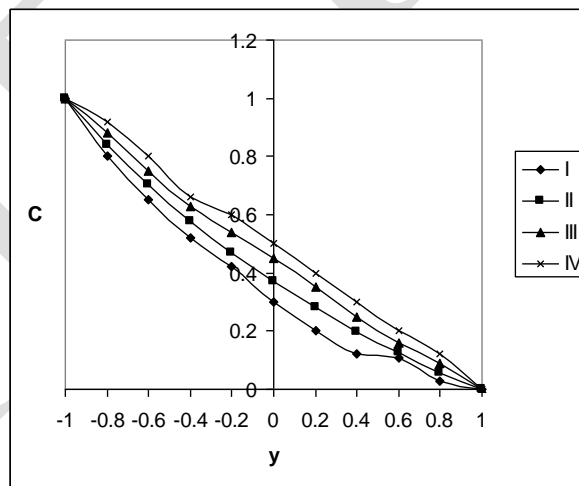
	I	II	III
α	2	4	6

Fig. 13 : Variation of C with β

	I	II	III	IV
β	-0.5	-0.3	-0.7	-0.9

The rate of heat transfer (Nusselt number) at $\eta = \pm 1$ is evaluated for different values of k, α, β, γ and is shown in tables 1-4. With respect to the chemical reaction parameter k we find that the rate of heat transfer at $\eta = +1$ enhances with k in the heating case and reduces in the cooling case. At $\eta = -1$, |Nu| reduces with increase in k≤1.5 and enhances with higher k≥2.5 for all G. Higher the strength of the heat source smaller |Nu| at $\eta = +1$ while at $\eta = -1$, |Nu| reduces with α≤4 and enhances with higher α≥6. It is found that higher the constriction of the channel walls/lesser the molecular diffusivity larger |Nu| at $\eta = +1$ and smaller at

η = -1 for all G. The variation of Nu with Wormsely number γ shows that an increase in $\gamma \leq 1.5$ enhances |Nu| at η = +1 and reduces at η = -1 and for higher $\gamma \geq 2.5$, we notice an enhancement in |Nu| at both the walls.

The rate of mass transfer (Sherwood number) at $\eta = \pm 1$ is shown in tables 5-8 for different values of k, α, γ, β. An increase in k enhances for G>0 and for G<0, |Sh| reduces with k$\leq$1.5 and enhances with higher k$\geq$2.5 at η = +1. At η = -1, |Sh| experiences an enhancement with k for all G. |Sh| at η = +1 reduces with increase in the strength of the heat source (α) in the heating case and enhances in the cooling case. At η = -1, |Sh| enhances with α for all G. Higher the constriction of the channel walls larger |Sh| at η = +1 and smaller at η=-1 for G>0 and for G<0, |Sh| enhances with |β| $\leq$ 0.5 and reduces with |β| $\geq$ 0.7 at η = +1 while at η = -1, |Sh| reduces. An increase in R leads to an enhancement in |Sh| at both the walls. |Sh| enhances with increase in γ at η = +1 and at η = -1, |Sh| enhances with $\gamma \leq 2.5$ and reduces with higher $\gamma \geq 3.5$.

Table – 1
Average Nusselt number (Nu) at η = 1

G	I	II	III	IV	V
10^3	3.50629	3.6760	3.8952	1.0377	0.5553
3×10^3	3.78016	4.2254	4.8519	1.13828	0.60681
-10^3	3.19764	3.0069	2.6649	0.9360	0.50360
-3×10^3	2.84714	2.1742	1.0242	0.8328	0.45179
k	0.5	1.5	2.5	0.5	0.5
α	2	2	2	4	6

Table – 2
Average Nusselt number (Nu) at η = 1

G	I	II	III	IV	V	VI	VII
10^3	1.7710	3.50629	6.27409	11.38266	3.51726	3.66129	3.78498
3×10^3	1.8928	3.78016	6.47401	10.5356	3.84149	3.9946	4.10848
-10^3	1.6454	3.19764	5.9944	13.2333	3.29129	3.4968	3.54154
-3×10^3	1.5159	2.84714	5.5753	20.4663	2.99146	3.01494	3.14545
β	-0.3	-0.5	-0.7	-0.9	-0.3	-0.3	-0.3
γ	0.5	0.5	0.5	0.5	1.5	2.5	3.5

Table – 3
Average Nusselt number (Nu) at η = -1

G	I	II	III	IV	V
10^3	-12.1363	-14.4641	-1635827	0.1096	-1.1541
3×10^3	-20.5552-	-42.3113	-4.1431	0.5512	-1.0204
-10^3	-7.5138	-6.0164	-4.5749	-0.3562	-1.2886
-3×10^3	-4.5923	-1.9335	0.4052	-0.8482	-1.4240
k	0.5	1.5	2.5	0.5	0.5
α	2	2	2	4	6

Table – 4
Average Nusselt number (Nu) at η = -1

G	I	II	III	IV	V	VI	VII
10^3	-14.6697	-12.1363	-11.6823	-10.1243	-12.4696	-13.9646	-14.1296
3×10^3	-16.6853	-20.5552	-22.7951	-12.4694	-21.1246	-22.0646	-23.1249
-10^3	-25.6921	-7.5138	-6.5101	-5.9512	-8.1264	-10.1246	-11.1455
-3×10^3	-11.1789	-4.5923	-3.5195	-2.9546	-6.0986	-8.0648	-9.1246
β	-0.3	-0.5	-0.7	-0.9	-0.3	-0.3	-0.3
γ	0.5	0.5	0.5	0.5	1.5	2.5	3.5

Table – 5
Sherwood number (Sh) at $\eta = 1$

G	I	II	III	IV	V
10^3	-0.22399	-0.27963	-0.54067	-0.2209	-0.22017
3×10^3	-0.22958	-0.52231	-1.22049	-0.21731	-0.21729
-10^3	-0.21841	-0.03692	0.14086	-0.22368	-0.220505
-3×10^3	-0.21282	0.20580	0.82413	-0.22686	-0.22893
k	0.5	1.5	2.5	0.5	0.5
α	2	2	2	4	6

Table – 6
Sherwood number (Sh)) at $\eta = 1$

G	I	II	III	IV	V	VI	VII
10^3	-0.21291	-0.22399	-0.24684	-0.30764	-0.22986	-0.23106	-0.23496
3×10^3	-0.211556	-0.22958	-0.28388	-0.30764	-0.23089	-0.23189	-0.23862
-10^3	-0.21342	-0.21841	-0.20982	-0.45380	-0.22106	-0.22186	-0.22646
-3×10^3	-0.21572	-0.21282	-0.17283	-0.16182	-0.21896	-0.22064	-0.22164
β	-0.3	-0.5	-0.7	-0.9	-0.3	-0.3	-0.3
γ	0.5	0.5	0.5	0.5	1.5	2.5	3.5

Table – 7
Sherwood number (Sh) at $\eta = -1$

G	I	II	III	IV	V
10^3	-0.78469	-1.59249	-2.55141	-0.78515	-0.78665
3×10^3	-0.77585	-1.57216	-2.40733	-0.77705	-0.77855
-10^3	-0.79354	-1.61283	-2.69673	-0.79326	-0.79474
-3×10^3	-0.86238	-1.63318	-2.84330	-0.80151	-0.80284
k	0.5	1.5	2.5	0.5	0.5
α	2	2	2	4	6

Table – 8
Sherwood number (Sh) at $\eta = -1$

G	I	II	III	IV	V	VI	VII
10^3	-0.88227	-0.78469	-0.70964	-0.67060	-0.78489	-0.78501	-0.77126
3×10^3	-0.87516	-0.77585	-0.71666	-0.68261	-0.77685	-0.78012	-0.77064
-10^3	-0.88938	-0.793531	-0.72017	-0.65864	-0.79454	-0.79654	-0.78124
-3×10^3	-0.809656	-0.80238	-0.67060	-0.64673	-0.80638	-0.80844	-0.79264
β	-0.3	-0.5	-0.7	-0.9	-0.3	-0.3	-0.3
γ	0.5	0.5	0.5	0.5	1.5	2.5	3.5

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