Effect of Radiation, Dissipation on Unsteady Convective Heat Transfer Flow of a Viscous Electrically Conducting Fluid through a Porous Medium in a Vertical Channel with Quadratic Density Temperature Variation

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Abstract:

We make an attempt to analyse the unsteady convective heat transfer of dissipative viscous electrically conducting fluid through a porous medium confined in a vertical channel on whose walls an oscillatory temperature is prescribed with quadratic density-temperature variation. Approximate solutions to coupled non-linear partial differential equations governing the flow and heat transfer are solved by a perturbation technique. The velocity, temperature, skin friction and rate of heat transfer are discussed for different variations of D^{-1}, M, α, γ, γ_1 and Ec.

Keywords: Heat Transfer, Vertical Channel, Oscillatory Temperature, quadratic density temperature variation

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INTRODUCTION

Flow through porous medium is very prevalent in nature and therefore the study of flow through porous medium has become of principal interesting in many scientific and engineering applications .In the theory of flow through a porous medium, the role of momentum equation or force balance is occupied by the numerous experimental observations summerised mathematically as the Darcy's law. It is observed that the Darcy's law is applicable as long as the Reynolds number based on average grain (pore)diameter does not exceed a value between 1 and 10.But in general the speed of specific discharge increases ,the convective forces get developed and the internal stress generated in the fluid due to its viscous nature produces distortion in the velocity field. Also in the case of highly porous media such as fiber glass, pappus dendilion etc., the viscous stress at the surface is able to penetrate into medium and produce gradient. Thus between the specific discharge and hydraulic gradient is inadequate in describing high speed flows or flows near surface which may be either permeable or not. Hence consideration for non-Darcian description for the viscous flow through a porous medium is warranted. Saffaman[25a]employing statistical method derived governing equation for the flow in a porous medium which takes into account the viscous stress. Later another modification has been suggested by Brinkman[3a]

$$0 = -\nabla p - (\frac{\mu}{k})\bar{v} + \mu\nabla^2\bar{v}$$

in which $\mu\nabla^2\bar{v}$ is intended to account for the distortion of the velocity profiles near the boundary. The same equation was derived analytically by Tam[32] to describe the viscous flow at low Reynolds number past a swam of small particles.

The process of free convection as a mode of heat transfer has wide applications in the fields of Chemical Engineering, Aeronautical and Nuclear power generation. It was shown by Gill and Casal [9a] that the buoyancy significantly affects the flow of a low Prandtal number fluids which is highly sensitive to gravitational force and the extent to which the buoyancy force influences a forced flow is a topic of interest. Free convection flows between two long vertical plates have been studied for many years because of their engineering applications in the fields of nuclear reactors, heat exchangers, cooling appliances in electronic instruments. These flow were studied by assuming the plates at two different constant temperatures or temperature of the plates varying linearly along the plates etc. The study of fully developed free convection flow between two parallel plates at constant temperature was initiated by Ostrach [19]. Combined natural and forced convection laminar flow with linear wall temperature profile was also studied by Ostrach [20]. The first exact solution for free convection in a vertical parallel plate channel with asymmetric heating for a fluid of constant properties was presented by Anug [2]. Many of the early works on free convection flows in open channels have been reviewed by Manca *et al.* [11]. Recently, Campo *et al.* [7] considered natural convection for heated iso-flux boundaries of the channel containing a low-Prandtl number fluid. Pantokratoras [21] studied the fully developed free convection flow between two asymmetrically heated vertical parallel plates for a fluid of varying thermophysical properties. However, all the above studies are restricted to fully developed steady state flows. Very few papers deal with unsteady flow situations in vertical parallel plate channels. Transient free convection flow between two long vertical parallel plates maintained at constant but unequal temperatures was studied by Singh *et al.*[28]. Jha *et al.* [10] extended the problem to consider symmetric heating of the channel walls. Narahari *et al.* [15] analyzed the transient free convection flow between two long vertical parallel plates with constant heat flux at one boundary, the other being maintained at a constant temperature. Singh and Paul [28] presented and analysis of the transient free convective flow of a viscous incompressible fluid between two parallel vertical walls occurring as a result of asymmetric heating / cooling of the walls. Narahari [16] presented an exact solution to the problem of unsteady free convective flow of a viscous incompressible fluid between two long vertical parallel plates with the plate temperature linearly varying with time at one boundary, that at the other boundary being held constant. There are many reasons for the flow to become unsteady. When the current is periodic due to on-off control mechanisms or due to partially rectified *a-c* voltage, there exist periodic heat inputs. Hence, it is important to study the effects of periodic heat flux on the unsteady natural convection, imposed on one of the plates of a channel formed by two long vertical parallel plates, the other being held at a constant initial fluid temperature. Recently Narahari[17]has discussed the unsteady free convection flow of dissipative viscous incompressible fluid between two long vertical parallel plates in which the temperature of one of the plates is oscillatory whereas that of the other plate is uniform.

In the context of space technology and in processes involving high temperatures, the effects of radiation are of vital importance. Recent developments in hypersonic flights, missile reentry, rocket combustion chambers, power plants for inter planetary flight and gas-cooled nuclear reactors have focused attention on thermal radiation as a mode of energy transfer and emphasize the need for improved understanding of radiative transfer in these processes. Mansour [12] studied the radiative and free convection effects on the oscillatory flow past a vertical plate. Raptis and Perdikis [24] considered the problem of thermal radiation and free convective flow past a moving plate. Das *et al.* [8] analyzed the radiation effects on the flow past an impulsively started infinite isothermal vertical plate. Prasad *et al.* [21a] considered the radiation and mass transfer effects on two dimensional flow past an impulsively started isothermal vertical plate. Chamkha *et al.* [4] studied the radiation effects on the free convection flow past a semi-infinite vertical plate with mass transfer..Raptis [25] analyzed the thermal radiation and free convection flow through a porous medium by using perturbation technique. Bakier and Gorla [3] investigated the effect of thermal radiation on mixed convection from horizontal surfaces in saturated porous media. Satapthy *et al.* [26] studied the natural convection heat transfer in a Darcian porous regime with Rosseland radiative flux effects. With regard to thermal radiation heat transfer flows in porous media, Chamkha [5] studied the solar radiation effects on porous media supported by a vertical plate. Forest fire spread also constitutes an important application of radiative convective heat transfer as described by Meroney [13]. More recently Chitraphiromsri and Kuznetsov [6] have studied the influence of high-intensity radiation in unsteady thermo-fluid transport in porous wet fabrics as a model of fire fighter protective clothing under intensive flash fires. Impulsive flows with thermal radiation effects and in porous media are important in chemical engineering systems, aerodynamic blowing processes and geophysical energy modeling. Such flows are transient and therefore temporal velocity and temperature gradients have to be included in the analysis. Raptis and Singh [23] studied numerically the natural convection boundary layer flow past an impulsively started vertical plate in a Darcian porous medium. The thermal radiation effects on heat transfer in magneto-aerodynamic boundary layers has also received some attention, owing to astronautical re-entry, plasma flows in astrophysics, the planetary magneto-boundary layer and MHD propulsion systems. Mosa [14] discussed one of the first models for combined radiative hydromagnetic heat transfer, considered the case of free convective channel flows with an axial temperature gradient. Nath *et al.* [18] obtained a set of similarity solutions for radiative – MHD stellar point explosion dynamics using shooting methods. Abd-El-Naby *et al.*[1] presented a finite difference solution of radiation effects on MHD unsteady free convection flow over a vertical porous plate.

In this chapter we make an attempt to analyse the unsteady convective heat transfer of dissipative viscous electrically conducting fluid through a porous medium confined in a vertical channel on whose walls an oscillatory temperature is prescribed with quadratic density-temperature variation. Approximate solutions to coupled non-linear partial differential equations governing the flow and heat transfer are solved by a perturbation technique. The velocity, temperature, skin friction and rate of heat transfer are discussed for different variations of D^{-1}, M, α, γ, γ_1 and Ec.

FORMULATION AND SOLUTION OF THE PROBLEM

We consider the non-Darcy unsteady flow of a viscous incompressible electrically conducting fluid through a porous medium in a vertical channel bounded by flat walls in the presence of constant heat sources. The unsteadiness in the flow is due to the oscillatory temperature prescribed on the boundaries. A uniform magnetic field of strength H0 is applied normal to the walls. Assuming the magnetic Reynolds number to be small we neglect the induced magnetic field in comparison to the applied field. We choose a Cartesian coordinate system 0(x y) with walls at $y = \pm 1$ by using Boussinesq approximation we consider the density variation only on the buoyancy term Also the kinematic viscosity υ,the thermal conductivity k are treated as constants. The equation governing the flow and heat transfer are

$$\frac{\partial u}{\partial t} = \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2} - (\frac{\mu}{k})u - (\frac{\sigma\mu_e^2 H_o^2}{\rho})u - \rho\bar{g} \quad (2.1)$$

$$\rho_0 C_p \frac{\partial T}{\partial t} = K_f \frac{\partial^2 T}{\partial y^2} + Q + 2\mu(u_y^2\ ) + \frac{\mu}{k}u^2 - \frac{\partial(q_R)}{\partial y} \quad (2.2)$$

$$\rho - \rho_0 = -\beta_0(T - T_0) - \beta_1(T - T_0)^2 \quad (2.3)$$

where u is a velocity component in x-direction, T is a temperature, p is a pressure, ρ is a density ,k is the permeability of the porous medium,μ is dynamic viscosity, k_f is coefficient of thermal conductivity, β is coefficient of volume expansion ,σ is the electrical conductivity ,μe is the magnetic permeability of the medium and Q is the strength of heat source

The boundary conditions are

$$\left.\begin{array}{l} u = 0,\ \ T = T_1 \text{ at } y = -L \\ u = 0,\ T = T_1 + \in(T_2 - T_1)\cos\omega t \end{array}\right\} \quad (2.4)$$

on introducing the non dimensional variables

$$y' = y/L, \qquad u' = \frac{u}{(\nu/L)} \qquad \theta = \frac{T - T_1}{T_2 - T_1}, \quad t' = \omega t,$$

Equations 2.1 & 2.2 reduce to (dropping the dashes)

$$\gamma_1^2 \frac{\partial u}{\partial t} = G(\theta + \gamma\theta^2) + \frac{\partial^2 u}{\partial y^2} - (D^{-1} + M^2)u \quad (2.5)$$

$$P\gamma_1^2 \frac{\partial\theta}{\partial t} = \frac{\partial^2\theta}{\partial y^2} + \alpha + PE_c u_y^2 + P\,E_c D^{-1}u^2 \quad (2.6)$$

where

$G = \beta g L^3 \frac{(T_2 - T_1)}{\gamma^2}$ (Grashof number), $M^2 = \frac{\sigma\mu_e^2 H_o^2 L^2}{\nu^2}$ (Hartmann Number)

$D^{-1} = \frac{L^2}{k}$ (Darcy parameter), $P = \frac{\mu C_P}{K_f}$ (Prandtl number)

$\alpha = \frac{Q.L^2}{(T_1 - T_2)K_f}$ (Heat source parameter), $Ec = \frac{\mu^2}{C_p L^2 (T_2 - T_1)}$ (Eckert Number)

$\gamma_1^2 = \frac{\omega L^2}{\nu}$ (Wormsely Number), $\gamma^2 = \frac{\beta_1(T_2 - T_1)}{\beta_0}$ (Density ratio),

$M_1^2 = M^2 + D^{-1}$

The transformed boundary conditions are

$$\left.\begin{array}{l} u = 0,\ \ \theta = 0,\ \ \text{at } y = -1 \\ u = 0,\ \ \theta = 1 + \in\cos(\omega t)\ \ \ \text{at } y = +1 \end{array}\right\} \quad (2.7)$$

In view of the boundary conditions (2.4) we assume

$$u = u_0 + \in e^{it} u_1$$
$$\theta = \theta_0 + \in e^{it}\theta_1 \quad (2.8)$$

Substituting the series expansion (2.8) in equations(2.5)&(2.6) and separating the steady and transient terms we get

$$\frac{\partial^2 u_0}{\partial y^2} - M_1^2 u_0 = -G(\theta_0 + \gamma\theta_o^2) \quad (2.9)$$

$$\frac{\partial^2 u_1}{\partial y^2} - (M_1^2 + i\gamma^2)u_1 = -G(\theta_1 + 2\theta_0\theta_1) \tag{2.10}$$

$$\frac{\partial^2 \theta_0}{\partial y^2} + \alpha_1 + P_1 E_c \frac{\partial^2 u_0}{\partial y^2} + P_1 E_c D^{-1} u_0^2 = 0 \tag{2.11}$$

$$\frac{\partial^2 \theta_1}{\partial y^2} - (iP\gamma^2)\theta_1 + (2P_1 Ec)\frac{\partial u_0}{\partial y}.\frac{\partial u_1}{\partial y} + (P_1 EcD^{-1})u_0 u_1 \tag{2.12}$$

Since the equations (2.9 – 2.12) are non-linear coupled equations., assuming Ec << 1 we take

$u_0 = u_{00} + Ec\, u_{01}$

$u_1 = u_{10} + Ec\, u_{11}$

$\theta_0 = \theta_{00} + Ec\, \theta_{01}$

$\theta_1 = \theta_{10} + Ec\, \theta_{11}$ (2.13)

Substitutimg (2.13) in equations (2.9) – (2.12) and separating the like terms we get

$u_{00}^{11} - M_1^2 u_{00} = -G(\theta_{00} + \gamma\theta_{00}\theta_{01})$, $u_{00}(\pm 1) = 0$ (2.14)

$\theta_{00}^{11} = -\alpha_1$, $\theta_{00}(-1) = 0$, $\theta_{00}(+1) = 1$ (2.15)

$u_{01}^{11} - M_1^2 u_{01} = -G(\theta_{01} + 2\theta_{00}\theta_{01})$, $u_{01}(\pm 1) = 0$ (2.16)

$\theta_{01}^{11} = -P_1 u_{00}^{12} - P_1 D^{-1} u_{00}^2$, $\theta_{01}(-1) = 0 = \theta_{01}$ (2.17)

$u_{10}^{11} - (M_1^2 + i\gamma^2)u_{10} = -G(\theta_{10} + 2\gamma\theta_{00}\theta_{10})$, $u_{10}(\pm 1) = 0$ (2.18)

$\theta_{10}^{11} - iP_1\gamma_1^2\theta_{10} = 0$, $\theta_{10}(-1) = 0$, $\theta_{10}(+1) = 1$ (2.19)

$u_{11}^{11} - (M_1^2 + i\gamma_1^2)u_{11} = -G\theta_{11}$, $u_{11}(\pm 1) = 0$ (2.20)

$\theta_{11}^{11} - (iP_1\gamma_1^2)\theta_{11} = -2P_1 u_{00}^1 u_{10}^1 - 2P_1 D^{-1} u_{00}u_{10}$, $\theta_{11}(\pm 1) = 0$ (2.21)

Solving the equations (2.14)-(2.21) subject to the relevant boundary conditions we obtain

$$\theta_{oo} = \frac{\alpha_1}{2}(1 - y^2) + 0.5(y+1)$$

$$u_{00} = a_8\left(1 - \frac{Ch(M_1 y)}{Ch(M_1)}\right) + a_{10}\left(y^2 - \frac{Ch(M_1 y)}{Ch(M_1)}\right) - a_{12}\left(y^4 - \frac{Ch(M_1 y)}{Ch(M_1)}\right) + \\ + a_9\left(y - \frac{Sh(M_1 y)}{Sh(M_1)}\right) - a_{11}\left(y^3 - \frac{Sh(M_1 y)}{Sh(M_1)}\right)$$

$$\theta_{01} = a_{51}y^2 + a_{52}y^3 + a_{53}y^4 + a_{54}y^5 + a_{55}y^6 + a_{56}y^7 + a_{57}y^8 + \\ + a_{58}y^9 + a_{59}y^{10} + a_{60}Ch(2M_1 y) + a_{61}Sh(2M_1 y) + \\ + (a_{62} + ya_{64} + y^2 a_{66} + y^3 a_{68} + y^4 a_{70})Sh(M_1 y) + \\ + (a_{63} + ya_{65} + y^2 a_{67} + y^3 a_{69} + y^4 a_{71})Ch(M_1 y) + \\ + a_{72}y + a_{73}$$

$$u_{01} = b_1(1-\frac{Ch(M_1y)}{Ch(M_1)}) + b_2(y-\frac{Sh(M_1y)}{Sh(M_1)}) + b_3((y^2-\frac{Ch(M_1y)}{Ch(M_1)}) +$$

$$+b_4((y^3-\frac{Sh(M_1y)}{Sh(M_1)}) + b_5(y^4-\frac{Ch(M_1y)}{Ch(M_1)}) + b_6(y^5-\frac{Sh(M_1y)}{Sh(M_1)}) +$$

$$+b_7(y^6-\frac{Ch(M_1y)}{Ch(M_1)}) + b_8((y^7-\frac{Sh(M_1y)}{Sh(M_1)}) + b_9(y^8-\frac{Ch(M_1y)}{Ch(M_1)}) +$$

$$+b_{10}(y^9-\frac{Sh(M_1y)}{Sh(M_1)}) + b_{11}(y^{10}-\frac{Ch(M_1y)}{Ch(M_1)}) + b_{12}(y^{11}-\frac{Sh(M_1y)}{Sh(M_1)}) +$$

$$+b_{13}((y^{12}-\frac{Ch(M_1y)}{Ch(M_1)}) + b_{14}(Ch(2M_1y) - Ch(2M_1)) + b_{15}(Sh(2M_1y) -$$

$$-ySh(2M_1)) + (b_{16}+yb_{18})(ySh(M_1y) - Sh(M_1)) + b_{17}y(Ch(M_1y) -$$

$$-Ch(M_1)) + (b_{19}+yb_{21})(y^2Ch(M_1y) - Ch(M_1)) + b_{20}(y^3Sh(M_1y -$$

$$-Sh(M_1) + b_{22}y(y^3Sh(M_1y) - Sh(M_1)) + b_{23}(y^4Ch(M_1y) - Ch(M_1)) +$$

$$+b_{24}(y^5Sh(M_1y) - Sh(M_1)) + b_{25}y(y^4Ch(M_1y) - Ch(M_1)) +$$

$$+b_{26}y(y^5Sh(M_1y) - Sh(M_1))$$

$$\theta_{10} = 0.5(\frac{Ch(\beta_2y)}{Ch(\beta_2)} + \frac{Sh(\beta_2y)}{Sh(\beta_2)})$$

$$u_{10} = b_{45}Ch(\beta_3y) + b_{46}Sh(\beta_3y) + \phi_4(y)$$

$$\phi_4(y) = (b_{39}+yb_{42}+y^2b_{44})Sh(\beta_2y) + (b_{40}+yb_{41}+y^2b_{43})Ch(\beta_2y)$$

where $a_1, a_2, \ldots\ldots\ldots\ldots a_{73}, b_1, \ldots\ldots\ldots\ldots\ldots b_{43}$ are constants given in the Appendix.

NUSSELT NUMBER

Knowing the temperature field (θ)the rate of heat transfer at y=±1 in the non-dimensional form are given by

$(Nu)_{y=\pm1} = (\frac{d\theta}{dy})_{y=\pm1}$ and the corresponding expressions are

$(Nu)_{y=+1} = b_{47} + Ecb_{49} + \varepsilon e^{it}b_{51}$ $\quad$ $(Nu)_{y=-1} = b_{48} + Ecb_{50} + \varepsilon e^{it}b_{52}$

where $b_{44},, b_{45}, \ldots\ldots\ldots., b_{52}$ are constants.

DISCUSSION OF THE NUMERICAL RESULTS:

We analyse the effect of convective heat transfer flow of a viscous, electrically conducting fluid through a porous medium in a vertical channel with walls maintained at an oscillatory temperature. In this analysis we consider non-linear density-temperature variation.

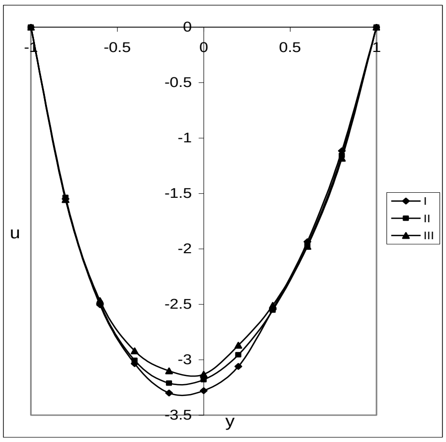

Fig.1 Variation of u with D^{-1}
$G=10^3, M=2, \alpha=2, \gamma=2, \gamma_1=2, P=0.71, E_c=0.004$
I II III
D^{-1} 10^2 $2x10^2$ $3x10^2$

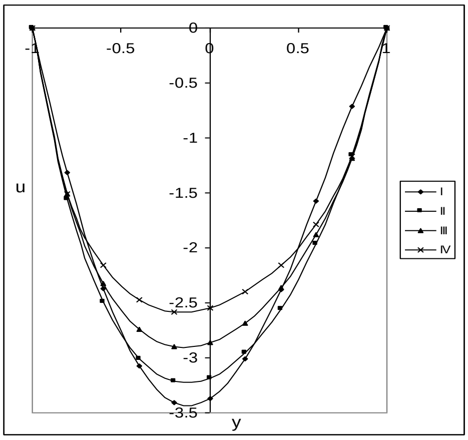

Fig.2 Variation of u with M
$G=10^3, D^{-1}=10^2, \alpha=2, \gamma=2, \gamma_1=2, P=0.71, E_c=0.004$
I II III IV
M 2 4 6 10

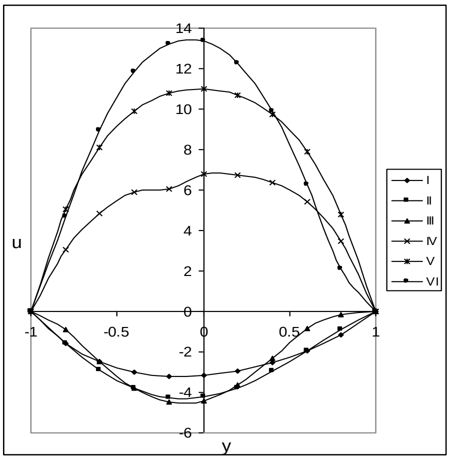

Fig.3 Variation of u with α
$G=10^3, M=2, \alpha=2, \gamma=2, \gamma_1=2, P=0.71, E_c=0.004$
I II III IV V VI
α 2 4 6 -2 -4 -6

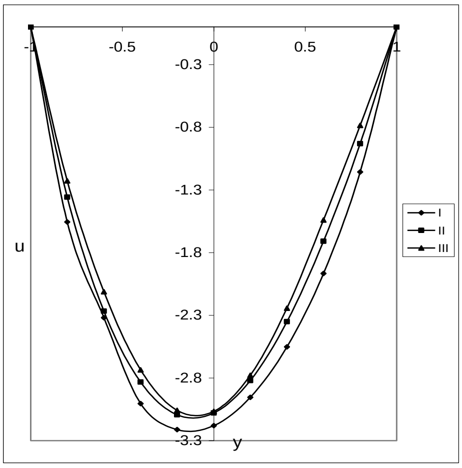

Fig.4 Variation of u with γ
$G=10^3, M=2, \alpha=2, \gamma_1=2, P=0.71, E_c=0.004$
I II III
γ 2 4 6

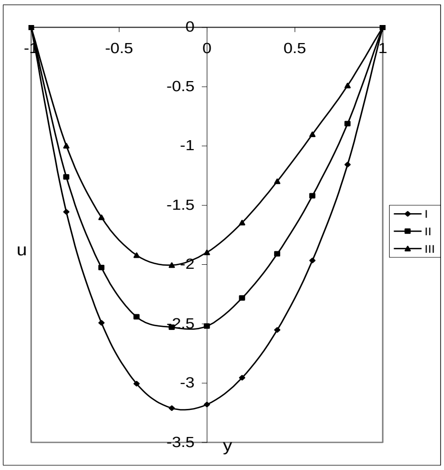

Fig.5 Variation of u with γ_1
$G=10^3, M=2, \alpha=2, \gamma=2, P=0.71, E_c=0.004$
I II III
γ_1 0.2 0.4 0.6

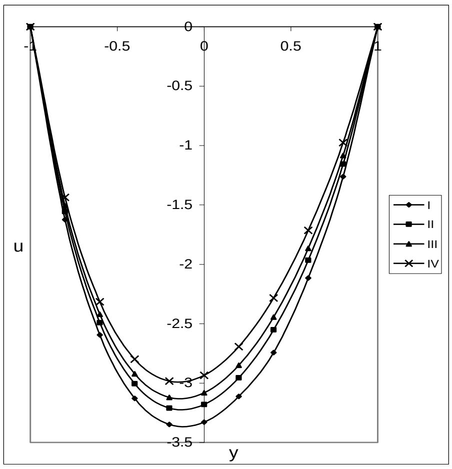

Fig.6 Variation of u with Ec
$G=10^3, M=2, \alpha=2, \gamma=2, \gamma_1=2, P=0.71$
I II III IV
E_c 0.001 0.004 0.006 0.009

The variation of u with Darcy parameter D^{-1} shows that lesser the permeability of the porous medium smaller |u| in the flow region and for further lowering of the permeability lesser the magnitude of u except in the vicinity of $y = \pm 1$ (fig.1). With respect to Hartman number M we find that higher the Lorentz force larger |u| in the flow resign and for further higher Lorentz force lesser |u| in the flow region (fig.2). The influence of heat sources on u is shown in fig4. It is noticed that |u| enhances with increase in $\alpha \leq 4$ and reduces with $\alpha \geq 6$ while in the case of heat sink, |u| enhances with $|\alpha|$ except in the vicinity of the $y= \pm 1$(fig 3) .An increase in the wormsely number γ results in an enhancement in |u| in the entire flow region (fig4). The effect non-linear density temperature variation on u is shown in (fig5). We find that axial velocity experiences an enhancement with increase in the density ratio γ. Thus the non-linearity in the density-temperature variation leads to an increase in the magnitude of the axial velocity .The affect of dissipation on u is exhibited in fig7. It is found that higher the dissipative heat lesser |u| in flow region (fig6).

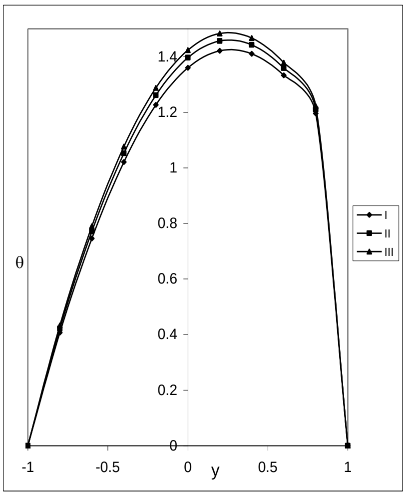

Fig.7 Variation of θ with D^{-1}
$G=10^3, M=2, \alpha=2, \gamma=2, \gamma_1=2, P=0.71, E_C=0.004$

	I	II	III
D^{-1}	10^2	$2x10^2$	$3x10^2$

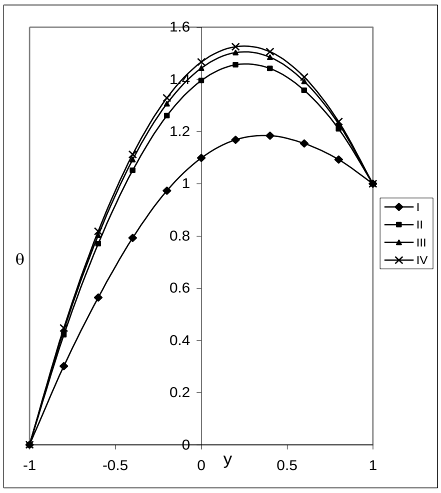

Fig.8 Variation of θ with M
$G=10^3, \alpha=2, \gamma=2, \gamma_1=0.2, P=0.71, E_C=0.004$

	I	II	III	IV
M	2	4	6	10

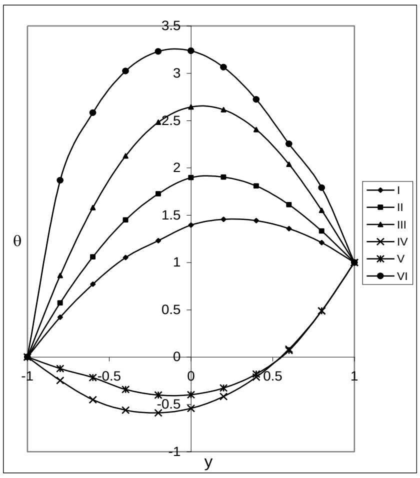

Fig.9 Variation of θ with α
$G=10^3, M=2, \gamma=2, \gamma_1=0.2, P=0.71, E_c=0.004$

	I	II	III	IV	V	VI
α	2	4	6	-2	-4	-6

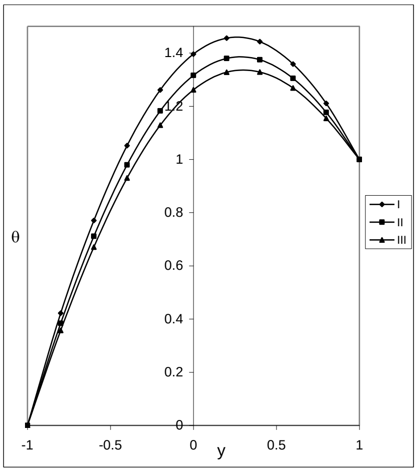

Fig.10 Variation of θ with γ
$G=10^3, M=2, \alpha=2, \gamma_1=0.2, P=0.71, E_C=0.004$

	I	II	III
γ	2	4	6

Lesser the permeability of the porous medium/higher the Lorentz force larger the actual temperature (fig.7&8). Fig.9 represents θ with heat source parameter α. It is found that the actual temperature enhances with increase in the strength of the heat source sink everywhere in the fluid region. It is found from fig.10 that higher the density ratio γ lesser the actual temperature and for further higher γ_1 larger that actual temperature (fig11). The effect of dissipation on θ is shown in fig.12. It is observed that higher the dissipative heat lesser the actual temperature.

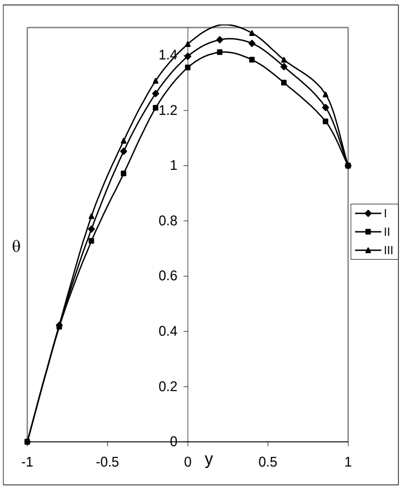

Fig.11 Variation of θ with γ_1
$G=10^3$, $M=2$, $\alpha=2$, $\gamma=2$, $P=0.71$, $E_C=0.004$

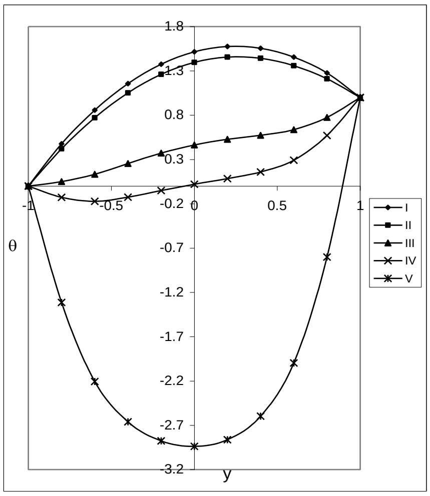

Fig.12 Variation of θ with Ec
$G=10^3$, $M=2$, $\gamma=2$, $\gamma_1=0.2$, $E_c=0.004$

The Nusselt number (Nu) at y =±1 is shown in tables 1-8 for different parametric values. The rate of heat transfer experiences an enhancement at both the walls with increase in |G|. Lesser the permeability of the porous medium higher the Lorentz force larger |Nu| for G>0 and lesser for G<0 at y = ±1. The variation of Nu with heat source parameter α shows that the Nusselt number enhances with increase in the strength of the heat source/sink at both the walls. The variation of Nu with γ and γ_1 shows that the rate of heat transfer deprecates with increase in the density ratio Υ_1 for G>0 and enhances for G<0. Thus the quadrate density temperature relation leads to on enhancement in the cooling case at y = ±1. Also it reduces with Wormsely number γ for G>0 and enhances for G<0. The variation of Nu with Eckert number Ec shows that higher the dissipative heat lesser in the heating case and larger in the cooling case at both the walls.

TABLE-1

NUSSELT NUMBER (Nu) at y = +1

G	I	II	III	IV	V
10^3	-1.30905	-1.35688	-1.44869	-0.24383	-0.85331
$2x10^3$	-1.29659	-1.38474	-1.57304	0.13037	-0.75339
$-1 x10^3$	-2.22663	-2.10338	-1.89928	-8.01374	-4.37822
$-2 x10^3$	-3.13175	-2.87774	-2.47422	-15.40945	-7.80321
D^{-1}	10^2	$2x10^2$	$3x10^2$	10^2	10^2
M	2	2	2	1.25	1.5

TABLE-2

NUSSELT NUMBER (Nu) at y = +1

G	I	II	III	IV	V	VI
10^3	-1.30905	-2.15987	-3.78633	2.85038	2.89569	-4.36126
$2x10^3$	-1.29659	0.61725	6.35543	3.99206	4.36053	-5.11468
$-1 x10^3$	-2.22663	-8.44526	-23.48902	2.80282	5.49176	7.11180
$-2 x10^3$	-3.13175	-11.95353	-33.04993	3.89693	9.55267	17.83145
α	2	4	6	-2	-4	-6

TABLE-3

NUSSELT NUMBER (Nu) at y = +1

G	I	II	III	IV	V
10^3	-1.30905	-1.12825	-1.00917	-1.33266	-1.31928
$2x10^3$	-1.29659	-0.77622	-0.31756	-1.40961	-1.35451
$-1 x10^3$	-2.22663	-2.72499	-3.28507	-2.07142	-2.14150
$-2 x10^3$	-3.13175	-3.96970	-4.86936	-2.88714	-2.99894
γ	2	4	6	2	2
γ_1	2	2	2	0.5	1.2

TABLE-4

NUSSELT NUMBER (Nu) at y = +1

G	I	II	III
10^3	-1.49791	-1.21126	-1.06458
$2x10^3$	-1.49774	-1.19258	-1.03656
$-1 x10^3$	-1.51083	-2.58763	-3.12914
$-2 x10^3$	-1.52358	-3.94532	-5.16567
P	0.01	0.71	0.71
E_C	0.04	0.06	0.09

TABLE-5

NUSSELT NUMBER (Nu) at y = -1

G	I	II	III	IV	V
10^3	2.25791	2.31662	2.42923	0.72211	1.60764
$2x10^3$	2.10267	2.24747	2.53994	-1.51792	0.69495
$-1 x10^3$	3.18640	3.09305	2.94022	8.14487	5.02233
$-2 x10^3$	3.95965	3.80034	3.56192	13.32760	7.52433
D^{-1}	10^2	$2x10^2$	$3x10^2$	10^2	10^2
M	2	2	2	1.25	1.5

TABLE-6

NUSSELT NUMBER (Nu) at y = -1

G	I	II	III	IV	V	VI
10^3	2.25791	2.88495	4.05090	-2.45127	-4.62510	-2.44344
$2x10^3$	2.10267	-0.76018	-7.79581	-4.64687	-9.79350	-10.18142
$-1 x10^3$	3.18640	8.69070	21.28157	-1.86411	-4.61694	-6.69643
$-2 x10^3$	3.95965	10.85131	26.6653	-3.47255	-9.77717	-18.68739
α	2	4	6	-2	-4	-6

TABLE-7

NUSSELT NUMBER (Nu) at y = -1

G	I	II	III	IV	V
10^3	2.25791	2.07316	1.93128	2.29411	2.27395
$2x10^3$	2.10267	1.59794	1.13609	2.24707	2.17643
$-1 x10^3$	3.18640	3.64160	4.13967	3.00619	3.08703
$-2 x10^3$	3.95965	4.73483	5.55288	3.67124	3.80258
γ	2	4	6	2	2
γ_1	2	2	2	0.5	1.2

TABLE-8

NUSSELT NUMBER (Nu) at y = -1

G	I	II	III
10^3	2.49739	2.13331	1.94641
$2x10^3$	2.49521	1.90045	1.59713
$-1\ x10^3$	2.51047	3.52604	4.03552
$-2\ x10^3$	2.52136	4.68593	5.77534
P	0.01	0.71	0.71
E_C	0.04	0.06	0.09

CONCLUSION:

An enhancement with increase in the density ratio γ. Thus the non-linearity in the density-temperature variation leads to an increase in the magnitude of the axial velocity. Higher the dissipative heat lesser |u| in flow region.

Higher the density ratio γ lesser the actual temperature and for further higher γ_1 larger that actual temperature. Higher the dissipative heat lesser the actual temperature.

The variation of Nu with Eckert number Ec shows that higher the dissipative heat lesser in the heating case and larger in the cooling case at both the walls

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