# SURFACE WAVES IN FIBRE-REINFORCED THERMOVISCO ELASTIC MEDIA OF HIGHER ORDER UNDER THE INFLUENCE OF GRAVITY

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**ABSTRACT:** The aim of paper is to investigate the stress and equation of motion in terms of displacement in fibre-reinforced thermoviscoelastic media of higher order under gravity. The equation for wave velocity of Stoneley waves is obtained. These results reduce to classical results when effects of gravity, fibre reinforcement, thermal field and viscosity are ignored.

**KEYWORDS:** Surface waves in fibre-reinforced elastic media, Effect of gravity, Thermal field and Viscosity, Stoneley waves.

## INTRODUCTION

The propagation of surface waves in elastic media is of considerable importance in earthquake engineering and seismology. The theory of surface waves has been developed by Stonely[4], Bullen[2] etc.

Love[3] has studied the effects of gravity on various wave problems and shown that velocity of Rayleigh waves increases significantly due to gravitational field when the wavelength of waves is large. Biot[1] developed a mathematical theory of initial stresses to investigate the effects of gravity on Rayleigh waves in elastic media.

The idea of continuous self reinforcement at every point of an elastic solid was introduced by Belfield[7].The characteristic property of a reinforced concrete member is that its components, namely concrete and steel act together as a single anisotropic unit as long as they remain in same elastic condition ,i.e the two components are bound together so that there can be no relative displacement between them.

In the present paper, the problem of nth order thermovisco-elastic surface waves in a homogeneous and isotropic fibre-reinforced medium under the influence of gravity is studied. The wave velocity equation for surface waves in presence of thermal viscous and gravitational effects is derived.

## BASIC EQUATIONS OF MOTION

Let $M_1$, $M_2$ be two homogenous isotropic semi infinite viscoelastic media in contact with each other where $M_2$ is above $M_1$ under the influence of thermal field and gravity along a common horizontal plane boundary. We choose rectangular cartesian coordinate system with the origin at any point on plane boundary and z axis normal to $M_1$. Consider possibility of a type of wave moving in positive x direction and assume that the disturbance is confined to neighbourhood of the boundary thus making it a surface wave. Assuming further that at any instance all the particles in a line parallel to x axis have equal displacement i.e all partial derivatives w.r.t y vanish.

Let u = (u,v,w) be the displacement vector at any point (x,y,z) at time t. Introducing two displacement potentials ϕ and ψ which are functions of x,z and t in the form

$$\tilde{N}^2 y = w_x - u_z$$

$$u=\phi_x-\psi_z,\ w=\phi_z+\psi_x \qquad \nabla^2\phi=u_x+v_y+w_z=\Delta$$

The component v is associated with purely distortional waves and ϕ ,ψ, v are associated with P waves, SV waves and SH waves resp. Equation of motion for three dimensional wave problem under influence of gravity are

$$\sigma_{xx,x}+\sigma_{xy,y}+\sigma_{xz,z}+\rho g w_{,x}=\rho u_{,tt} \qquad \sigma_{yx,x}+\sigma_{yy,y}+\sigma_{yz,z}+\rho g w_{,y}=\rho v_{,tt}$$

$$\sigma_{zx,x}+\sigma_{zy,y}+\sigma_{zz,z}-\rho g(u_{,x}+v_{,y})=\rho w_{,tt}$$

where $\sigma_{xy}$ are stress components, ρ is density and g is acceleration due to gravity.
According to Voigt's definition, stress strain relation in higher order thermovisco elastic medium are

$$\sigma_{ij}=D_\lambda e_{kk}\delta_{ij}+2D_{\mu_T}e_{ij}+D_\alpha(a_k a_m e_{km}\delta_{ij}+a_i a_j e_{kk})+2(D_{\mu_L}-D_{\mu_T})(a_i a_k e_{kj}+a_j a_k e_{ki})+D_\beta a_k a_m e_{km}a_i a_j-D_\gamma(T-T_0)\delta_{ij}$$

where $\sigma_{ij}$ are components of stress, $e_{ij}$ are components of strain, $D_\lambda, D_{\mu_T}, D_{\mu_L}$ are elastic parameters, $D_\alpha, D_\beta, (D_{\mu_L}-D_{\mu_T})$ are reinforced elastic parameters and $\vec{a}=(a_1,a_2,a_3)$ where $a_1^2+a_2^2+a_3^2=1$ if $\vec{a}$ =(1,0,0) i.e x-axis. Also

$$D_{\mu_L}=\sum_{k=0}^{n}(\mu_L)_k\frac{\partial^k}{\partial t^k} \qquad D_{\mu_T}=\sum_{k=0}^{n}(\mu_T)_k\frac{\partial^k}{\partial t^k} \qquad D_\alpha=\sum_{k=0}^{n}\alpha_k\frac{\partial^k}{\partial t^k} \qquad D_\gamma=\sum_{k=0}^{n}\gamma_k\frac{\partial^k}{\partial t^k}$$

$$D_\lambda=\sum_{k=0}^{n}\lambda_k\frac{\partial^k}{\partial t^k} \qquad D_\beta=\sum_{k=0}^{n}\beta_k\frac{\partial^k}{\partial t^k}$$

in which $\lambda_0,\mu_0$ and $\lambda_k,\mu_k$ (k=1,2,..,n) are the parameters representing effect of viscosity, $\beta_k$(k=0,1,.....,n) are thermal parameters , T is absolute temperature over initial temperature $T_0$.Also $\beta_k=(3\lambda_k+2\mu_k)\,\alpha_i$ , $\alpha_i$ being coefficient of linear expansion.

We get following dynamical equations of motion

$$[D_\lambda+2D_\alpha+4D_{\mu_L}-2D_{\mu_T}+D_\beta]u_{,xx}+[D_\alpha+D_\lambda+D_{\mu_L}]w_{,xz}+D_{\mu_L}u_{,zz}-D_\gamma\frac{\partial T}{\partial x}+\rho g w_{,x}=\rho u_{,tt}$$

$$[(D_{\mu_L}-D_{\mu_T})]v_{,xx}+D_{\mu_T}[v_{,xx}+v_{,zz}]=\rho v_{,tt}$$

$$D_{\mu_L}w_{,xx}+[D_\alpha+D_\lambda+D_{\mu_L}]u_{,xz}+[D_\lambda+2D_{\mu_T}]w_{,zz}-D_\gamma\frac{\partial T}{\partial z}-\rho g u_{,x}=\rho w_{,tt}$$

Introducing displacement potentials ϕ and ψ, we get

$$A_t \frac{\partial^2 \phi}{\partial x^2} + A_r \frac{\partial^2 \phi}{\partial z^2} - A_v T + g \frac{\partial \psi}{\partial x} = \frac{\partial^2 \phi}{\partial t^2}$$

$$A_s \frac{\partial^2 \psi}{\partial x^2} + A_q \frac{\partial^2 \psi}{\partial z^2} - g \frac{\partial \phi}{\partial x} = \frac{\partial^2 \psi}{\partial t^2}$$

$$A_q \frac{\partial^2 v}{\partial x^2} + A_p \frac{\partial^2 v}{\partial z^2} = \frac{\partial^2 v}{\partial t^2}$$

Where

$$A_r = \sum_{k=0}^{n} M_{kr} \frac{\partial^k}{\partial t^k} \qquad A_q = \sum_{k=0}^{n} M_{kq} \frac{\partial^k}{\partial t^k} \qquad A_p = \sum_{k=0}^{n} M_{kp} \frac{\partial^k}{\partial t^k}$$

$$A_t = \sum_{k=0}^{n} M_{kt} \frac{\partial^k}{\partial t^k} \qquad A_v = \sum_{k=0}^{n} M_{kv} \frac{\partial^k}{\partial t^k} \qquad A_s = \sum_{k=0}^{n} M_{ks} \frac{\partial^k}{\partial t^k}$$

$$M_{kt} = \frac{\lambda_k + 2\alpha_k + 4(\mu_L)_k - 2(\mu_T)_k + \beta_k}{\rho}, \quad M_{kr} = \frac{\lambda_k + \alpha_k + 2(\mu_L)_k}{\rho}, \quad M_{ks} = \frac{\beta_k + \alpha_k + 3(\mu_L)_k - 2(\mu_T)_k}{\rho}$$

$$M_{kp} = \frac{(\mu_T)_k}{\rho}, \quad M_{kq} = \frac{(\mu_L)_k}{\rho}, \quad M_{kv} = \frac{\gamma_k}{\rho}$$

Similarly in $M_2$ replace $\rho$, $\lambda_k$, $\mu_k$, $\beta_k$ by dashes over the quantities. The generalized law of heat conduction in absence of heat source is

$$K\nabla^2 T = \rho C_v \frac{\partial T}{\partial t} + T_0 D_b \frac{\partial}{\partial t}(\nabla^2 \phi)$$

Where K is thermal conductivity, $C_v$ is specific heat of body at constant volume. Using above, we have

$$K\nabla^2 T = \rho C_v \frac{\partial T}{\partial t} + \rho T_0 A_v \frac{\partial}{\partial t}(\nabla^2 \phi)$$

## SOLUTION OF THE PROBLEM

Let solutions for medium $M_1$ are of the form

$$(\phi, \psi, T, v) = [\phi_1(z), \psi_1(z), T_1(z), v_1] \exp\{i\omega(x - ct)\}$$

Substituting in above equations, we get a set of differential equations for medium $M_1$ as follows

$$(\frac{d^2}{dz^2} + E_k)\phi_1 - \frac{\sum_{k=0}^{n}(-i\omega c)^k M_{kv}}{\sum_{k=0}^{n}(-i\omega c)^k M_{kr}} T_1 + \frac{g i \omega \psi_1}{\sum_{k=0}^{n}(-i\omega c)^k M_{kr}} = 0$$

$$(\frac{d^2}{dz^2}+B_k)\psi_1-\frac{gi\omega\phi_1}{\sum_{k=0}^{n}(-i\omega c)^k M_{kq}}=0$$

$$(\frac{d^2}{dz^2}+D_k)v_1=0$$

$$(\frac{d^2}{dz^2}-F_k)T_1+\frac{i\omega c\rho T_0}{k}\sum_{k=0}^{n}(-i\omega c)^k M_{kv}(\frac{d^2}{dz^2}-\omega^2)\phi_1=0$$

$$E_k=\frac{\omega^2(c^2-\sum_{k=0}^{n}M_{kt}(-i\omega c)^k)}{\sum_{k=0}^{n}M_{kr}(-i\omega c)^k}$$

$$B_k=\frac{\omega^2(c^2-\sum_{k=0}^{n}M_{ks}(-i\omega c)^k)}{\sum_{k=0}^{n}M_{kq}(-i\omega c)^k}$$

$$D_k=\frac{\omega^2(c^2-\sum_{k=0}^{n}M_{kq}(-i\omega c)^k)}{\sum_{k=0}^{n}M_{kp}(-i\omega c)^k}$$

$$F_k=\omega^2-\frac{i\omega c\rho C_v}{K}$$

It is ensure that these equations have exponential solution and in order that ϕ,ψ, T& v shall describe surface waves they must become vanishingly small as z→-∞. For $M_1$

$$\phi=[A_1\exp(i\omega p_1 z)+A_2\exp(i\omega p_2 z)+A_3\exp(i\omega p_3 z)]\exp(i\omega(x-ct))$$

$$T=[\alpha_{k1}A_1\exp(i\omega p_1 z)+\alpha_{k2}A_2\exp(i\omega p_2 z)+\alpha_{k3}A_3\exp(i\omega p_3 z)]\exp(i\omega(x-ct))$$

$$\psi=[\beta_{k1}A_1\exp(i\omega p_1 z)+\beta_{k2}A_2\exp(i\omega p_2 z)+\beta_{k3}A_3\exp(i\omega p_3 z)]\exp(i\omega(x-ct))$$

$$v=C\exp[i\omega(I_{k1}z+x-ct)]$$

Where

$$I_{k1}^2=\frac{\rho c^2-\sum_{k=0}^{n}(-i\omega c)^k(\mu_L)_k}{\sum_{k=0}^{n}(-i\omega c)^k(\mu_T)_k}$$

$$\alpha_{kj}=\frac{-i\omega^3\rho c T_0\sum_{k=0}^{n}(-i\omega c)^k M_{kv}(p_j{}^2+1)}{K(\omega^2 p_j{}^2+F_k)}$$

$$\beta_{kj}=\frac{-gi\omega}{\sum_{k=0}^{n}M_{ks}(-i\omega c)^k(\omega^2 p_j{}^2-B_k)}$$

## BOUNDARY CONDITIONS

1) The displacement components,tempraure & normal flux at the boundary must be continuous.

2) The stress components at the boundary must satisfy be continuous.

Using above boundary conditions we obtain

$$(1-\beta_{k1}p_1)A_1 + (1-\beta_{k2}p_2)A_2 + (1-\beta_{k3}p_3)A_3 = (1+\beta'_{k1}p'_1)A'_1 + (1+\beta'_{k2}p'_2)A'_2 + (1+\beta'_{k3}p'_3)A'_3$$

$$C = C'$$

$$(p_1+\beta_{k1})A_1 + (p_2+\beta_{k2})A_2 + (p_3+\beta_{k3})A_3 = (-p'_1+\beta'_{k1})A'_1 + (-p'_2+\beta'_{k2})A'_2 + (-p'_3+\beta'_{k3})A'_3$$

$$\alpha_{k1}A_1 + \alpha_{k2}A_2 + \alpha_{k3}A_3 = \alpha'_{k1}A'_1 + \alpha'_{k2}A'_2 + \alpha'_{k3}A'_3$$

$$K[\alpha_{k1}A_1p_1 + \alpha_{k2}A_2p_2 + \alpha_{k3}A_3p_3] = -K'[\alpha'_{k1}A'_1p'_1 + \alpha'_{k2}A'_2p'_2 + \alpha'_{k3}A'_3p'_3]$$

$$(\mu_L)^*_k[A_1(\beta_{k1}p_1^2 - 2p_1 - \beta_{k1}) + A_2(\beta_{k2}p_2^2 - 2p_2 - \beta_{k2}) + A_3(\beta_{k3}p_3^2 - 2p_3 - \beta_{k3})]$$

$$= (\mu_L)^{*'}_k[A'_1(2p'_1 + \beta'_{k1}p'^2_1 - \beta'_{k1}) + A'_2(2p'_2 + \beta'_{k2}p'^2_2 - \beta'_{k2}) + A'_3(2p'_3 + \beta'_{k3}p'^2_3 - \beta'_{k3})]$$

$$(\mu_T)^*_k C I_{k1} = -(\mu_T)^{*'}_k C' I'_{k1}$$

$$A_1[\lambda^*_k(1+p_1^2) + 2(\mu_T)^*_k p_1(p_1+\beta_{k1}) - \alpha^*_k(\beta_{k1}p_1 - 1) + \gamma^*_k\alpha_{k1}] + A_2[\lambda^*_k(1+p_2^2) + 2(\mu_T)^*_k p_2(p_2+\beta_{k2}) - \alpha^*_k(\beta_{k2}p_2 - 1) + \gamma^*_k\alpha_{k2}]$$

$$+A_3[\lambda^*_k(1+p_3^2) + 2(\mu_T)^*_k p_3(p_3+\beta_{k3}) - \alpha^*_k(\beta_{k3}p_3 - 1) + \gamma^*_k\alpha_{k3}] = A'_1[\lambda^{*'}_k(1+p'^2_1) + 2(\mu_T)^{*'}_k p'_1(p'_1-\beta'_{k1}) + \alpha^{*'}_k(\beta'_{k1}p'_1 + 1) + \gamma^{*'}_k\alpha'_{k1}]$$

$$+A'_2[\lambda^{*'}_k(1+p'^2_2) + 2(\mu_T)^{*'}_k p'_2(p'_2-\beta'_{k2}) + \alpha^{*'}_k(\beta'_{k2}p'_2 + 1) + \gamma^{*'}_k\alpha'_{k2}] + A'_3[\lambda^{*'}_k(1+p'^2_3) + 2(\mu_T)^{*'}_k p'_3(p'_3-\beta'_{k3}) + \alpha^{*'}_k(\beta'_{k3}p'_3 + 1) + \gamma^{*'}_k\alpha'_{k3}]$$

It follows from above equations that both C and C' vanish and hence there is no displacement in y-direction, thus no SH wave occur in this case.

The wave velocity equation is obtained by eliminating $A_1$, $A_2$, $A_3$, $A_1'$, $A_2'$, $A_3'$

The roots of the above equation determine the wave velocity of surface wave propagation along the common boundary between two thermovisco elastic solid media of Voigt type in the presence of gravity in fibre-reinforced medium. This equation will give the wave velocity of Stonely waves in presence of thermal, viscous and gravitational effects. In absence of these effects above equation will reduce to classical Stonely waves.

## CONCLUSION

The present study reveals the effect of temperature, viscosity, fibre-reinforcement and gravity are reflected in the wave velocity equations corresponding to the Stonely waves. So the result of this analysis seems to be useful in circumstances where these effects cannot be neglected.

## REFERENCES

1) Biot, M.A, Mechanics of Incremental Deformation, John Wiley, New York (1965)

2) Bullen, K.E., An Introduction to Theory of Seismology, Cambridge university Press

3) Love, A.E.H, Some problems of Geodynamics, Dover Publications, New York (1911)

4) Stonely, R., Proc. Roy. Soc. London A 106: 416-428 (1924)

5) P.R. Sengupta, Journal of applied mathematics and Stochastic Analysis, 1991.

6) Sethi M and Gupta K.C (2010). Surface waves in non-homogeneous, general visco-elastic media of higher order. Int. J. of appl. Math. and Mech. 6(20): pp. 50-65

7) Belfield , Rogers , A. Spencer, Stress in elastic plates reinforced by fibres lying in concrete circles , Journal of the Mechanics and Physics of Solids, 31 ,25(1983)