

BAYESIAN CHAIN SAMPLING PLAN USING BINOMIAL DISTRIBUTION

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ABSTRACT

This paper is concerned with the set of tables for the selection of Bayesian Chain Sampling Plan (BChSP-4(0,2)2) plan on the basis of different combinations of entry parameters. Beta distributions is considered as prior distribution. Comparison is made with conventional Chain Sampling Plan.

KEY WORDS

Bayesian Chain Sampling , Beta Binomial Distribution , Acceptance Quality Level(AQL), Limiting Quality Level(LQL) , Indifference Quality Level (IQL), Probabilistic Quality Region (PQR), Indifference Quality Region (IQL).

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INTRODUCTION BAYESIAN ACCEPTANCE SAMPLING

Bayesian acceptance sampling approach is associated with the utilization of prior process history for the selection of distribution (viz., gamma poisson , beta binomial) to describe the random fluctuations involved in acceptance sampling, Bayesian sampling plan requires the user to specify explicitly the distribution of defective from lot to lot. the prior distribution is the expected distribution of a lot quality on which the sampling plan is going to operate. The distribution is called prior, because it is formulated prior to the taking of samples. The combination of prior knowledge, represented with the prior distribution and the empirical knowledge based on the sample leads to the decision on the lot.

To improve the quality for any product and services, it is customary to modernize the quality practices and simultaneously reduce the cost for inspection and quality improvement. As a result of increasing customer quality requirements and development for new product technology many

existing quality assurance practices and techniques need to be modified. The need for such statistical and analytical techniques in quality assurance is rapidly increasing owing to stiff competition in industry towards product quality improvement.

This paper introduces a method for selection of Bayesian Chain Sampling Plan based on range of quality instead of point wish description of quality by invoking a novel approach called quality interval sampling (QIS) plan. This method seems to be versatile and can be adopted in the elementary production process where the stipulated quality level is advisable to fix at later stage and provides a new concept for selection of Bayesian ChSP-4(0,2)2 plan involving quality levels.

The sampling plan provides both vendor and buyer decision rules for the product acceptance to meet the present product quality requirement. Due to rapid advancement of manufacturing technology. Suppliers require their products to be of high quality with very low fraction defectives often measured in parts per million. Unfortunately, traditional methods in some particular situations fail to find out a minute defect in the product. In order to overcome such problems quality interval sampling (QIS) plan is introduced. This paper designs the parameters for the plan indexed with quality regions involving QIS.

Case and Keats have examined the relationship between defectives in the sample and defectives in the remaining lot for each of the five prior distributions; they observe that the use of a binomial prior renders sampling useless and inappropriate. These results serve to make the designers and users of Bayesian sampling plans more aware of the consequence associated with selection of particular prior distribution. Calvin has presented in a clear and concise treatment by means of ‘‘how and when to perform Bayesian acceptance sampling’’. These procedures are suited to the sampling of lots from process or assembly operations, which contain assignable causes. These causes may be unknown and awaiting isolation, known but irremovable due to the state of the art limitations, or known but uneconomical to remove. He has considered the Bayesian sampling in which primary concern is with the process average function non conforming p_1 with lot fraction non-conforming p and its limitations being discussed.

Hald has derived optimal solutions for the cost function $k(n,c)$ in the cases where the prior distribution is rectangular, polya and binomial. Tables are given for optimum n,c and $k(n,c)$ for various values of the parameters, which is an important result on Bayesian acceptance sampling (BAS). Hald has given a rather system of single sampling attribute plans obtained by minimizing average cost, under the assumptions that the cost linear in the fraction defective P . and that the depends on six parameters namely N, p_r, p_1, p_2 and w_2 cost parameters and p_1, p_2, w_2 , are however, that the weight combine with the p 's is such a way that only five independent parameters are left out.

Soundararajan (1978) gives plans approximation satisfying the condition specified such as given Acceptance Quality Level (AQL), and Producer's risk (2), Limiting Quality Level (LQL), and Consumer's risk β . Raju (1984) Contribution to the study of Chain Sampling Plans. Suresh and Latha (2001) discussed the Construction and Evaluation of Performance Measures of Bayesian Chain Sampling Plan using Gamma Distribution as the prior distribution.

Latha and jayabharathi (2013) have studied the Performance Measures for Bayesian Chain Sampling Plan using Binomial Distribution. Suresh and Sangeetha have studied the selection of Repetitive Deferred Sampling Plan with Quality Regions.

This paper designs the parameters of the plan indexed with AQL, LQL and PQR, IQR for specified s and k the parameter of the prior distribution with numerical illustrations are also provided.

ChSP-4A (c_1, c_2)r Plan:

Frishman (1960) presents extended Chain Sampling Plans designated as ChSP-4A (c_1, c_2)r. These plans evolve from an application in the sampling inspection of torpedoes for Naval Ordnance (1954) as a check on the control of production process and test equipment (including 100 percent inspection). Features of the plans include a basic acceptance number greater than zero, an option for forward or backward cumulating of results for an acceptance – rejection decision on the current lot, and provision for rejecting a lot on the basis of the results of a single sample ChSP-4A (c_1, c_2)r.

The Conditions for application and the Operating Procedure of these plans are as follows:

Conditions for application of ChSP-4A (c_1, c_2)r :

1. The product to be inspected or tested comprises a series of successive lots or batches (of material of individual units) produced by an essentially continuing process.
2. Under normal conditions, the lots are expected to be of essentially the same quality.
3. The lots are statistically independent of each other, and the sample size is small enough in comparison with the lots size, to permit the computing of probabilities by use of the binomial distribution.

Operating Procedure of ChSP-4A (c_1, c_2)r :

Step 1 : For each lot, Select a sample of n units and test each unit for conformance to the specified requirement

Step 2 : Accept the lot if d (the observed number of defectives) is less than or equal to c_1 .

Step 3 : If d is greater than or equal to r , reject the lot. This is the first stage.

Step 4 : If $c_1 < d < r$, either of the following procedures called the second stage can be followed'

(i) Accept the lot if d' (the total number of defectives arising out of lot under investigation plus the previous $(k-1)$ lots) is less than or equal to c_2 . Reject the lots if $d' > c_2$.

(or)

(ii) Defer action until an additional $(k-1)$ lots have been tested. Accept the lot under consideration if d' (the total number of defectives for the k lots) is less than or equal to c_2 .

Reject the lot if $d' > c_2$.

Frishman has presented OC curves for several plans and has illustrated the effects of the changes in sample sizes, Changes in the parameter k and the rejection number r . He observes the following properties.

1. Tighter Plans with greater discrimination are obtained for larger sample sizes.

2. Somewhat tighter Plans are obtained for increased values of the parameter k.
3. Slightly tighter plans in the region of the good quality are obtained for smaller values of r.
4. Adding the second stage to the first one results in higher probability of acceptance in the region of principal interest. The first stage is an ordinary single sampling plan with n and c₁. The second stage is the Chain Sampling feature using cumulative results.

THE OC FUNCTION OF CHSP-4A (C₁,C₂)R PLANS ARE RESPECTIVELY GIVEN AS (FRISHMAN (1960))

$$P_a(p) = P[d \leq c_1/n, p] + P\left[d' \leq \frac{c_2}{c_1} < d < r, kn, p\right]$$

The Condition of the Binomial Model the OC curve of ChSP-4 (0,2)2 plan is given by

$$P_a(p) = (1 - p)^n + np(1 - p)^{nk-1} + (k - 1)n^2p^2(1 - p)^{nk-2} \tag{1}$$

BETA DISTRIBUTION

$$f(p) = \beta(s, t, p) = \frac{p^{s-1}(1-p)^{t-1}}{\beta(s, t)}, \quad 0 < p < 1, \quad s, t > 0, \quad q = 1 - p \tag{2}$$

BAYESIAN AVERAGE PROBABILITY OF ACCEPTANCE

Under the proposed Chain Sampling Plans, the Probability of Acceptance of Chain Sampling Plan of type ChSP-4(0,2)2 plan based on the Beta Binomial Distribution is given by,

$$\begin{aligned} \bar{p} &= \int_0^{\infty} p_a(p)f(p)dp \tag{3} \\ &= \int_0^{\infty} [(1 - p)^n + np(1 - p)^{nk-1} + (k - 1)n^2 p^2(1 - p)^{nk-2}] \frac{p^{s-1}(1 - p)^{t-1}}{\beta(s, t)} dp \\ &= \frac{1}{\beta(s, t)} [\beta(s, n + t) + n\beta(s + 1, nk + t - 1) + n^2(k - 1)\beta(s + 2, nk + t - 2)] \tag{4} \end{aligned}$$

Here we assume the prior distribution as beta distribution. Hence the above equation is mixed distribution of beta and binomial distribution.

CONSTRUCTION OF TABLE :

If s=1, \bar{p} is reduced and \bar{x}_0 is the point of control The above equation (4) can be reduced to

$$\begin{aligned} \bar{p} &= \frac{(1 - \bar{x})}{(n\bar{x} + 1 - \bar{x})} + \frac{n\bar{x}(1 - \bar{x})}{(kn\bar{x} + 1 - \bar{x})(kn\bar{x} + 1 - 2\bar{x})} \\ &+ \frac{2n\bar{x}^2(k - 1)(1 - \bar{x})}{(kn\bar{x} + 1 - \bar{x})(kn\bar{x} + 1 - 2\bar{x})(kn\bar{x} + 1 - 3\bar{x})} \end{aligned}$$

Where $\mu = \frac{s}{s+t}$ (5)

If $s=2$, \bar{p} is reduced to,

$$\bar{p} = \frac{(2 - \mu)(2 - \mu)}{(n\mu + 2 - \mu)(n\mu + 2 - 2\mu)} + \frac{2n\mu(2 - \mu)(2 - 2\mu)}{(kn\mu + 2 - \mu)(kn\mu + 2 - 2\mu)(kn\mu + 2 - 3\mu)}$$

$$+ \frac{6(n\mu)^2(k - 1)(2 - \mu)(2 - 2\mu)}{(kn\mu + 2 - \mu)(kn\mu + 2 - 2\mu)(kn\mu + 2 - 3\mu)(kn\mu + 2 - 4\mu)}$$
(6)

If $s=3$, \bar{p} is reduced to ,

$$\bar{p} = \frac{(3 - \mu)(3 - 2\mu)(3 - 3\mu)}{(n\mu + 3 - \mu)(n\mu + 3 - 2\mu)(n\mu + 3 - 3\mu)}$$

$$+ \frac{3n\mu(3 - \mu)(3 - 2\mu)(3 - 3\mu)}{(kn\mu + 3 - \mu)(kn\mu + 3 - 2\mu)(kn\mu + 3 - 3\mu)(kn\mu + 3 - 4\mu)}$$

$$+ \frac{12n^2\mu^2(k - 1)(3 - \mu)(3 - 2\mu)(3 - 3\mu)}{(kn\mu + 3 - \mu)(kn\mu + 3 - 2\mu)(kn\mu + 3 - 3\mu)(kn\mu + 3 - 4\mu)(kn\mu + 3 - 5\mu)}$$
(7)

If $s=4$, \bar{p} is reduced to ,

$$\bar{p} = \frac{(4 - \mu)(4 - 2\mu)(4 - 3\mu)(4 - 4\mu)}{(n\mu + 4 - \mu)(n\mu + 4 - 2\mu)(n\mu + 4 - 3\mu)(n\mu + 4 - 4\mu)}$$

$$+ \frac{4n\mu(4 - \mu)(4 - 2\mu)(4 - 3\mu)(4 - 4\mu)}{(kn\mu + 4 - \mu)(kn\mu + 4 - 2\mu)(kn\mu + 4 - 3\mu)(kn\mu + 4 - 4\mu)(kn\mu + 4 - 5\mu)}$$

$$+ \frac{20n^2\mu^2(k - 1)(4 - \mu)(4 - 2\mu)(4 - 3\mu)(4 - 4\mu)}{(kn\mu + 4 - \mu)(kn\mu + 4 - 2\mu)(kn\mu + 4 - 3\mu)(kn\mu + 4 - 4\mu)(kn\mu + 4 - 5\mu)(kn\mu + 4 - 6\mu)}$$
(8)

If $s=5$, \bar{p} is reduced to ,

$$\bar{p} = \frac{(5 - \mu)(5 - 2\mu)(5 - 3\mu)(5 - 4\mu)(5 - 5\mu)}{(n\mu + 5 - \mu)(n\mu + 5 - 2\mu)(n\mu + 5 - 3\mu)(n\mu + 5 - 4\mu)(n\mu + 5 - 5\mu)}$$

$$+ \frac{5n\mu(5 - \mu)(5 - 2\mu)(5 - 3\mu)(5 - 4\mu)(5 - 5\mu)}{(kn\mu + 5 - \mu)(kn\mu + 5 - 2\mu)(kn\mu + 5 - 3\mu)(kn\mu + 5 - 4\mu)(kn\mu + 5 - 5\mu)(kn\mu + 5 - 6\mu)}$$

$$+ \frac{30n^2\mu^2(k - 1)(5 - \mu)(5 - 2\mu)(5 - 3\mu)(5 - 4\mu)(5 - 5\mu)}{(kn\mu + 5 - \mu)(kn\mu + 5 - 2\mu)(kn\mu + 5 - 3\mu)(kn\mu + 5 - 4\mu)(kn\mu + 5 - 5\mu)(kn\mu + 5 - 6\mu)(kn\mu + 5 - 7\mu)}$$
(9)

The Indifference Quality Level (IQL) or point of control \bar{p}_0 can be calculated by equating the above equations to 0.50 for various values of s, n using Newton's method approximation and those values are presented in the Table 1(a).

Table 1 (a): Certain μ values for specified values of $P(\mu)$

		Probability of Acceptance						
S	k	0.99	0.95	0.90	0.50	0.10	0.05	0.01
1	1	0.00112	0.00289	0.00463	0.02369	0.15669	0.27896	0.66612
	2	0.00065	0.00169	0.00273	0.01452	0.10340	0.19402	0.55432
	3	0.00051	0.00134	0.00220	0.01237	0.09225	0.17555	0.52456
	4	0.00043	0.00117	0.00193	0.01147	0.08815	0.16876	0.51313
	5	0.00039	0.00106	0.00177	0.01099	0.08619	0.16554	0.50762
2	1	0.00010	0.00054	0.00117	0.01195	0.06344	0.09858	0.22668
	2	0.00010	0.00054	0.00115	0.00902	0.04176	0.06464	0.15232
	3	0.00010	0.00053	0.00111	0.00720	0.03138	0.04847	0.11557
	4	0.00010	0.00052	0.00107	0.00627	0.02701	0.04183	0.10063
	5	0.00010	0.00052	0.00102	0.00572	0.02485	0.03865	0.09374
3	1	0.00132	0.00325	0.00500	0.01870	0.06131	0.08588	0.16360
	2	0.00122	0.00283	0.00420	0.01437	0.04563	0.06392	0.12114
	3	0.00106	0.00235	0.00343	0.01166	0.03874	0.05509	0.10878
	4	0.00093	0.00202	0.00294	0.01028	0.03617	0.05208	0.10456
	5	0.00083	0.00179	0.00261	0.00948	0.03507	0.05090	0.10307
4	1	0.00136	0.00332	0.00506	0.01818	0.05448	0.07357	0.12932
	2	0.00126	0.00290	0.00428	0.01397	0.04030	0.05435	0.09616
	3	0.00110	0.00241	0.00348	0.01130	0.03429	0.04707	0.08585
	4	0.00097	0.00207	0.00299	0.00994	0.03215	0.04481	0.08329
	5	0.00086	0.00183	0.00265	0.00914	0.03130	0.04400	0.08252
5	1	0.00138	0.00336	0.00511	0.01787	0.05077	0.06708	0.11229
	2	0.00129	0.00295	0.00433	0.01373	0.03740	0.04932	0.10055
	3	0.00113	0.00245	0.00354	0.01109	0.03187	0.04289	0.09155
	4	0.00099	0.00211	0.00303	0.00974	0.02999	0.04102	0.07298
	5	0.00089	0.00187	0.00268	0.00895	0.02928	0.04042	0.07252

Table 1(b): Values of μ_1/μ_2 tabulated against s and k for given α and β for Bayesian Chain Sampling Plan

s	k	μ_2/μ_1 for $\alpha=0.05$ $\beta=0.10$	μ_2/μ_1 for $\alpha=0.05$ $\beta=0.05$	μ_2/μ_1 for $\alpha=0.05$ $\beta=0.01$	μ_2/μ_1 for $\alpha=0.01$ $\beta=0.10$	μ_2/μ_1 for $\alpha=0.01$ $\beta=0.05$	μ_2/μ_1 for $\alpha=0.01$ $\beta=0.01$
1	1	54.21799	96.52595	230.49135	139.90179	249.07143	594.75000
	2	61.18343	114.80473	328.00000	159.07692	298.49231	852.80000
	3	68.84328	131.00746	391.46269	180.88235	344.21569	1028.54902
	4	75.34188	144.23932	438.57265	205.00000	392.46512	1193.32558
	5	81.31132	156.16981	478.88679	221.00000	424.46154	1301.58974
2	1	117.48148	182.55556	419.77778	634.40000	985.80000	2266.80000
	2	77.33333	119.70370	282.07407	417.60000	646.40000	1523.20000
	3	59.20755	91.45283	218.05660	313.80000	484.70000	1155.70000
	4	51.94231	80.44231	193.51923	270.10000	418.30000	1006.30000
	5	47.78846	74.32692	180.26923	248.50000	386.50000	937.40000
3	1	18.86462	26.42462	50.33846	46.44697	65.06061	123.93939
	2	16.12368	22.58657	42.80565	37.40164	52.39344	99.29508
	3	16.48511	23.44255	46.28936	36.54717	51.97170	102.62264
	4	17.90594	25.78218	51.76238	38.89247	56.00000	112.43011
	5	19.59218	28.43575	57.58101	42.25301	61.32530	124.18072
4	1	16.40964	22.15964	38.95181	40.05882	54.09559	95.08824
	2	13.89655	18.74138	33.15862	31.98413	43.13492	76.31746
	3	14.22822	19.53112	35.62241	31.17273	42.79091	78.04546
	4	15.53140	21.64734	40.23672	33.14433	46.19588	85.86598
	5	17.10383	24.04372	45.09290	36.39535	51.16279	95.95349
5	1	15.11012	19.96429	33.41964	36.78986	48.60870	81.36957
	2	12.67797	16.71864	34.08475	28.99225	38.23256	77.94574
	3	13.00816	17.50612	37.36735	28.20354	37.95575	81.01770
	4	14.21327	19.44076	34.58768	30.29293	41.43434	73.71717
	5	15.65775	21.61497	38.78075	32.89888	45.41573	81.48315

1 DESIGNING PLANS FOR GIVEN AQL, LQL, A AND B

Tables 1(a) and 1(b) are used to design Bayesian Chain Sampling Plan for given AQL, LQL, α and β .

The steps utilized for selecting Bayesian chain sampling plan (BChCP-4) are as follows:

1. To design a plan for given (AQL, $1-\alpha$) and (LQL, β) first calculate the operating ratio μ_2/μ_1
2. Find the value in Table 1(b) under the column for the appropriate α and β , which is closest to the desired ratio.

3. Corresponding to the located value of μ_2/μ_1 the value of s , k can be obtained.

Table1(c): Values of tabulated μ_0 , μ_1 , μ_2 and μ_2/μ_1 against sand k for given $P(\mu)$ for

Bayesian Chain Sampling Plan

s	k	μ_1	μ_0	μ_2	OR
1	1	0.00289	0.02369	0.15669	54.21799
	2	0.00169	0.01452	0.10340	61.18343
	3	0.00134	0.01237	0.09225	68.84328
	4	0.00117	0.01147	0.08815	75.34188
	5	0.00106	0.01099	0.08619	81.31132
2	1	0.00054	0.01195	0.06344	117.48148
	2	0.00054	0.00902	0.04176	77.33333
	3	0.00053	0.00720	0.03138	59.20755
	4	0.00052	0.00627	0.02701	51.94231
	5	0.00052	0.00572	0.02485	47.78846
3	1	0.00325	0.01870	0.06131	18.86462
	2	0.00283	0.01437	0.04563	16.12368
	3	0.00235	0.01166	0.03874	16.48511
	4	0.00202	0.01028	0.03617	17.90594
	5	0.00179	0.00948	0.03507	19.59218
4	1	0.00332	0.01818	0.05448	16.40964
	2	0.00290	0.01397	0.04030	13.89655
	3	0.00241	0.01130	0.03429	14.22822
	4	0.00207	0.00994	0.03215	15.53140
	5	0.00183	0.00914	0.03130	17.10383
5	1	0.00336	0.01787	0.05077	15.11012
	2	0.00295	0.01373	0.03740	12.67797
	3	0.00245	0.01109	0.03187	13.00816
	4	0.00211	0.00974	0.02999	14.21327
	5	0.00187	0.00895	0.02928	15.65775

Example :1

For $s=2$, $k=1$, $n=100$, and $\bar{p} = 0.50$ the corresponding IQL value $\mu_0=0.01195$

For $s=4$, $k=5$, $n=100$, and $\bar{p} = 0.50$ the corresponding IQL value $\mu_0=0.00914$

From Table 1(a) for the given variation Average Probability of acceptance of the above equations. The average product quality level μ using Newton's approximation method is obtained.

Example :2

For $s=1$, $k=4$, $n=100$, and $\bar{p} = 0.95$ the average product quality $\mu_1=0.00117$

For $s=2$, $k=2$, $n=100$, and $\bar{p} = 0.10$ the average product quality $\mu_2=0.04176$

From the above examples, we can understand that when s and k are increased, the average product quality is decreased.

Example :3

For $s=1$, $k=2$, $n=100$, and AQL value $\mu_1= 0.00169$ and LQL values $\mu_2=0.10340$

For $s=5$, $k=3$, $n=100$, and AQL value $\mu_1= 0.00245$ and LQL values $\mu_2=0.03187$

Example :4

Suppose the value for μ_1 is assumed as 0.0033 and value for μ_2 is assumed as 0.062 then the operating ratio is calculate as 18.79. Now the integer approximately equal to this calculated operating ratio and their corresponding parametric values are observed from the table 1(b). The actual $\mu_1=0.00325$ and $\mu_2=0.06131$ at ($\alpha=0.05$ and $\beta=0.10$),

In the similar way, the above equations are equated to the average probability of acceptance 0.95 and 0.10, AQL(μ_1) and IQL(μ_2) are obtained in Table1(c).

Table1(c): Values of tabulated μ_0 , μ_1 , μ_2 and μ_2/μ_1 against sand k for given P(μ) for Bayesian Chain Sampling Plan

s	k	μ_1	μ_0	μ_2	OR
1	1	0.00289	0.02369	0.15669	54.21799
	2	0.00169	0.01452	0.10340	61.18343
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5	1	0.00336	0.01787	0.05077	15.11012
	2	0.00295	0.01373	0.03740	12.67797
	3	0.00245	0.01109	0.03187	13.00816
	4	0.00211	0.00974	0.02999	14.21327
	5	0.00187	0.00895	0.02928	15.65775

2 Designing of Quality interval Bayesian Chain Sampling Plan (ChSP-4(0, 2)2plan) as follows:

2.1 Probabilistic Quality Region (PQR)

It is an interval of quality ($\mu_1 < \mu < \mu_2$) in which product is accepted with a minimum probability 0.10 and maximum probability 0.95

Probability Quality Range denoted as $d_2 = (\mu_2 - \mu_1)$ is derived from the average Probability of acceptance

$$\bar{p}(\mu_1 < \mu < \mu_2) = \frac{1}{\beta(s, t)} [\beta(s, n + t) + n\beta(s + 1, nk + t - 1) + n^2(k - 1)\beta(s + 2, nk + t - 2)]$$

Where $\mu = \frac{s}{s+t}$, is the expectation of beta distribution and approximately the mean values of product quality.

2.2 Indifference Quality Region (IQR):

It is an interval of quality ($\mu_1 < \mu < \mu_0$) in which product is accepted with a minimum probability 0.50 and maximum probability 0.95

Indifference Quality Range denoted as $d_0 = (\mu_0 - \mu_1)$ is derived from the average Probability of acceptance

$$\bar{p}(\mu_1 < \mu < \mu_0) = \frac{1}{\beta(s, t)} [\beta(s, n + t) + n\beta(s + 1, nk + t - 1) + n^2(k - 1)\beta(s + 2, nk + t - 2)]$$

Where $\mu = \frac{s}{s+t}$, is the expectation of beta distribution and approximately the mean values of product quality.

2.3 Selection of the Sampling Plan :

Table 1(d) gives unique values of T for different values of 's' and 'k'. Here Operating Ratio

$T = \frac{\alpha_2 - \alpha_1}{\alpha_0 - \alpha_1} = \frac{d_2}{d_0}$, Where $d_2 = (\alpha_2 - \alpha_1)$ and $d_0 = (\alpha_0 - \alpha_1)$ is used to characterize the sampling plan. For any given values of PQR(d_2) and IQR(d_0) one can find the ratio $T = \frac{d_2}{d_0}$,

Find the value in the Table 1(d) under the column T, which is equal to or just less than the specified ratio, Corresponding 's' and 'k' values are noted. From this ratio one can determine the parameters for the BChSP-4(0,2)2 Plan.

Table1(d): Values of PQR in IQR for specified values of ‘s’ and ‘k’

s	k	μ_1	μ_0	μ_2	d_2	d_1	T
1	1	0.00289	0.02369	0.15669	0.15380	0.02080	7.39423
	2	0.00169	0.01452	0.10340	0.10171	0.01283	7.92751
	3	0.00134	0.01237	0.09225	0.09091	0.01103	8.24207
	4	0.00117	0.01147	0.08815	0.08698	0.01030	8.44466
	5	0.00106	0.01099	0.08619	0.08513	0.00993	8.57301
2	1	0.00054	0.01195	0.06344	0.06290	0.01141	5.51271
	2	0.00054	0.00902	0.04176	0.04122	0.00848	4.86085
	3	0.00053	0.00720	0.03138	0.03085	0.00667	4.62519
	4	0.00052	0.00627	0.02701	0.02649	0.00575	4.60696
	5	0.00052	0.00572	0.02485	0.02433	0.00520	4.67885
3	1	0.00325	0.01870	0.06131	0.05806	0.01545	3.75793
	2	0.00283	0.01437	0.04563	0.04280	0.01154	3.70884
	3	0.00235	0.01166	0.03874	0.03639	0.00931	3.90870
	4	0.00202	0.01028	0.03617	0.03415	0.00826	4.13438
	5	0.00179	0.00948	0.03507	0.03328	0.00769	4.32770
4	1	0.00332	0.01818	0.05448	0.05116	0.01486	3.44280
	2	0.00290	0.01397	0.04030	0.03740	0.01107	3.37850
	3	0.00241	0.01130	0.03429	0.03188	0.00889	3.58605
	4	0.00207	0.00994	0.03215	0.03008	0.00787	3.82211
	5	0.00183	0.00914	0.03130	0.02947	0.00731	4.03146
5	1	0.00336	0.01787	0.05077	0.04741	0.01451	3.26740
	2	0.00295	0.01373	0.03740	0.03445	0.01078	3.19573
	3	0.00245	0.01109	0.03187	0.02942	0.00864	3.40509
	4	0.00211	0.00974	0.02999	0.02788	0.00763	3.65400
	5	0.00187	0.00895	0.02928	0.02741	0.00708	3.87147

Example :5

Given $s=1$, $k=3$ and $\mu_1=0.0013$ compute the values of PQR and IQR then compute T.

Select the respective values from Table1(d). The nearest values of PQR and IQR corresponding to $s=1$, $k=3$, and $\mu_1=0.00134$ are $d_2=0.09091$ and $d_0=0.01103$, Then $T=8.24207$.

Corresponding to $s=1$, $k=3$, one can obtain the values of μ_1 from Table 1(c). Hence the required plan has parameters $n=100$, $s=1$, $k=3$, through Quality Interval.

Example :6

Given $s=3$, $k=2$ and $\mu_1=0.0028$ compute the values of PQR and IQR then compute T. Select the respective values from Table1(c). The nearest values of PQR and IQR corresponding to $s=3$, $k=2$, and $\mu_1=0.00283$ are $d_2= 0.04280$ and $d_0= 0.01154$, Then $T= 3.70884$.

Corresponding to $s=3$, $k=2$, one can obtain the values of μ_1 from Table 1(b). Hence the required plan has parameters $n=100$, $s= 3$, $k= 2$, through Quality Interval.

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