Radiation And Dissipation On The Convective Heat Transfer Flow Of A Viscous Fluid Through A Porous Medium In A Rectangular Cavity Using Darcy Model

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ABSTRACT:

In this chapter an attempt has been made to discuss the combined influence of radiation and dissipation on the convective heat transfer flow of a viscous fluid through a porous medium in a rectangular cavity using Darcy model. Making use of the incompressibility the governing non-linear coupled equations for the momentum, energy and diffusion are derived in terms of the non-dimensional stream function, temperature. The Galerkin finite element analysis with linear triangular elements is used to obtain the Global stiffness matrices for the values of stream function, temperature. These coupled matrices are solved using iterative procedure and expressions for the stream function, temperature are obtained as linear combinations of the shape functions. The behavior of temperature and Nusselt number are discussed computationally for different values of the governing Parameters Ra, α, N_1 and Ec.

Keywords: Heat Transfer, Porous Medium, Rectangular duct, Finite Element Analysis

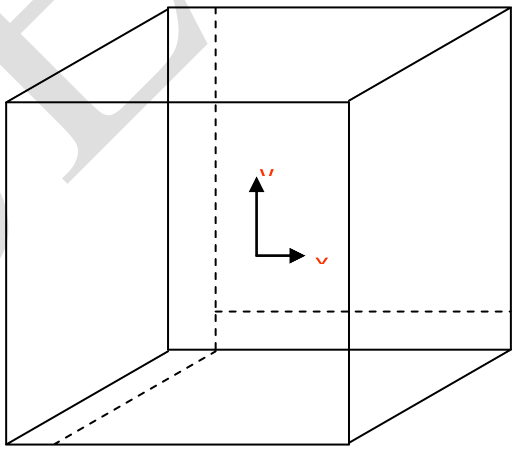

Fig. i

SCHEMATIC DIAGRAM OF THE FLOW MODEL

FORMULATION OF THE PROBLEM

We consider the mixed convective heat transfer flow of a viscous incompressible fluid in a saturated porous medium confined in the rectangular duct (Fig. 1) whose base length is a and height b. The heat flux on the base and top walls is maintained constant. The Cartesian coordinate system O (x,y) is chosen with origin on the central axis of the duct and its base parallel to x-axis.

We assume that

i) The convective fluid and the porous medium are everywhere in local thermodynamic equilibrium.
ii) There is no phase change of the fluid in the medium.
iii) The properties of the fluid and of the porous medium are homogeneous and isotrophic.
iv) The porous medium is assumed to be closely packed so that Darcy's momentum law is adequate in the porous medium.
v) The Boussinesq approximation is applicable.

Under these assumption the governing equations are given by

$$\frac{\partial u'}{\partial x'}+\frac{\partial v'}{\partial y'}=0 \qquad (2.1)$$

$$u'=-\frac{k}{\mu}\left(\frac{\partial p'}{\partial x'}\right) \qquad (2.2)$$

$$v'=-\frac{k}{\mu}\left(\frac{\partial p'}{\partial y'}+\rho' g\right) \qquad (2.3)$$

$$\rho_\sigma c_p\left(u'\frac{\partial T'}{\partial x'}+v'\frac{\partial T'}{\partial y'}\right)=K_1\left(\frac{\partial^2 T'}{\partial x'^2}+\frac{\partial^2 T'}{\partial y'^2}\right)+Q(T_0-T)+\left(\frac{\mu}{K}\right)(u^2+v^2)-\frac{\partial(q_r)}{\partial x} \qquad (2.4)$$

$$\rho'=\rho_0\{1-\beta(T'-T_0)\} \qquad (2.5)$$

$$T_0=\frac{T_h+T_c}{2},$$

Where u′ and v′ are Darcy velocities along direction of θ (x, y), T′, p′ and g′ are the temperature, pressure and acceleration due to gravity, T_c , and T_h are the temperature on the cold and warm side walls respectively. ρ′, μ, ν, and β are the density, coefficients of viscosity, kinematic viscosity and thermal expansion of he fluid, k is the permeability of the porous medium, K_1 is the thermal conductivity, C_p is the specific heat at constant pressure , Q is the strength of the heat source,k_{11} is the cross diffusivity , μ_e is the magnetic permeability of the medium and q_r is the radiative heat flux.

The boundary conditions are

$$
\begin{aligned}
&u' = v' = 0 && \text{on the boundary of the duct} \\
&T' = T_c, && \text{on the side wall to the left} \\
&T' = T_h, && \text{on the side wall to the right} \qquad (2.6)\\
&\frac{\partial T'}{\partial y} = 0, && \text{on the top (y = 0) and bottom} \\
&u = v = 0 && \text{walls (y = 0)which are insulated.}
\end{aligned}
$$

Invoking Rosseland approximation for radiation

$$q_r = \frac{4\sigma^*}{3\beta_R}\frac{\partial T'^4}{\partial y}$$

Expanding T^4 in Taylor's series about T_e and neglecting higher order terms

$$T'^4 \cong 4T_e^3 T - 3T_e^4$$

We now introduce the following non-dimensional variables

$$x' = ax; \quad y' = by \quad ; \quad c = b/a; \quad u' = (\nu/a)\,u;$$
$$v' = (\nu/a)v; \quad p' = (\nu^2\rho/a^2)p; \quad T' = T_0 + \theta\,(T_h - T_c) \qquad (2.7)$$

The governing equations in the non-dimensional form are

$$u = -\left(\frac{K}{a^2}\right)\frac{\partial p}{\partial x} \qquad (2.8)$$

$$v = -\frac{k}{a^2}\frac{\partial p}{\partial y} - \frac{kag}{\nu^2} + \frac{kag\beta(T_h - T_c)\theta}{\nu^2} - \left(\frac{\sigma\mu_e^2 H_o^2 k}{\mu}\right)v \qquad (2.9)$$

$$P\left(u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y}\right) = \left(1 + \frac{4N}{3}\right)\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right) - \alpha\theta + E_C\left(u^2 + v^2\right) \qquad (2.10)$$

In view of the equation of continuity we introduce the stream function ψ as

$$u = \frac{\partial\psi}{\partial y} \quad ; \quad v = -\frac{\partial\psi}{\partial x} \qquad (2.11)$$

Eliminating p from the equation (2.8) and (2.9) and making use of (2.10) the equations in terms of ψ and θ are

$$\nabla^2\psi = -Ra(\frac{\partial\theta}{\partial x}) \tag{2.12}$$

$$P\left(\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}\right) = \left(1+\frac{4}{3N_1}\right)\left(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right) - \alpha\theta + E_C\left(\left(\frac{\partial\psi}{\partial y}\right)^2 + \left(\frac{\partial\psi}{\partial x}\right)^2\right) \tag{2.13}$$

Where

$G = \frac{g\beta(T_h - T_c)a^3}{\nu^2}$ (Grashoff number)

$D^{-1} = \frac{a^2}{k}$ (Darcy parameter)

$P = \mu\, c_p / K_1$ (Prandtl number)

$\alpha = Qa^z/K_1$ (Heat source parameter)

$Ra = \frac{\beta g(T_g - T_c)ka}{\nu^2}$ (Rayleigh Number)

$N_1 = \frac{3\beta_R K_1}{4\sigma^{\bullet} : T_e^3}$ (Radiation parameter)

$Ec = \left(\frac{a^4}{\mu K K_1 \Delta T}\right)$ (Eckert number)

The boundary conditions are

$$\frac{\partial\psi}{\partial x} = 0, \frac{\partial\psi}{\partial y} = 0 \;\; on \;\; x = 0 \;\&1 \tag{2.14}$$

$$\theta = 1 \quad on \quad x = 0$$

$$\theta = 0 \quad on \quad x = 1 \tag{2.15}$$

FINITE ELEMENT ANALYSIS and SOLUTION OF THE PROBLEM

The region is divided into a finite number of three node triangular elements, in each of which the element equation is derived using Galerkin weighted residual method. In each element f_i the approximate solution for an unknown f in the variational formulation is expressed as a linear combination of shape function. $\left(N_k^i\right) k = 1,2,3,$ which are linear polynomials in x and y. This approximate solution of the unknown f coincides with actual values at each node of the element. The variational formulation results in a 3 x 3 matrix equation (stiffness matrix) for the unknown local nodal values of the given element. These stiffness matrices are assembled in terms of global nodal values using inter element continuity and boundary conditions resulting in global matrix equation.

In each case there are r distinct global nodes in the finite element domain and f_p (p = 1,2,……r) is the global nodal values of any unknown f defined over the domain then

$$f = \sum_{i=1}^{8} \sum_{p=1}^{r} f_p \ \Phi_p^i ,$$

where the first summation denotes summation over s elements and the second one represents summation over the independent global nodes and

$\Phi_p^i = N_N^i$, if p is one of the local nodes say k of the element e_i

= 0, otherwise.

f_p' s are determined from the global matrix equation. Based on these lines we now make a finite element analysis of the given problem governed by (2.12) - (2.13) subjected to the conditions (2.14) – (2.15).

Let ψ^i and θ^i be the approximate values of ψ and θ in an element θ_i.

$$\psi^i = N_1^i \ \psi_1^i + N_2^i \ \psi_2^i + \ N_3^i \ \psi_3^i \tag{3.1a}$$

$$\theta^i = N_1^i \ \theta_1^i + N_2^i \ \theta_2^i + \ N_3^i \ \theta_3^i \tag{3.1b}$$

Substituting the approximate value ψ^i and θ^i for ψ and θ respectively in (2.13), the error

$$E_1^i = \left(1+\frac{4}{3N_1}\right)\frac{\partial^2\theta^i}{\partial x^2}+\frac{\partial^2\theta^i}{\partial y^2}-P\left(\frac{\partial\psi^i}{\partial y}\ \frac{\partial\theta^i}{\partial x}\ -\frac{\partial\psi^i}{\partial x}\ \frac{\partial\theta^i}{\partial y}\right)-\alpha\theta+E_C\left[\left(\frac{\partial\psi}{\partial y}\right)^2+\left(\frac{\partial\psi}{\partial x}\right)^2\right] \qquad (3.2)$$

Under Galerkin method this error is made orthogonal over the domain of e_i to the respective shape functions (weight functions) where

$$\int_{ei} E_1^i\ \ N_k^i\ \ d\Omega = 0\,,$$

$$\int_{ei} E_2^i N_k^i d\Omega = 0$$

$$\int_{ei=} N_k^i(\left(1+\frac{4}{3N_1}\right)\left(\frac{\partial^z\theta^i}{\partial x^2}+\frac{\partial^z\theta^i}{\partial y^2}\right)-P\left(\frac{\partial\psi^i}{\partial y}\ \frac{\partial\theta^i}{\partial x}\ -\frac{\partial\psi^i}{\partial x}\ \frac{\partial\theta^i}{\partial y}\right)$$

$$-\alpha\theta+\left[E_C\left(\frac{\partial\psi}{\partial y}\right)^2+\left(\frac{\partial\psi}{\partial x}\right)^2\right]d\Omega = 0 \qquad (3.4)$$

Using Green's theorem we reduce the surface integral (3.4)&(3.5)without affecting ψ terms and obtain

$$\int_{ei} N_k^i\left\{\begin{matrix}\left(1+\frac{4}{3N_1}\right)\frac{\partial N_k^i}{\partial x}\ \frac{\partial\theta^i}{\partial x}+\frac{\partial N_k^i}{\partial y}\ \frac{\partial\theta^i}{\partial y}-p^iN_k\left(\frac{\partial\psi^i}{\partial y}\ \frac{\partial\theta^i}{\partial x}\ -\frac{\partial\psi^i}{\partial x}\ \frac{\partial\theta^i}{\partial y}\right)\\ -\alpha\theta+E_C\left(\frac{\partial\psi}{\partial y}\right)^2+\left(\frac{\partial\psi}{\partial x}\right)^2\end{matrix}\right\}d\Omega$$

$$=\int_{\Gamma i} N_k^i\left(\frac{\partial\theta^i}{\partial x}n_x+\frac{\partial\theta^i}{\partial y}n_y\right)d\Gamma_i \qquad (3.6)$$

where Γ_I is the boundary of e_i.

Substituting L.H.S. of (3.1a)- (3.1b) for ψ^i and θ^i in (3.6) we get

$$\sum_1\int_{ei}\left(1+\frac{4N}{3}\right)\frac{\partial N_k^i}{\partial x}\ \frac{\partial N_L^i}{\partial x}+\frac{\partial N_L^i}{\partial y}\ \frac{\partial N_k^i}{\partial y}-P\sum_1\ \psi_m^i\int_{ei}\left(\frac{\partial N_m^i}{\partial y}\ \frac{\partial N_L^i}{\partial x}-\frac{\partial N_m^i}{\partial x}\ \frac{\partial N_L^i}{\partial y}\right)d\Omega$$

$$-\alpha\sum\theta^i\int_{ei} N\underset{k}{N}_l\,d\Omega_i+E_C\int_{ei}\left(\left(\frac{\partial\psi}{\partial y}\right)^2+\left(\frac{\partial\psi}{\partial x}\right)^2\right)d\Omega$$

$$= \int_{\Gamma_i} N_k^i \left(\frac{\partial \theta^i}{\partial x} \; n_x + \frac{\partial \theta^i}{\partial y} n_y \right) d\Gamma_i = Q_k^i \qquad (l, m, k = 1,2,3) \qquad (3.8)$$

where

$Q_k^i = Q_{k1}^i + Q_{k2}^i + Q_{k3}^i, Q_k^i$'s being the values of Q_k^i on the sides s = (1,2,3) of the element e_i. The sign of Q_k^i 's depends on the direction of the outward normal w.r.t the element.

Choosing different N_k^i 's as weight functions and following the same procedure we obtain matrix equations for three unknowns (Q_p^i) viz.,

$$(a_p^i)(\theta_p^i) = (Q_k^i) \qquad (3.10)$$

where (a_{pk}^i) is a 3 x 3 matrix, $(\theta_p^i),(Q_k^i)$ are column matrices.

Repeating the above process with each of s elements, we obtain sets of such matrix equations. Introducing the global coordinates and global values for θ_p^i and making use of inter element continuity and boundary conditions relevant to the problem the above stiffness matrices are assembled to obtain a global matrix equation. This global matrix is r x r square matrix if there are r distinct global nodes in the domain of flow considered.

Similarly substituting ψ^i and θ^i in (2.12) and defining the error

$$E_3^i = \nabla^2 \psi + \mathrm{Ra}\,(\frac{\partial \theta}{\partial \mathrm{x}} + N \frac{\partial \phi}{\partial \mathrm{x}}) \qquad (3.11)$$

and following the Galerkin method we obtain

$$\int_\Omega E_3^i \psi_j^i d\Omega = 0 \qquad (3.12)$$

Using Green's theorem (3.8) reduces to

$$\int_\Omega \left(\frac{\partial N_k^i}{\partial x} \; \frac{\partial \psi^i}{\partial x} + \frac{\partial N_k^i}{\partial y} \; \frac{\partial \psi^i}{\partial y} + \mathrm{Ra}\,(\theta^i \, \frac{\partial N_k^i}{\partial x} \right) d\Omega$$

$$= \int_\Gamma N_k^i \left(\frac{\partial \psi^i}{\partial x} \; n_x + \frac{\partial \psi^i}{\partial y} n_y \right) d\Gamma_i \; + \int_\Gamma N_k^i n_x \theta^i \, d\Gamma_i \qquad (3.13)$$

In obtaining (3.13) the Green's theorem is applied w.r.t derivatives of ψ without affecting θ terms.

Using (3.1) and (3.2) in (3.13) we have

$$\sum_m \psi_m^i \left\{ \int_\Omega \left(\frac{\partial N_k^i}{\partial x} \frac{\partial N_m^i}{\partial x} + \frac{\partial N_m^i}{\partial y} \frac{\partial N_k^i}{\partial y} \right) d\Omega_i + \mathrm{Ra} \sum_L (\theta_L^i \int_{\Omega^i} \mathrm{N}_k^i \frac{\partial N_L^i}{\partial x} d\Omega \right\}$$

$$= \int_\Gamma N_k^i \left(\frac{\partial \psi^i}{\partial x} n_x + \frac{\partial \psi^i}{\partial y} n_y \right) d\Gamma_i + \int_\Gamma N_k^i \theta^i d\Omega_i = \Gamma_k^i \qquad (3.14)$$

In the problem under consideration, for computational purpose, we choose uniform mesh of 10 triangular elements. The domain has vertices whose global coordinates are (0,0), (1,0) and (1,c) in the non-dimensional form. Let $e_1, e_2 \ldots e_{10}$ be the ten elements and let $\theta_1, \theta_2, \ldots \theta_{10}$ be the global values of θ and $\psi_1, \psi_2, \ldots \psi_{10}$ be the global values of ψ at the ten global nodes of the domain

SHAPE FUNCTIONS and STIFFNESS MATRICES

Range functions in $n_{i,j}$; i = element, j = node.

$n_{1,1} = 1-3x$ $\quad n_{1,2} = 3x - \frac{3y}{C}$ $\quad n_{2,1} = 1 - \frac{3y}{C}$ $\quad n_{2,2} = -1 + \frac{3y}{C}$

$n_{2,3} = 1-3x+\frac{3y}{C}$ $\quad n_{3,1} = 2-3x$ $\quad n_{3,2} = -1+3x-\frac{3y}{C}$ $\quad n_{3,3} = \frac{3y}{C}$

$n_{4,1} = 1-\frac{3y}{C}$ $\quad n_{4,2} = -2+3x$ $\quad n_{4,3} = 2-3x+\frac{3y}{C}$ $\quad n_{5,1} = 2-3x$

$n_{5,2} = -1+3x-\frac{3y}{C}$ $\quad n_{5,3} = \frac{3y}{C}$ $\quad n_{6,1} = 2-3x$ $\quad n_{6,2} = 3x-\frac{3y}{C}$

$n_{6,3} = 1+\frac{3y}{C}$ $\quad n_{7,1} = 2-\frac{3y}{C}$ $\quad n_{7,2} = -2+3x$ $\quad n_{7,3} = 1-3x+\frac{3y}{C}$

$n_{8,1} = 3-3x$ $\quad n_{8,2} = -1+3x-\frac{3y}{C}$ $\quad n_{9,2} = 3x-\frac{3y}{C}$ $\quad n_{9,3} = -1+\frac{3y}{C}$

Substituting the vabove shape functions in (3.8),(3.9)&(3.14) w.r.t each element and integrating over the respective triangular domain we obtain the element in the form (3.8).The 3x3 matrix equations are assembled using connectivity conditions to obtain a 8x8 matrix equations for the global nodes ψ_p, θ_p and ϕ_p.

The global matrix equation for θ is

$$A_3 X_3 = B_3 \qquad (4.1)$$

The global matrix equation for ψ is

$$A_5 X_5 = B_5 \qquad (4.3)$$

The global matrix equations are coupled and are solved under the following iterative procedures. At the beginning of the first iteration the values of (ψ_i) are taken to be zero and the global equations (4.1)&(4.2) are solved for the nodal values of θ and ϕ.These nodal values (θi) and (ϕi) obtained are then used to solve the global equation (4.3) to obtain(ψ_i).In the second iteration these (ψ_i)values are obtained are used in (4.1)&(4.2) to calculate (θi) and (ϕi) and vice versa. The three equations are thus solved under iteration process until two consecutive iterations differ by a pre-assigned percentage.

The domain consists three horizontal levels and the solution for Ψ & θ at each level may be expressed in terms of the nodal values as follows,

In the horizontal strip $0 \le y \le \frac{c}{3}$

$$\Psi = (\Psi_1 N^1_1 + \Psi_2 N^1_2 + \Psi_7 N^1_7)\, H(1-\tau_1)$$

$$= \Psi_1 (1-4x) + \Psi_2 4(x-\frac{y}{c}) + \Psi_7 (\frac{4y}{c}(1-\tau_1)) \qquad (0 \le x \le \frac{1}{3})$$

$$\Psi = (\Psi_2 N^3_2 + \Psi_3 N^3_3 + \Psi_6 N^3_6)\, H(1-\tau_2)$$

$$+ (\Psi_2 N^2_2 + \Psi_7 N^2_7 + \Psi_6 N^2_6) H(1-\tau_3) \qquad (\frac{1}{3} \le x \le \frac{1}{3})$$

$$= (\Psi_2 2(1-2x) + \Psi_3 (4x-\frac{4y}{c}-1) + \Psi_6 (\frac{4y}{c})) H(1-\tau_2)$$

$$+ (\Psi_2 (1-\frac{4y}{c}) + \Psi_7 (1+\frac{4y}{c}-4x) + \Psi_6 (4x-1)) H(1-\tau_3)$$

$$\Psi = (\Psi_3 N^5_3 + \Psi_4 N^5_4 + \Psi_5 N^5_5)\, H(1-\tau_3)$$

$$+ (\Psi_3 N^4_3 + \Psi_5 N^4_5 + \Psi_6 N^4_6) H(1-\tau_4) \qquad (\frac{2}{3} \le x \le 1)$$

$$= (\Psi_3 (3-4x) + \Psi_4 2(2x-\frac{2y}{c}-1) + \Psi_6 (\frac{4y}{c}-4x+3)) H(1-\tau_3)$$

$$+ \Psi_3 (1-\frac{4y}{c}) + \Psi_5 (4x-3) + \Psi_6 (\frac{4y}{c})) H(1-\tau_4)$$

Along the strip $\frac{c}{3} \le y \le \frac{2c}{3}$

$$\Psi = (\Psi_7 N^6_7 + \Psi_6 N^6_6 + \Psi_8 N^6_8)\, H(1-\tau_2) \qquad (\frac{1}{3} \le x \le 1)$$

$$+ (\Psi_6 N^7_6 + \Psi_9 N^7_9 + \Psi_8 N^7_8)\, H(1-\tau_3) + (\Psi_6 N^8_6 + \Psi_5 N^8_5 + \Psi_9 N^8_9)\, H(1-\tau_4)$$

$$\Psi = (\Psi_7\, 2(1-2x) + \Psi_6\,(4x-3) + \Psi_8\,(\frac{4y}{c}-1))H(1-\tau_3)$$

$$+\Psi_6\,(2(1-\frac{2y}{c}) + \Psi_9\,(\frac{4y}{c}-1) + \Psi_8\,(1+\frac{4y}{c}-4x))H(1-\tau_4)$$

$$+\Psi_6\,(4(1-x) + \Psi_5\,(4x-\frac{4y}{c}-1) + \Psi_9 2(\frac{2y}{c}-1))H(1-\tau_5)$$

Along the strip $\frac{2c}{3} \le y \le 1$

$$\Psi = (\Psi_8 N^9{}_8 + \Psi_9 N^9{}_9 + \Psi_{10} N^9{}_{10})\, H(1-\tau_6) \qquad (\frac{2}{3} \le x \le 1)$$

$$= \Psi_8\,(4(1-x) + \Psi_9 4(x-\frac{y}{c}) + \Psi_{10} 2(\frac{4y}{c}-3))H(1-\tau_6)$$

where $\tau_1 = 4x$, $\tau_2 = 2x$, $\tau_3 = \frac{4x}{3}$,

$\tau_4 = 4(x-\frac{y}{c})$, $\tau_5 = 2(x-\frac{y}{c})$, $\tau_6 = \frac{4}{3}(x-\frac{y}{c})$

and H represents the Heaviside function.

The expressions for θ are

In the horizontal strip $0 \le y \le \frac{c}{3}$

$$\theta = [\theta_1(1-4x) + \theta_2\, 4(x-\frac{y}{c}) + \theta_7\,(\frac{4y}{c}))\, H(1-\tau_1) \qquad (0 \le x \le \frac{1}{3})$$

$$\theta = (\theta_2(2(1-2x) + \theta_3\,(4x-\frac{4y}{c}-1) + \theta_6(\frac{4y}{c}))\, H(1-\tau_2)$$

$$+\theta_2(1-\frac{4y}{c}) + \theta_7(1+\frac{4y}{c}-4x) + \theta_6(4x-1))H(1-\tau_3) \qquad (\frac{1}{3} \le x \le \frac{2}{3})$$

$$\theta = \theta_3(3-4x) + 2\,\theta_4(2x-\frac{2y}{c}-1) + \theta_6(\frac{4y}{c}-4x+3)\, H(1-\tau_3)$$

$$+(\theta_3(1-\frac{4y}{c}) + \theta_5(4x-3) + \theta_6(\frac{4y}{c}))\, H(1-\tau_4) \qquad (\frac{2}{3} \le x \le 1)$$

Along the strip $\frac{c}{3} \le y \le \frac{2c}{3}$

$$\theta = (\theta_7(2(1-2x) + \theta_6(4x-3) + \theta_8(\frac{4y}{c}-1))\,H(1-\tau_3) \qquad (\frac{1}{3} \le x \le \frac{2}{3})$$

$$+(\theta_6(2(1-\frac{2y}{c}) + \theta_9(\frac{4y}{c}-1) + \theta_8(1+\frac{4y}{c}-4x))\,H(1-\tau_4)$$

$$+(\theta_6(4(1-x) + \theta_5(4x-\frac{4y}{c}-1) + \theta_9\,2(\frac{4y}{c}-1))\,H(1-\tau_5)$$

Along the strip $\frac{2c}{3} \le y \le 1$

$$\theta = (\theta_8 4(1-x) + \theta_9 4(x-\frac{y}{c}) + \theta_{10}(\frac{4y}{c}-3)\,H(1-\tau_6) \qquad (\frac{2}{3} \le x \le 1)$$

The dimensionless Nusselt numbers(Nu) and Sherwood Numbers (Sh) on the non-insulated boundary walls of the rectangular duct are calculated using the formula

$$Nu = (\frac{\partial\theta}{\partial x})_{x=}$$

Nusselt Number on the side wall x=1in different regions are

$Nu_1 = 2-4\theta_3 \qquad (0 \le y \le h/3)$

$Nu_2 = 2-4\theta_6 \qquad (h/3 \le y \le 2h/3)$

$Nu_3 = 2-4\theta_8 \qquad (2h/3 \le y \le h)$

DISCUSSION OF THE NUMERICAL RESULTS

In this analysis we investigate the effect of chemical reaction on the mixed convective heat transfer flow of a viscous electrically conducting fluid through a porous medium n a rectangular cavity.

The non-dimensional temperature (θ) is shown in figs 1-4 at different horizontal and vertical levels with variations in Ra, N_1, α and Ec. The variation of non dimensional temperature (θ) with Rayleigh number Ra at horizontal and vertical levels, it is found that the actual temperature enhances with Ra ≤ 2×10^2 and depreciates with higher Ra ≥ 3×10^2 also it reduces with |Ra|at all horizontal levels (fig 1and 2). At the higher vertical level $x=\frac{2}{3}$, it enhances with Ra≤ $2x10^2$ and depreciates Ra ≥ $3x10^2$. Also it reduces with |Ra| at $x=\frac{1}{3}$ and $x=\frac{2}{3}$ levels (figs. 3 and 4). It is found that the temperature at the horizontal levels is greater than that at the vertical levels. The variation of non-dimensional temperature (θ) with radiation parameter $N_1$is shown in fig 5-8 at different levels, it is found that higher the radiative heat flux larger the actual temperature at $y=\frac{2c}{3}$ level and smaller at $y=\frac{c}{3}$ level. Whole at the vertical it depreciates with N_1(fig 7&8).

Fig 9-12 represent non dimensional temperature (θ) with heat source parameter α it is found that the actual temperature experiences depreciation at all horizontal and vertical levels with increase in the strength of the heat source while the increase in the strength of the heat sink enhances the actual temperature at all the levels. It is found that the temperature at the vertical levels is greater than that at the horizontal source. The variation of non-dimensional temperature (θ) with Eckert number E_c is shown in fig 13-16 at horizontal and vertical levels. it is found that the higher the dissipation heat smaller the actual temperature at all the levels fig(13-16). It is noticed that the variation of θ at the vertical levels is greater than that at the horizontal levels.

The rate of heat transfer for different values Ra, N_1, α and E_c is shown in table 1-4. The variation of N_u with Rayleigh number Ra at different level shows that the N_u at the lower and upper quadrant enhances with increase in Ra while in the middle quadrant it depreciates with Ra(Table-1). Table-2 shows that the variation of N_u with radiation parameter N_1. It is shown that the rate heat transfer at the lower and middle quadrant enhances with $N_1 \leq 0.07$ and reduces with $N_1 \geq 0.09$whole at the upper quadrant it reduces with N_1.

The variation of N_u with heat source parameter α is shows in Table-3. In entire region it is found that the rate of heat transfer enhances with increase in the strength of the heat source and depreciates with that of heat sink at all quadrants. The variation of N_u with Eckert number E_c shows in the Table-4 that higher the dissipative heat larger $|N_u|$ at all quadrant.

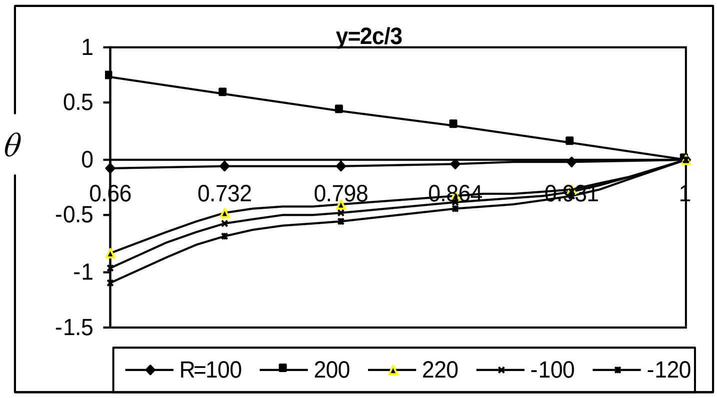

Fig. 1 Variation of (θ) with Ra at y = 2c/3 level
Ec = 0.001, α =2,N_1=0.01

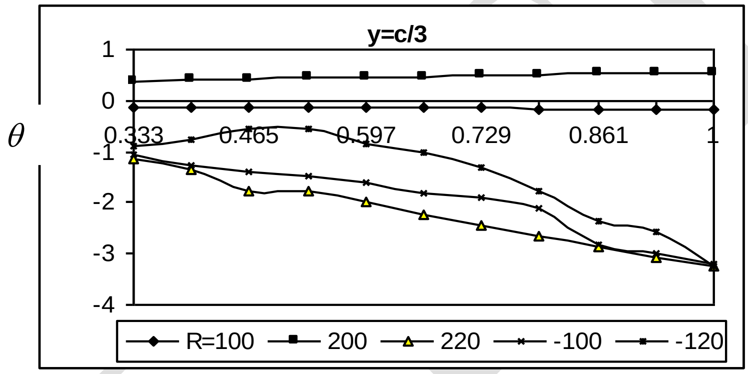

Fig. 2 variation of θ with Ra at y = c/3 level
Ec = 0.001, α =2, N_1=0.01

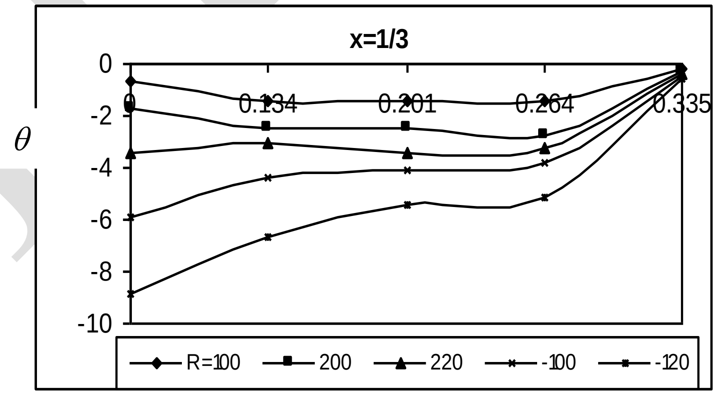

Fig. 3 variation of θ with Ra at x = 1/3 level
Ec = 0.01, α =2, N_1=0.01

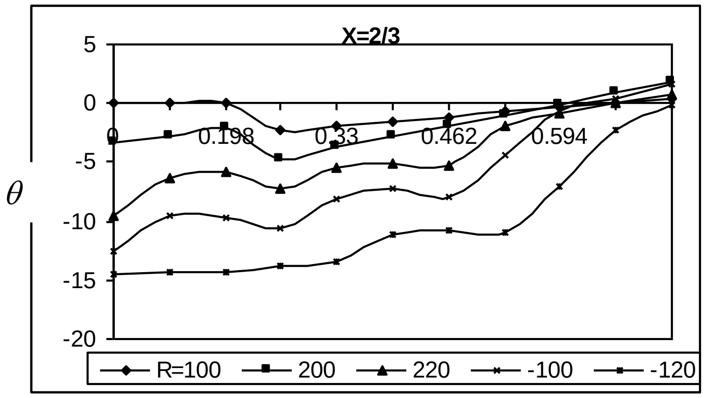

Fig. 4 variation of θ with Ra at x = 2/3 level

Ec = 0.01, α =2, N_1=0.01

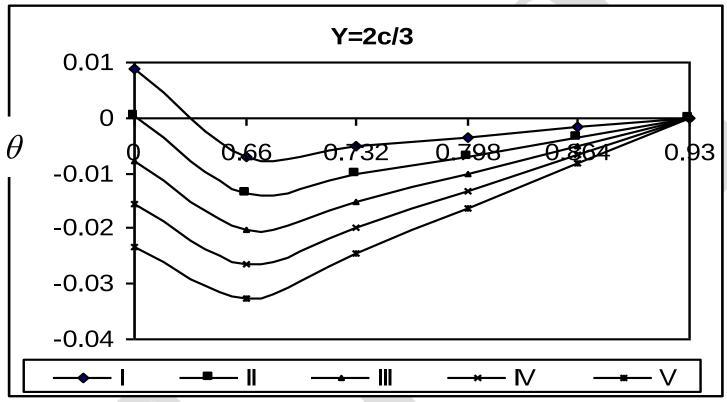

Fig. 5 variation of θ with N_1 at y =2 c/3 level

Ra = 100, Ec=0.001, α =2

	I	II	III	IV	V
N_1	0.01	0.03	0.05	0.07	0.09

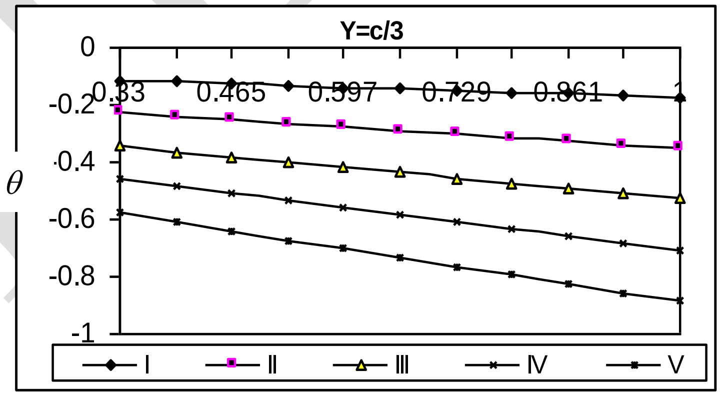

Fig. 6 variation of θ with N_1 at y = c/3 level

Ec=0.001, Ra = 100, α =2

	I	II	III	IV	V
N_1	0.01	0.03	0.05	0.07	0.09

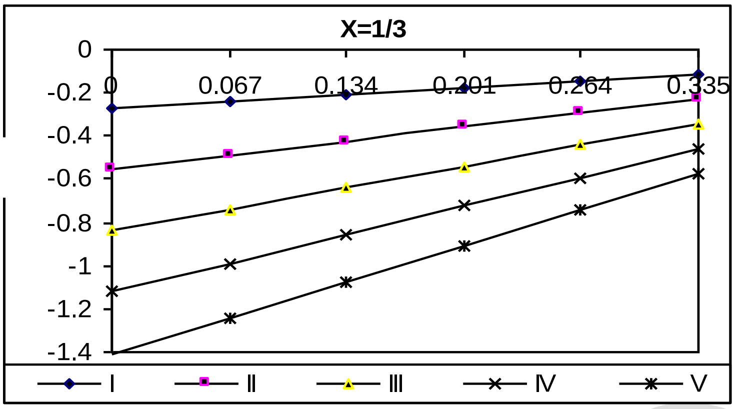

Fig. 7 θ variation with N_1 at x = 1/3 level
Ec=0.001, Ra = 100, α =2

	I	II	III	IV	V
N_1	0.01	0.03	0.05	0.07	0.09

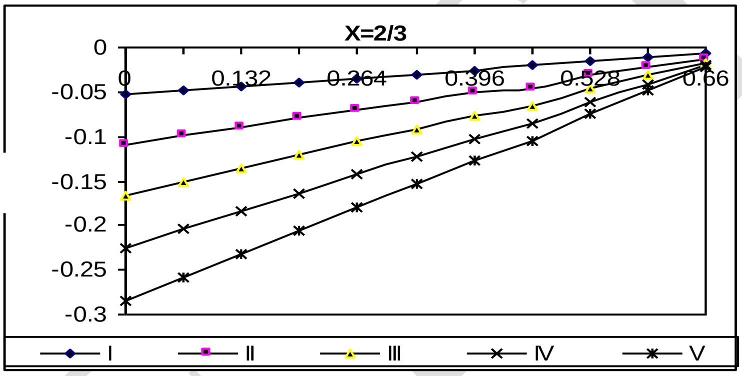

Fig. 8 variation of θ with N_1 at x = 2/3 level
Ec=0.001, Ra = 100, α =2

	I	II	III	IV	V
N_1	0.01	0.03	0.05	0.07	0.09

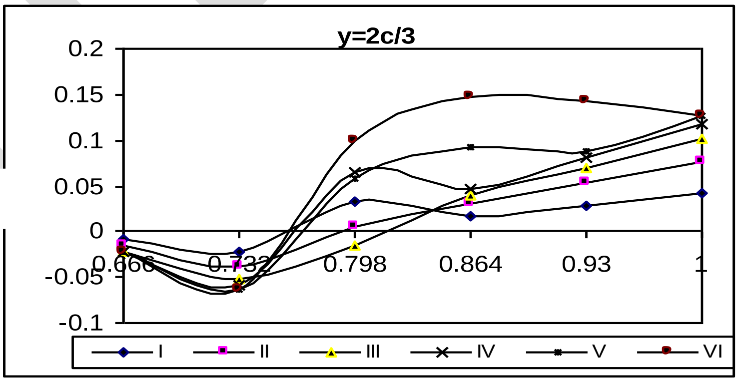

Fig. 9 variation of θ with α at y =2c/3 level
Ra = 100, Ec=0.001, N_1=0.01

	I	II	III	IV	V	VI
α	2	4	6	-2	-4	-6

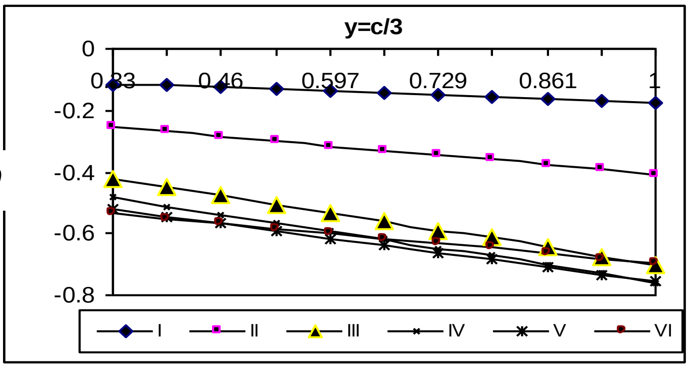

Fig. 10 variation of θ with at α y = c/3 level

Ra = 100, Ec=0.001, N_1 =0.01

	I	II	III	IV	V	VI
α	2	4	6	-2	-4	-6

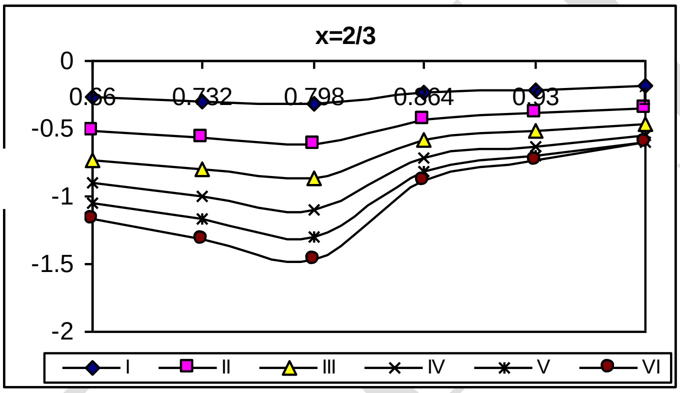

Fig. 11 variation of θ with at α x = 2/3 level

Ra = 100, Ec=0.001, N_1 =0.01

	I	II	III	IV	V	VI
α	2	4	6	-2	-4	-6

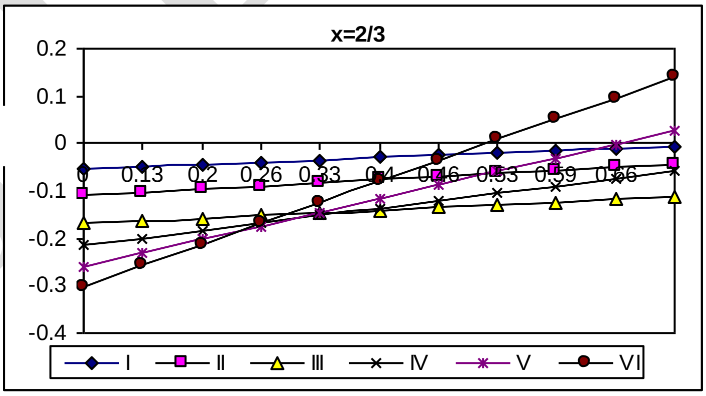

Fig. 12 variation of θ with α at x =2/3 level

Ra = 100, Ec=0.001, N_1 =0.01

	I	II	III	IV	V	VI
α	2	4	6	-2	-4	-6

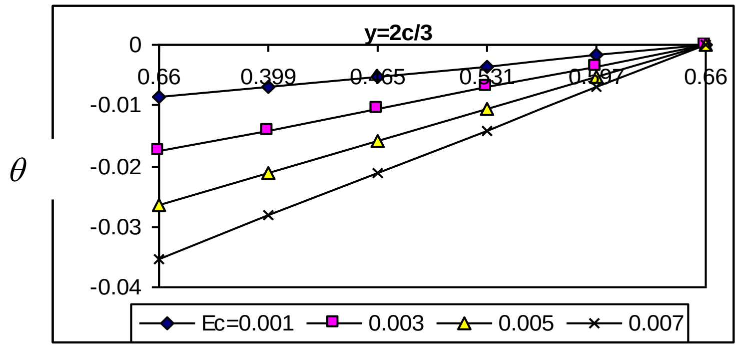

Fig. 13 variation of θ with Ec at y =2c/3 level
Ra = 100, α =2, N_1=0.01

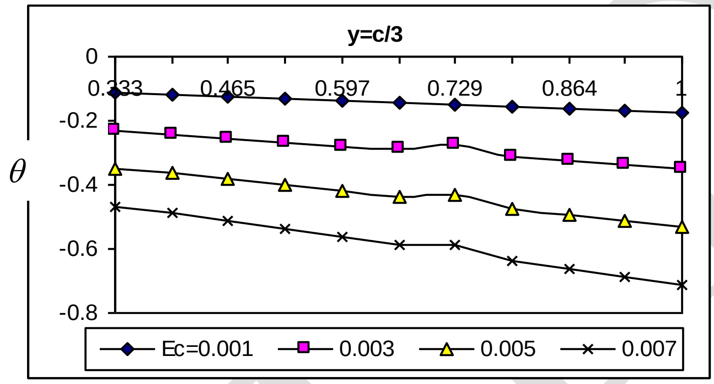

Fig. 14 variation of θ with Ec at y =c/3 level
Ra = 100, α =2, N_1=0.01

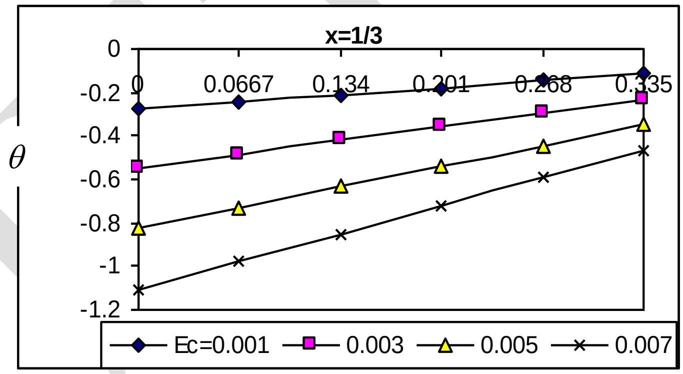

Fig. 15 variation of θ with Ec at x=1/3 level
Ra = 100,, α =2, N_1=0.01

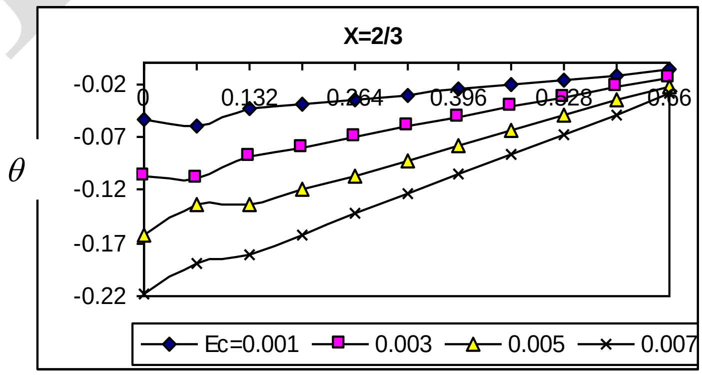

Fig. 16 variation of θ with Ec at x=1/3 level
Ra = 100,, α =2, N_1=0.01

Table-1
Nusselt Number (Nu) at different levels

Nu_1	2.06412	1.528	4.756	1.2352	1.582
Nu_2	8.1628	7.188	6.43524	4.7528	4.3602
Nu_3	5.036	-3.4624	5.92	-1.0643	-1.4126
Ra	$1x10^2$	$2x10^2$	$3x10^2$	$-1x10^2$	$-2x10^2$

Table-2
Nusselt Number (Nu) at different levels

Nu_1	2.2128	2.237	2.268	2.364	2.0323
Nu_2	2.1198	2.1296	2.142	2.258	2.175
Nu_3	2.0269	2.0224	2.018	2.0136	2.0096
N_1	0.01	0.03	0.05	0.07	0.09

Table-3
Nusselt Number (Nu) at different levels

Nu_1	2.2128	2.2248	2.2368	2.1891	2.1778	2.16692
Nu_2	2.11988	2.1870	2.2544	2.0135	1.9202	1.8546
Nu_3	2.0269	2.1493	2.2720	2.2163	1.6627	1.54244
α	2	4	6	-2	-4	-6

Table-4
Nusselt Number (Nu) at different levels

Nu_1	2.2128	2.2242	2.2356	2.2431
Nu_2	2.11988	2.13052	2.1592	2.1818
Nu_3	2.0268	2.0408	2.0706	2.0808
E_c	0.001	0.003	0.005	0.007

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