GENERALIZED INTUITIONISTIC FUZZY TOPOLOGY

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ABSTRACT

This paper is devoted to the study of intuitionistic fuzzy topological spaces. We introduce the concept of generalized intuitionistic fuzzy topology in intuitionistic fuzzy topological spaces.

Key words: Intuitionistic fuzzy topology, γ - intuitionistic fuzzy open sets.

INTRODUCTION

Most of the real life problems have various uncertainities. In order to deal with many complicated problem in the fields of engineering, social science, economics, medical science etc. involving uncertainities, various theories have been developed such as the theory of probability, fuzzy set theory, intuitionistic fuzzy set theory, Rough set theory etc.

The concept of fuzzy sets was introduced by Zadeh [8]. Fuzzy topology was introduced by Chang in 1967 [6]. Later Atanassov [2] generalized this idea to intuitionistic fuzzy sets using the notion of fuzzy sets.

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Fuzzy sets are intuitionistic fuzzy sets but the converse is not necessarily true [1]. Coker [7] introduced the intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets.

Intuitionistic fuzzy set theory has been applied in different areas like logical programming [3,4], decision making problems [5]. In intuitionistic fuzzy sets we consider membership and non-membership values of elements of the topological space. In the last few years, various concepts in fuzzy were extended to intuitionistic fuzzy sets. Recently the concept of generalized fuzzy topology was introduced in fuzzy topology. We have extended the concept of generalized intuitionistic fuzzy topology to intuitionistic fuzzy topological spaces and studied some of their properties.

2 PRELIMINARIES

Definition 2.1 [2] An intuitionistic fuzzy set (IFS in short) A in X is an object having the form A={ $x, \mu_A(x), \nu_A(x) / x \in \mathrm{X}$ } where the functions $\mu_A(x) : \mathrm{X} \to [0,1]$ and $\nu_A(x) : \mathrm{X} \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\nu_A(x)$) of each element $x \in$ X of the set A, respectively and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each $x \in$ X.

Definition 2.2 [2] Let A and B be IFSs of the form

A = { < x, $\mu_A(x), \nu_A(x)$ > / $x \in$ X } and B = { < x, $\mu_B(x), \nu_B(x)$ > / $x \in$ X }. Then

(a) A$\subseteq$B if and only if $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for all $x \in$ X

(b) A=B if and only if A$\subseteq$B and B $\subseteq$ A

(c) A^c = { < x, $\nu_A(x), \mu_A(x)$ > / $x \in$ X}

(d) A $\cap$ B = { < x, $\mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)$ > / $x \in$ X }

(e) A $\cup$ B = { < x, $\mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)$ > / $x \in$ X }

Definition 2.3: [7] An Intuitionistic fuzzy topology (IFT in short) on X is a family of IFSs in X satisfying the following conditions

(i) $0\sim, 1\sim \in \tau$

(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$

(iii) $\cup G_i \in \tau$ for any family $\{G_i / i \in J\} \subseteq \tau$

In this case, the pair (X,τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement of an IFOS A in an IFTS (X,τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X and it is denoted by A^c.

Definition 2.4: Let X be a non empty set. Let $\gamma : I^X \to I^X$ be a function. γ is said to be monotonic if $\lambda_1 \le \lambda_2$ implies $\gamma(\lambda_1) \le \gamma(\lambda_2)$ for all $\lambda_1, \lambda_2 \in I^X$. Let $\Gamma(X)$ denote the collection of all such monotonic functions. An intuitionistic fuzzy subset $\lambda \in I^X$ is said to be γ - intuitionistic fuzzy open if $\lambda \le \gamma(\lambda)$ such that $\lambda \in \Gamma(X)$.

3 PROPERTIES OF GENERALIZED INTUITIONISTIC FUZZY TOPOLOGY

Proposition 3.1: Union of γ - intuitionistic fuzzy open sets is γ - intuitionistic fuzzy open set and zero intuitionistic fuzzy set is γ - intuitionistic fuzzy open set.

Proof: Let $\{\lambda_\alpha\}_{\alpha\in\Delta}$ be a family of γ - intuitionistic fuzzy open sets. Then for each $\alpha \in \Delta, \lambda_\alpha \le \gamma(\lambda_\alpha)$.Also $\lambda_\alpha \le \vee \lambda_\alpha$.Since γ is monotonic, $\gamma(\lambda_\alpha) \le \gamma(\vee \lambda_\alpha)$.

Now $\vee \lambda_\alpha \le \vee \gamma(\lambda_\alpha) < \vee \gamma(\vee \lambda_\alpha) = \gamma(\vee \lambda_\alpha)$.This implies, $\vee \lambda_\alpha \le \gamma(\vee \lambda_\alpha)$.This implies, $\vee \lambda_\alpha$ is γ - intuitionistic fuzzy open set. Also the zero intuitionistic fuzzy set $\bar{0}$ is clearly γ - intuitionistic fuzzy open.

Proposition 3.2: Let $\mathcal{G}$ be any generalized intuitionistic fuzzy topology and function $\gamma : I^X \to I^X$ be monotonic such that $\mathcal{G}$ is the collection of all γ - intuitionistic fuzzy sets, then

(i) $\gamma(\bar{0}) = \bar{0}$, where $\bar{0}$ is the zero intuitionistic fuzzy set in $\mathcal{G}$

(ii) $\gamma(\lambda) < \lambda$

(iv) $\gamma\gamma(\lambda) = \gamma(\lambda)$ for $\lambda \in I^X$

Proof: Clearly $\bar{0} \in \mathcal{G}$. Define $\gamma(\lambda) = \vee\{\lambda_\alpha \in \mathcal{G} / \lambda_\alpha \le \lambda\}$. This implies $\gamma(\lambda) \in \mathcal{G}$. Also $\gamma(\lambda) \le \lambda$ and $\gamma(\bar{0}) = \bar{0}$. Now suppose that $\omega \in \mathcal{G}$. Then $\gamma(\omega) = \omega \ge \omega$. This means ω is γ - intuitionistic fuzzy open.

Conversely, if λ is γ - intuitionistic fuzzy open set, then $\gamma(\lambda) \ge \lambda$. Therefore $\gamma(\lambda) = \lambda$ and $\lambda \in \mathcal{G}$. Thus the family of γ - intuitionistic fuzzy open sets are members of $\mathcal{G}$. Now if $\lambda \in I^X$, $\gamma(\lambda) \in \mathcal{G}$ and therefore $\gamma(\gamma(\lambda)) = \gamma(\lambda)$.

Definition 3.3: We define γ - intuitionistic fuzzy interior of λ as $i_\gamma(\lambda) = \vee\{\omega / \omega \le \lambda, \omega$ is γ - intuitionistic fuzzy open$\}$

Definition 3.4: We define γ - intuitionistic fuzzy closure of λ as $c_\gamma(\lambda) = \wedge\{\omega / \lambda \le \omega, \omega$ is γ - intuitionistic fuzzy closed$\}$

Proposition 3.5: For any $\gamma \in \Gamma(X)$, we have $i_\gamma(\lambda) \le \lambda \wedge \gamma(\lambda)$ for all $\lambda \in I^X$.

Proof: If $\omega \le \lambda$ and ω is γ - intuitionistic fuzzy open, then $\omega \le \gamma(\omega) \le \gamma(\lambda)$
Therefore, we have $\omega \le \lambda \wedge \gamma(\lambda)$. Since this is valid for any γ - intuitionistic fuzzy open set such that $\omega < \lambda$, it is also valid for the largest γ - intuitionistic fuzzy open set contained in λ that is $i_\gamma(\lambda)$, $i_\gamma(\lambda) \le \lambda \wedge \gamma(\lambda)$.

Remark 3.6: In order to obtain the condition for equality in proposition 3.5, consider
$i \in \Gamma(X)$, satisfying $i^2\lambda = i\lambda \le \lambda$, $\lambda \in I^X$ and $\kappa \in \Gamma(X)$ satisfying
$i\lambda \le \kappa i\lambda \le \lambda$, $\lambda \in I^X$.

In any intuitionistic fuzzy topological space, i = interior of intuitionistic fuzzy topology, c = κ, the closure of intuitionistic fuzzy topology satisfy the conditions in remark 3.6.

Proposition 3.7: If i, $\kappa \in \Gamma(X)$ satisfying the conditions $i^2\lambda = i\lambda \le \lambda$, $\lambda \in I^X$ and $i\lambda \le \kappa i\lambda \le \lambda$, $\lambda \in I^X$ and $\gamma = (\kappa i)^n$, $\gamma = i(\kappa i)^n$, $n \in N$, then $i_\gamma = \lambda \wedge \gamma(\lambda)$.

Proof: Let λ be γ - intuitionistic fuzzy open. Then we have $\lambda \leq \gamma(\lambda)$. Given $i\lambda \leq \lambda \leq \gamma(\lambda)$. This means $i\lambda \leq (\kappa i)^n \lambda$ for $\lambda \in I^X$ and $n \in N$. This means $i\lambda \leq \lambda \wedge (\kappa i)^n \lambda$ and $i\lambda \leq \lambda \wedge i(\kappa i)^n \lambda$.But $i^2 \lambda = i\lambda \leq i(\lambda \wedge (\kappa i)^n \lambda)$. Therefore $(\kappa i)^{\kappa} \lambda \leq (\kappa i)^{\kappa}(\lambda \wedge (\kappa i)^n \lambda)$ and $i(\kappa i)^{\kappa} \lambda \leq i(\kappa i)^{\kappa}(\lambda \wedge (\kappa i)^n \lambda)$ for any $\kappa \in N$. When k = n, $\lambda \wedge (\kappa i)^n \lambda \leq (\kappa i)^n[\lambda \wedge (\kappa i)^n \lambda]$ and $\lambda \wedge i(\kappa i)^n \lambda \leq i(\kappa i)^n[\lambda \wedge i(\kappa i)^n \lambda]$ .Thus $\lambda \wedge \gamma(\lambda)$ is γ - intuitionistic fuzzy open for $\gamma = (\kappa i)^n$ and $\gamma = i(\kappa i)^n$. Hence $\lambda \wedge \gamma(\lambda) \leq i_\gamma(\lambda)$. By proposition 3.5, $i_\gamma(\lambda) \leq \lambda \wedge \gamma(\lambda)$. Therefore, $i_\gamma(\lambda) = \lambda \wedge \gamma(\lambda)$.

4 γ - INTUITIONISTIC FUZZY CLOSURE

Definition 4.1: Given $\gamma \in \Gamma(X)$, define $\gamma^*(\lambda) = 1 - \gamma(1-\lambda)$ for $\lambda \in I^X$ .Then $\gamma^* \in \Gamma(X)$.

Property 4.2: For any $\lambda \in I^X$, $(\gamma^*)^*(\lambda) = \gamma(\lambda)$.

Proof: $(\gamma^*)^*(\lambda) = 1 - \gamma^*(1-\lambda) = 1 - \{(1-\gamma) - \gamma[1-(1-\lambda)]\}$

$= 1 - (\lambda^c - \gamma(1-\gamma^c)) = 1 - (\lambda^c - \gamma(\lambda)) = 1 - (\lambda^c - \mu)$ where $\mu = \gamma(\lambda)$

$= 1 - (\lambda^c \wedge \mu^c) = (1-\lambda^c)(1-\mu^c) = \lambda \vee \mu = \lambda \vee \gamma(\lambda) = \gamma(\lambda)$

Therefore $(\gamma^*)^* = \gamma$ and also $c_\gamma = (i_\gamma)^*$. That is $c_\gamma(\lambda) = (1 - i_\gamma(1-\lambda))$.

Proposition 4.3: $i_\gamma(\lambda) = \lambda \wedge \gamma(\lambda)$ holds for $\gamma \in \Gamma(X)$ and for every $\lambda \in I^X$ if and only if $c_\gamma(\lambda) = \lambda \vee \gamma^*(\lambda)$.

Proof: Suppose $i_\gamma(\lambda) = \lambda \wedge \gamma(\lambda)$.

Now $c_\gamma(\lambda) = (i_\gamma)^*(\lambda)$

$= 1 - i_\gamma(1-\lambda) = 1 - ((1-\lambda) \wedge \gamma(1-\lambda)) = 1 - (\lambda^c \wedge \gamma(1-\lambda))$

$= (1-\lambda^c) \vee (1-\gamma(1-\lambda)) = \lambda \vee \gamma^*(\lambda)$

Conversely suppose that $c_{\gamma}(\lambda) = \lambda \vee \gamma^*(\lambda)$

Now $i_{\gamma}(\lambda) = (c_{\gamma})^*(\lambda) = 1 - c_{\gamma}(1-\lambda) = 1 - [(1-\lambda) \vee \gamma^*(1-\lambda)]$

$=1 - [\lambda^c \vee \gamma^*(1-\lambda)] = (1-\lambda^c) \wedge [1-\gamma^*(1-\lambda)] = \lambda \wedge \gamma(\lambda)$. This completes the proof.

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