A New Heuristic Approach for Specially Structured Two Stage Flow Shop Scheduling to Minimize the Rental Cost, Processing Time, Associated With Probabilities Including Transportation Time, and Job Weightage

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Abstract

The present paper is attempt to develop a new heuristic algorithm, an alternative to the traditional algorithm proposed by johnson's (1954) to find the optimal sequence to minimize the utilization time of the machines and hence their rental cost for two stage specially structured flow shop scheduling under specified rental policy in which processing times are associated with probabilities including transportation time . Further jobs are attached with weights to indicate their relative importance. The proposed method is very simple and easy to understand and also provide an important tool for the decision maker. Algorithm is justified by numerical illustration.

Keywords: Specially structured flow shop scheduling, Rental Policy, Processing time, Weightage of jobs and Transportation time.

1. Introduction

Scheduling models deals with the determination of an optimal sequence in which to service customers or to perform a set of jobs, in order to minimize total elapsed time or another suitable measure of performance. Some widely studied classical models comprise single machine, parallel machine, flow shop scheduling, job shop scheduling, open shop scheduling etc. The objective of flow shop scheduling problem is to find a permutation schedule that minimizes the maximum completion time of a sequence. Scheduling has become a major field with in operation research with several hundred publications appearing each year. Johnson [9] first of all gave a method to minimize the make span for n-jobs, two machine

scheduling problems. Practically scheduling problem depends upon the significant factors namely, Transportation time, weight in jobs, break down effect, relative importance of a job over another job etc. These concepts were separately studied by Ignall and Schrage[8], Gupta Deepak[16], Singh T.P.[14] Maggu & Das [11] Yoshida & Hitomi [18] etc.. In a flow shop scheduling each job has the same routing throw machines and the sequence of operations is fixed. In a specially structured flow shop scheduling the data is not merely random but bears a well defined structural relation. Gupta J.N.D. [7] gave an algorithm to find the optimal schedule for specially structured flow shop scheduling. for specially structured flow shop scheduling. Gupta [6] studied specially structured flow shop problem to minimize the rental cost of the machine under predefined rental policy in which the probabilities have been associated with processing time. Yoshida and Hitomi [18] further considered the problem with set up time. The basic concept of equivalent job for a job block has been introduced by Maggu & Das [11]. Singh T.P. and Gupta Deepak [15] studied the optimal two stage production schedule in which processing time and set up time both were associated with probabilities including job block criteria. . Miyazaki [20] associated weights with the jobs. The transportation times (loading time, moving time and unloading etc.) from one machine to another are also not negligible and therefore must be included in the job processing. However, in some application, transportation time have major impact on the performance measures considered for the scheduling problem so they need to considered separately.
Gupta & Singla [19] studied 2-stage specially structured flow shop problem to minimize rental cost under the pre-defined rental policy with job weightage. This paper is an attempt to extend the study made by Gupta & Singla (2012) by introducing transportation time.
Thus the problem discussed in this paper become wider and very close to practical situation in manufacturing/ process industry. We have obtained an algorithm which gives minimum possible rental cost while minimizing total utilization time.

2. Practical Situation

The practical situation of specially structured flow shop scheduling, in a readymade garment manufacturing plant which has mainly two machines. viz, cutting and sewing , in which the time taken by the 2^{nd} machine(sewing machine) will always be grater then the time taken by first machine(cutting machine). Moreover different quality of garment are to be produced with relative importance i.e. weight of jobs become significant. In our day to day working in factories and industrial production concern different jobs are processed on various machines. These jobs are required to process in machines A,B,C,----- in a specified order. When the machine on which jobs are to be processed are planted at different places the transportation time (which include loading time, moving time, and unloading time etc.) has a significant role in production concern .Various practical situations occur in real life when one has got the assignment but does not have one’s own machine or does not have enough money to purchase machine. Under such circumstances the machine has to be taken on rent in order to complete the assignment. Rental of various equipments is an affordable and quick solution for a businessman, a manufacturer or a company, which presently constrained by the availability of limited funds due to recent global economic recession. Renting enables saving working capital, gives option for having the equipment and allows up-gradation to new technology.

3. Notations

S : Sequence of jobs 1, 2, 3,….,n
S_k : Sequence obtained by applying Johnson's procedure, k = 1, 2, 3, ------ r.
M_j : Machine j, j= 1,2.
a_{ij} : Processing time of i^{th} job on machine M_j
p_{ij} :Probability associated to the processing time a_i
A_{ij} : Expected processing time of i^{th} job on machine.
$t_{i1\rightarrow 2}$: Transportation time of i^{th} job on 1st machine to 2nd machine
$t_{ij}(S_k)$: Completion time of i^{th} job of sequence S_k on machine M_j
w_i : weight of i^{th} job.
G_i : weighted flow time of i^{th} job on machine M_1.
H_i : weighted flow time of i^{th} job on machine M_2.
$U_j(S_k)$: Utilization time for which machine M_j is required.
Cj : Renal cost per unit time of j^{th} machine.
$R(S_k)$: Total rental cost for the sequence S_k of all machine

4. Definition

Completion time of i^{th} job on machine M_j is denoted by t_{ij} and is defined as:
$t_{ij} = max\ (t_{i-1,,j},+\ ,\ t_{i,,j-1+}\ t_{i1\rightarrow 2})\quad :+A_{ij}\ ;\ \ j\geq 2.$
where A_{ij}=*Expected* processing time of i^{th} job on j^{th} machine.

5. Rental Policy (P)

The machines will be taken on rent as and when they are required and are returned as and when they are no longer required. i.e. the first machine will be taken on rent in the starting of the processing the jobs, 2nd machine will be taken on rent at time when 1st job is completed on the 1st machine.

6. Problem Formulation

Let some job i (i = 1,2,……..,n) are to be processed on two machines M_j (j = 1,2) under the specified rental policy P. Let A_{ij} be the expected processing time of i^{th} job on j^{th} machine. Let w_i be weight of the i^{th} job..And t_i be the transportation time of ith job from machine M_1 to M_2

Our aim is to find the sequence $\{S_k\}$ of jobs which minimize the rental cost of the machines while minimizing the utilization time of machines.

The mathematical model of the problem in matrix form can be stated as:

Jobs	MachineM_1			MachineM_2		Weight of jobs
i	a_{i1}	p_{i1}	t_i	a_{i2}	p_{i2}	w_i
1	a_{11}	p_{11}	5	a_{12}	p_{12}	w_1
2	a_{21}	p_{21}	3	a_{22}	p_{22}	w_2
3	a_{31}	p_{31}	6	a_{32}	p_{32}	w_3
-	-	-	-	-	-	-
5	a_{n1}	p_{n1}	4	a_{n2}	p_{n2}	w_n

Table -1

Mathematically, the problem is stated as:

Minimize $U_2(S_k)$ and hence

Minimize $R(S_k) = \sum_{i=1}^{n} A_{i1} \times C_1 + U_j(S_k) \times C_2$

Subject to constraint: Rental Policy (P).

i.e. our objective is to minimize utilization time of machine and hence rental cost of machines.

7. Theorem

If $A_{i1} \le A_{i2}$ *for all i, j, i ≠ j,* then $k_1, k_2 \ldots\ldots k_n$ is a monotonically decreasing sequence, where $K_n = \sum_{i=1}^{n} A_{i1} - \sum_{i=1}^{n-1} A_{i2}$.

Proof : Let $A_{i1} \le A_{j2}$ for all $i, j, i \ne j$
i.e., max $A_{i1} \le$ min A_{j2} for all $i, j, i \ne j$

Let $K_n = \sum_{i=1}^{n} A_{i1} - \sum_{i=1}^{n-1} A_{i2}$

Therefore, we have $k_1 = A_{11}$
Also $k_2 = A_{11} + A_{21} - A_{12} = A_{11} + (A_{21} - A_{12}) \le A_{11}$ $(\because A_{21} \le A_{12})$
$\therefore k_1 \le k_2$
Now, $k_3 = A_{11} + A_{21} + A_{31} - A_{12} - A_{22}$
$= A_{11} + A_{21} - A_{12} + (A_{31} - A_{22}) = k_2 + (A_{31} - A_{22}) \le k_2$ $(\because A_{31} \le A_{22})$
Therefore, $k_3 \le k_2 \le k_1$ or $k_1 \ge k_2 \ge k_3$.
Continuing in this way, we can have $k_1 \ge k_2 \ge k_3 \ge \ldots\ldots \ge k_n$, a monotonically decreasing sequence.

Corollary 1: The total rental cost of machines is same for all the sequences, if
$A_{i1} \le A_{i2},$ for all i, j, i ≠ j.

Proof: The total elapsed time $T(S) = \sum_{i=1}^{n} A_{i2} + k_1 = \sum_{i=1}^{n} A_{i2} + A_{11}$.

It implies that under rental policy P the total elapsed time on machine M_2 is same for all the sequences thereby the rental cost of machines is same for all the sequences.

8. Theorem

If $A_{i1} \ge A_{j2}$ for all *i, j, i ≠ j***,** then $K_1, K_2 \ldots\ldots K_n$ is a monotonically increasing sequence, where $K_n = \sum_{i=1}^{n} A_{i1} - \sum_{i=1}^{n-1} A_{i2}$.

Proof: Let $K_n = \sum_{i=1}^{n} A_{i1} - \sum_{i=1}^{n-1} A_{i2}$

Let $A_{i1} \ge A_{j2}$ for all $i, j, i \ne j$ i.e., min $A_{i1} \ge$ max A_{j2} for all $i, j, i \ne j$
Here $k_1 = A_{11}$
$k_2 = A_{11} + A_{21} - A_{12} = A_{11} + (A_{21} - A_{12}) \ge k_1$ $(\because A_{21} \ge A_{j2})$
Therefore, $k_2 \ge k_1$.
Also, $k_3 = A_{11} + A_{21} + A_{31} - A_{12} - A_{22} = A_{11} + A_{21} - A_{12} + (A_{31} - A_{22})$
$= k_2 + (A_{31} - A_{22}) \ge k_2$ $(\because A_{31} \ge A_{22})$
Hence, $k_3 \ge k_2 \ge k_1$.
Continuing in this way, we can have $k_1 \le k_2 \le k_3 \ldots\ldots \le k_n$, a monotonically increasing sequence.

Corollary 2: The total elapsed time of machines is same for all the possible sequences, if $A_{i1} \geq A_{j2}$ for all i, j, $i \neq j$.

Proof: The total elapsed time

$$T(S) = \sum_{i=1}^{n} A_{i2} + k_n = \sum_{i=1}^{n} A_{i2} + \left(\sum_{i=1}^{n} A_{i1} - \sum_{i=1}^{n-1} A_{i2} \right) = \sum_{i=1}^{n} A_{i1} + \left(\sum_{i=1}^{n} A_{i2} - \sum_{i=1}^{n-1} A_{i2} \right) = \sum_{i=1}^{n} A_{i1} + A_{n2}$$

Therefore total elapsed time of machines is same for all the sequences.

9. Assumptions

1. Jobs are independent to each other. Let n jobs be processed thorough two machines M_1 and M_2 in order M_1M_2
2. Machine breakdown is not considered.
3. Pre-emption is not allowed.
4. $0 \leq p_{i1} \leq 1$, $0 \leq p_{i2} \leq 1$, $\sum p_{i1} = 1$ and $\sum p_{i2} = 1$.
5. Weighted flow time has the following structural relation
   i.e. Either $G_i \geq H_i$
   or $G_i \leq H_i$ for all i

10. Algorithm

Step 1: Calculate the expected processing times, $A_{ij} = a_{ij} \times p_{ij}$;

Step 2: Compute $A'_{i1} = A_{i1} + t_i$

$A'_{i2} = A_{i2} + t_i$

Step 3: Calculate weighted flow time G_i & H_i as follow

If min $(A'_{i1}, A'_{i2}) = A'_{i1}$

Then $G_i = (A'_{i1} + w_i) / w_i$, $H_i = A'_{i2}/w_i$

And

If min $(A'_{i1}, A'_{i2}) = A'_{i2}$

Then $G_i = A'_{i1}/w_i$ $H_i = (A'_{i2} + w_i) / w_i$

Step 4: Check the structural relationship

Either $G_i \geq H_i$

or $G_i \leq H_i$, for all i

if the structural relation hold good go to Step 7 other wise modify the problem.

Step 5: If $J_1 \neq J_n$ then put J_1 on the first position and J_n as the last position and go to step 9 otherwise go to step 7.

Step 6: Take the difference of processing time of job J_1 on M_1 from job J_2 (say) having next maximum processing time on M_1 call this difference as G_i. also take the difference of processing time of job J_n on M_2 from job J_{n-1} (say) having next minimum processing time on M_2 . Call the difference as G_2.

Step 7: If $G_1 \leq G_2$ put J_n on the last position and J_2 on the first position otherwise put J_1 on 1^{st} position and J_{n-1} on the last position.

Step 8: Arrange the remaining (n-2) jobs between 1^{st} job & last job in any order, thereby we get the sequences $S_1, S_2 \ldots S_r$.

Step 9: Compute in - out table for any one (say S_1) of the sequence $S_1, S_2, \ldots S_n$.

Step 10: Compute the total completion time CT(S_k).
(S_k)
Step 11: Calculate utilization time U_2 of 2nd machine where
$U_2(S_1) = CT(S_k) – A_{i1}(S_1)$;
Step 12: Find rental cost

$$R(S_1) = \sum_{i=1}^{n} A_{i1}(S_1) \times C_1 + U_2(S_1) \times C_2$$

where C_1 & C_2 are the rental cost per unit time of 1st & 2nd machine respectively.

11. Numerical Illustration

Consider 5 jobs, 2 machines problem to minimize the rental cost. The processing times with probabilities ,transportation time t_i of ith job from machine M_1 to machine M_2 and weight in jobs w_i are given in the following table. Let β = (2,4) as equivalent job for job block (2,4). The rental cost per unit time for machines M_1 and M_2 are 10 units and 5 units respectively.

Jobs	MachineM_1			MachineM_2		Weight of jobs
i	a_{i1}	p_{i1}	t_i	a_{i2}	p_{i2}	w_i
1	140	0.2	5	90	0.2	1
2	160	0.3	3	110	0.1	2
3	130	0.2	6	70	0.2	3
4	180	0.2	2	80	0.2	1
5	220	0.1	4	50	0.3	2

Table :2

Solution : As per step 1: The expected processing time & expected set up times for machines M_1 and M_2 are as follow

Jobs	Machine M_1	$T_{i1\to2}$	Machine M_2	Weight of jobs
i	A_{i1}	t_i	A_{i2}	Wi
1	28.0	5	18.0	1
2	48.0	3	11.0	2
3	26.0	6	14.0	3
4	36.0	2	16.0	1
5	22.0	4	15.0	2

Table : 3

As per step 2: Expected flow time for two machines M_1 and M_2 as follow :

Jobs	Machine M_1	Machine M_2	Weight
i	A_{i1}'	A_{i2}'	w_i
1	33.0	23.0	1
2	51.0	14.0	2
3	32.0	20.0	3
4	38.0	18.0	1
5	26.0	19.0	2

Table : 4

As per step 3: Weighted flow time for machines M_1 and M_2 as follow :

Jobs	Machine M_1	Machine M_2
i	G_i	H_i
1	33.0	24.0
2	25.5	8.0
3	10.66	7.66
4	38.0	19.0
5	13.0	10.5

Table : 4

As per step 3: the new reduced problem become as under:

Here, $G_i \geq H_i$ for all *i*.
As per step 7 max G_i = 38.0 which is for job 4 i.e. $J_1 = 4$
And min H_i = 7.66 which is for job 3 i.e. $J_n = 3$.
Since $J_1 \neq J_n$. we put $J_1 = 4$ on the first position
And $J_n = 3$ on the last position
Therefore the optimal sequences are S_1= 4 –2- 1 – 5 – 3 .
S_2= 4 – 2 – 5 – 1 – 3, S_3= 4 −1−2−5−3, S_4= 4 −1−5−2−3, S_5= 4 −5−2−1−3, S_6= 4 −5−2−1−3,
Due our structural conditions the total elapsed time is same for all these 6 possible sequences S_1, S_2; S_3, S_4 S_5, S_6 say for S_1 = 4 – 2 – 1 – 5 – 3 is :

Jobs	Machine M_1	Machine M_2
i	In-Out	In-Out
4	0-36	38-54
2	36-84	87-98
1	84-112	117-135
5	112-134	138-153
3	134-160	166-180

Table : 5

Therefore, the total elapsed time = $CT(S_1)$ = 180 units
Utilization time of machine $M_2 = U_2(S_1)$ = 180 – 51
= 129 units

Also $\sum_{i=1}^{n} A_{i1}$ =160 units.

Therefore the total rental cost for each of the sequence (S_k); k = 1, 2 is
$R(S_k)$ = 160 × 10 + 129 × 5
= 1600 + 645
= 2245 units.

12.Remarks

a. If we solve the same problem by Johnson's methods we get the optimal sequence as S= 1 – 4 – 5 – 2 – 3.

The in – out flow table is:

Jobs	Machine M_1	Machine M_2
I	In - Out	In - Out
1	0-28	33-51
4	28-64	66-82
5	64-90	94-109
2	90-138	141-152
3	138-164	170-184

Table:6

Therefore, the total elapsed time = CT(S) = 184 units
Utilization time of machine $M_2 = U_2(S)$ = 151 units

Also $\sum_{i=1}^{n} A_{i1}$ =164 units.

Therefore the total rental cost is

$R(S_k)$ = 164 × 10 + 151 × 5
= 1640 + 755
= 2395 unit

b. If assumptions 5 and job weightage and transportation is not included then result tally with [6].

13.Conclusion

The algorithm proposed here for specially structured two stage flow shop scheduling problem in which processing time associated with probabilities including transportation time and job weightage is more efficient as compared to the algorithm proposed by Johnson (1954) to find an optimal sequence to minimize the utilization time of the machines and hence their rental cost. The study may further be extended by considering various parameters like breakdown effect, set up time etc.

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