FINITE ELEMENT ANALYSIS OF COMPOSITE SHELLS

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ABSTRACT

Composite materials and structures are finding wide acceptance because of their stiffness-to-weight ratio that is particularly favorable. This project deals with the bending analysis of composite doubly curved, laminated composite shells having spherical shells is presented & also compare results with cylindrical shells with the material properties graded in the thickness direction. The present formulation is based on the stress resultant type Koiter's shell theory. The curvilinear coordinates are employed to describe the basic shell equations. Transverse shear deformation is also considered according to Mindlin's hypotheses which is also known as the First order Shear Deformation Theory (FSDT). Both thick and thin shell panels have been solved. A four- noded quadrilateral shell element for geometric and material linear analysis is presented. Four noded element with five degrees of freedom per node, three translational and two rotations have been used. The effect of grading on the deformation of the composite shells in a given boundary conditions has been studied. Composite shell panels having different types of transverse loads, shell thickness (a/h = 0.01 and 0.1), curvature to length ratios (R/a), ply orientation (0/90)n and boundary conditions have been analyzed .

KEY WORDS: Composite materials, doubly curved shells, FEA, FSDT.

I. INTRODUCTION

The shell type structures are known as most desired structural elements in the modern construction engineering, aircraft construction, ship building, rocket construction, nuclear, aerospace as well as petrochemical industries.. Therefore, with the development of finite element method a large amount of resources have been devoted to the innovative analytical tools in computational shell mechanics. The requirements for higher strength-to-weight ratios, better corrosion resistance, longer fatigue life, greater stealth characteristics over metals as well as the directional properties, have resulted in increasing demand for laminated composite structures in many challenging applications. A number of stress resultant theories [1,2,5,13,16] are now available for thin elastic shells, in linear domain, based on the Love –

kirchoff assumptions, but each theory is different since it either neglects or approximates one or more terms. Amongst them , Koiter's shell theory [17] has been found to satisfying following necessary invariants i.e strain equations remain

invariant under the transformation of the middle surface coordinates & also remain unaffected by any arbitrary rigid-motion body. Thus Koiter's shell theory may be considered to be appropriate, which has followed in the present investigation. In the recent days, application of fibre reinforced laminated composite materials have become popular in the construction of structural element requiring high strength, stiffness & dimensional stability. & Also the anisotropy leads to linear elastic coupling among bending, torsion, extension & shear deformations which is utilized to tailor the structural stiffness & strength according to the desired requirement. However, laminated shells made of advanced filamentary composite materials are susceptible to thickness effects because the ratios of elastic moduli to shear moduli very large. For this material transverse shear deformation plays a very important role in the reducing the effective flextural stiffness. Both twist & normal curvature components are incorporated to keep the strain equations complete. The present five DOF shell formulation is kept sufficiently general to capture both the bending –dominated & membrane-dominated situations. Though numerous theoretical models have been developed and applied to various practical circumstances, no single theory has proven to be general and comprehensive enough for the entire range of applications.

Composite materials

Composite materials are engineered materials made from two or more constituent materials with significantly different physical or chemical properties which remain separate and distinct on a macroscopic level within the finished structure. They have varied applications including army and aerospace vehicles, nuclear reactor vessels, turbines, buildings, smart highways as well as in sports equipment and medical prosthetics. Our area of interest are Laminated composite structures consisting of several layers of different fibre -reinforced laminate bonded together to obtain desired structural properties (e.g. stiffness, strength, wear resistance, damping, etc). Varying the lamina thickness, lamina material properties, and stacking sequence the desired structural properties can be achieved. Composite materials exhibit high strength-to weight and stiffness-to-weight ratios, which make them ideally suited for use in weight sensitive structures. This weight reduction of structures leads to improvement of their structural performance especially in aerospace applications. Composite materials have been widely applied to various fields of work as the replacement of traditional monolithic materials such as metals, ceramics or polymers due to its advantages and better performances. Generally, composite are materials that combine two or more conventional monolithic materials into one. Such a material contains the characteristics of both its origin materials and other new characteristics that might be useful.

Composites can be divided into various groups depends on their criterion such as metals and non-metals, natural or manufactured, usage, and application. The most primitive composites that can be obtained easily from market are brick for

The major steps in the finite element analysis of a typical problem are :

1. Discretization of the domain into a set of finite elements (mesh generation).
2. Weighted-integral or weak formulation of the differential equation over a typical finite element (subdomain).
3. Development of the finite element model of the problem using its weightedintegral or weak form. The finite element model
consists of a set of algebraic equations among the unknown parameters of the element.
4. Assembly of finite elements to obtain the global system (i.e., for the total problem) of algebraic equations.
5. Imposition of boundary conditions.
6.Solution of equations.
7.Post-computation of solution and quantities of interest.

The above steps of the finite element method make it a modular technique that can be implemented on a computer, independent of the shape of the domain and boundary conditions. In addition, the method allows coupling of various physical problems because finite elements based on different physical problems can be easily generated in the same computer program. In this chapter, we develop finite element models of the linear equations governing laminated composite shells. The objective is to introduce the reader to the finite element formulations of laminated composite structures. While the coverage is not exhaustive in terms of solving complicated problems, for this is primarily a textbook, it helps the reader in gaining an understanding of the plate and shell finite elements used in the analysis of practical problems. It is important to note that any numerical or computational method is a means to analyze a practical engineering problem and that analysis is not an end in itself but rather an aid to design. Even to develop proper input data to a computer program requires a good understanding of the underlying theory of the problem as well as the method on which the program is based.

First order shear deformation Theory

The classical laminate shell theory is based on the Kirchhoff Love hypothesis, in which transverse normal and shear stresses are neglected. Although such stresses can be post computed through 3-D elasticity equilibrium equations, they are not always accurate. The equilibrium-derived transverse stress field is sufficiently accurate for homogeneous and thin shell; they are not accurate when shell are relatively thick (i.e., $a/h < 20$). In the first-order shear deformation theory (FSDT), a constant state of transverse shear stresses is accounted for, and often the transverse normal stress is neglected. The FSDT allows the computation of interlaminar shear stresses through constitutive equations, which is quite simpler than deriving them through equilibrium equations. It should be noted that the interlaminar stresses derived from constitutive equations do not match, in general, those derived from equilibrium equations. In fact, the transverse shear stresses derived from the equilibrium equations are quadratic through lamina thickness, for CLPT, whereas those computed from constitutive equations are constant. The more significant difference between the classical and first-order theories is the effect of including transverse shear deformation on the predicted deflections, frequencies, and buckling loads. the classical laminate theory under predicts deflections and overpredicts frequencies as well as buckling loads with plate side-to-thickness ratios of the order of 20 or less. For this reason alone it is necessary to use the first-order theory in the analysis of relatively thick laminated shells. We develop analytical solutions of rectangular laminates using the first-order shear deformation theory. The primary objective is to bring out the effect of shear deformation on deflections, stresses, frequencies, and buckling load.

II.LITERATURE REVIEW

Fiber composite materials are increasingly used in variety of systems, such as aircrafts and submarine structures, space structures, automobiles, sport equipment, medical prosthetic devices and electronic circuit boards. It is materially efficient in applications that required high strength to weight and stiffness to weight ratios. With the increasing use of fiber reinforced composites in structures components, studies involving the behavior made of composites are getting considerable attention. The analytical study and design of composites requires knowledge of anisotropic elasticity, structural theories and failure mode.

Finite Element (FE) Method is one of the preferable methods used in the analysis of the structural and mechanical behavior of materials, especially when the researchers are dealing with the non-linear structure or a complex structure which cannot be analyzed with merely hand calculation or even a computer program if the structure is not properly formulated. It is considerably powerful numerical techniques devised for solving solid, structural mechanics, and even multidisciplinary problems in geometrically complicated regions.

Shells are common structural elements in many engineering structures, including pressure vessels, submarine hulls, ship hulls, wings and fuselages of airplanes, pipes, exteriors of rockets, missiles, automobile tires, concrete roofs, containers of liquids, and many other structures. The theory of laminated shells includes the theories of ordinary shells, flat plates, and curved beams as special cases. Therefore, in the present study, we consider the theory of laminated composite shells. A number of theories exist for layered anisotropic shells. Many of these theories were developed originally for thin shells and are based on the Kirchhoff-Love [1] kinematic hypothesis that straight lines normal to the undeformed midsurface remain straight and normal to the middle surface after deformation. Other shell theories can be found in the works of Naghdi [4,9] and a detailed study of thin isotropic shells can be found in the monographs by Ambartsumyan [3}, Fliigge [6] and Kraus [12]. The first-order shear deformation theory of shells, also known as the Sanders shell theory [5] can be found in Kraus [12].

The first analysis that incorporated the bending-stretching coupling (owing to unsymmetric lamination in composites) is due to Ambartsumyan [3]. In his analyses, Ambartsumyan assumed that the individual orthotropic layers were oriented such that the principal axes of material symmetry coincided with the principal coordinates of the shell reference surface. Thus, Ambartsurnyan's work dealt with what is now known as laminated orthotropic shells, rather than laminated anisotropic shells; in laminated anisotropic shells, the individual layers are, in general, anisotropic, and the principal axes of material symmetry of the individual layers coincide with only one of the principal coordinates of the shell (the thickness normal coordinate).

Dong, Pister, and Taylor [7] formulated a theory of thin shells laminated of anisotropic material that is an extension of the theory developed by Stavsky [8] for laminated anisotropic plates to Donnell's shallow shell theory (see Donne [2]) Cheng and Ho [11] presented an analysis of laminated anisotropic cylindrical shells using Fliigge's shell theory [6]. A shell theory for the unsymmetric deformation of no homogeneous, anisotropic, elastic cylindrical shells was derived by Widera and Churig [14] by means of the asymptotic integration of the elasticity equations. For a homogenous, isotropic material, the theory reduces to Donnell's equations. All of the theories listed above are based on Kirchhofi-Love's hypotheses, in which the transverse shear deformation is neglected. These theories, known as the Love's first-approximation theories (see Love [1]), are expected to yield sufficiently accurate results when (1) the radius-to-thickness ratio is large, (2) the dynamic excitations are within the low-frequency range, and (3) the material anisotropy is not severe. However, the application of such theories to layered anisotropic composite shells could lead to 30% or more errors in deflections, stresses, and frequencies. Reddy [17] presented a generalization of the Sanders shell theory [5] to laminated, doubly-curved anisotropic shells. The theory accounts for transverse shear strains and the von Karman (or Sanders J.L) nonlinear strains. For additional references and applications to composite shells, see Bert [15,16].

III.OBJECTIVES & MATHEMATICAL FORMULATION

Objectives

The objective of the present work has been to:

- Develop a finite element modeling for doubly curved shells, spherical shells & cylindrical shell subjected to a uniform distributed loading & sinusoidal loading integrated with layers of composite shell on its surface.
- Develop a matlab program to determine the bending analysis of a composite shells
- To analyze and compare the results between three different composite shells
- Validate the matlab codes

MATHEMATICAL FORMULATION

This chapter explains the flow and the methods that are used in my research. Beside the general formulae used in deriving the basic composite parameters, the finite element method will also be applied in my study beginning with the formulation stage. Figure 1 and 2 show the flows of the research in brief. This project develops FEM models of a simply supported, anti-symmetric, composite laminate plate under a uniformly distributed load and sinusoidal load. The first step is to develop an FEA model of a composite shell that consists of number layers. The model will be subjected to uniformly and sinusoidally distributed load. Deflection results will be analyzed.

Shell Element

A doubly curved laminated composite shallow shell of uniform thickness h made of homogeneous linearly elastic material is considered. The radii of curvature of the shell along x & y directionsare R_x & R_y respectively and the twist radius of curvature R_{xy}. The projection of shell on the xy plane is rectangle of dimension of a & b

Assumptions in the model development

In developing the working model we have taken into account some assumptions they are:

- Composite shell is assumed to be graded in thickness direction

- The material used for the shell is Linear & elastic.
- The deformations follow Mindlin's hypothesis, i.e. normal to the middle surface of the shell before deformation may not remain normal after deformation but remains straight and inextensional.
- The in-plane displacement components are assumed to vary linearly along the thickness direction to yield constant transverse shear strain.

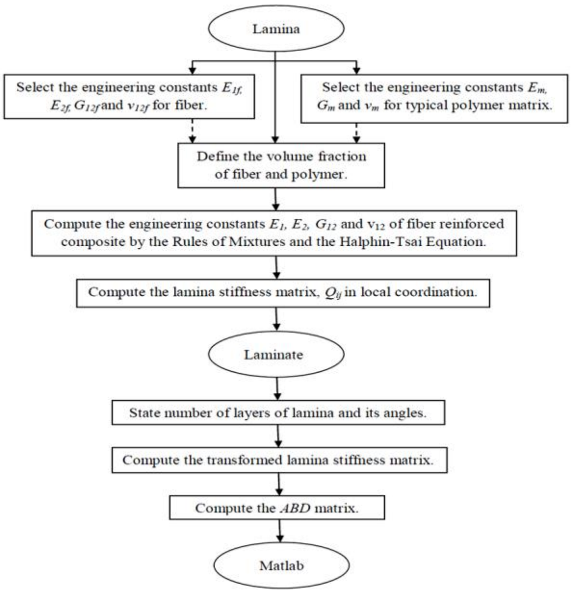


Fig 1. Definition of the *ABD* matrix

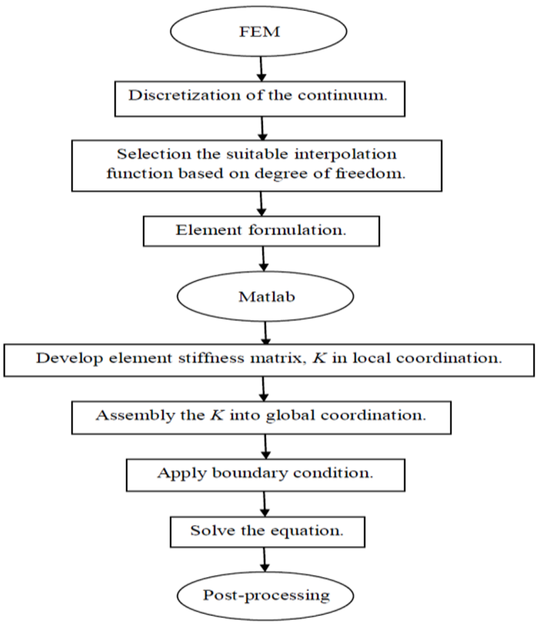


Fig 2. Finite element formulation steps.

Displacement field

The displacement field assumed for the composite shell element is based on the first order shear deformation theory. The following simplifying assumptions are used, which provide a reasonable description of the behavior of thin elastic shells:

(1) The thickness of shell is small compared to the radii of curvature (h/R_x, h/R_y<<1)

(2) The transverse normal stress theory is negligible.

(3) Normal to the middle plane of the shell before deformation remains straight but not necessarily normal after deformation (a relaxed form of Kirchoff-Love's hypothesis). The displacements U, V and W at any point (x,y,z) are given by

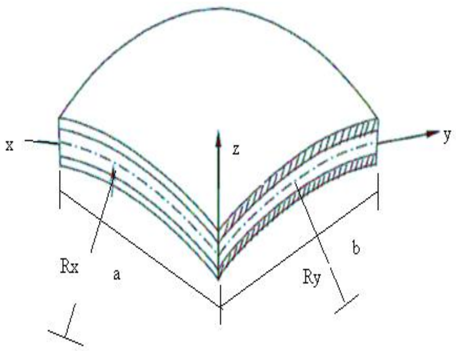


Fig 3. Doubly curved laminated composite shell element.

IV.RESULT AND DISCUSSION

Matlab

In this stage, we are required to define all the values of the variables in order to run the program and perform the analysis. As mentioned before, the program used here is MATLAB. Matlab is a fourth generation computer program which is used widely for a matrix operation. The word 'Matlab' stand for matrix laboratory. As usual, Matlab contains toolbar with normal function like file, edit, debug, desktop, window and help. Even though Matlab is a program designed for the matrix operation, it provides others functions such as plotting of functions and data, implementation of algorithms, creation of user interfaces, and interfacing with programs written in other languages including C, C++, and FORTRAN. Moreover, Matlab is an advanced computer program of Maple which is also a programming tool used for defining the mathematics related problems including the matrix. Nowadays, Matlab is still not a common or famous programming tool for public use. It is only famous in certain fields such as mathematics, researches, program development, etc. It is because the Matlab has some limitations and specifications for general users.

Boundary conditions

The two boundary conditions used in the present investigation are

(1) Simply supported:

(a) Cross ply shell panel

v=w=θ_y = 0 at x= 0& a

u=w= θx= 0 at y= 0 & b

(b) Angle ply shell panel

u=w= θ_y = 0 at x= 0 & a

v=w= θx = 0 at y= 0 & b

(2) Clamped:

(3) u=v=w= θx = θ_y = 0 at x=0 & a

u=v=w= θx = θ_y = 0 at y=0 & b

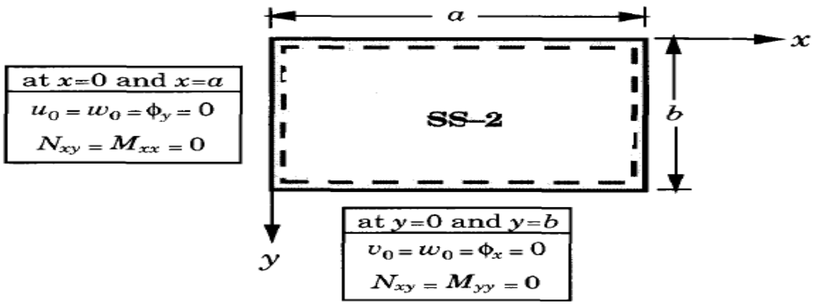


Fig 5. Crossply Laminates

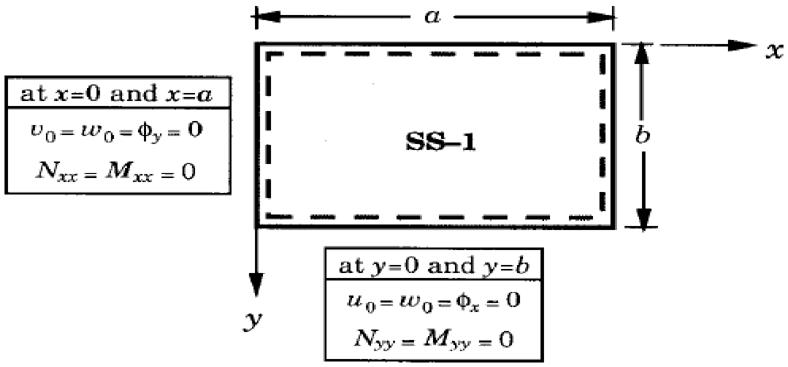


Fig 6.Angleply Laminates

Results of Spherical shell:

Effect of a/h ratio (Shell thickness ratio):

In this part we take fibre orientation $[0/90]_1$ as a constant.

The fig.7 & Table shows that as a variation of shell thickness ratio along the increase of different Radius of curvature (R/a) ratio then deflection goes on decreases.

Effect of shell thickness (a/h) ratio for different radius of curvature (R/a) ratio.

a/h ratio	5	10	20	50	100	1e30
10	10.5545	10.9600	11.0661	11.0962	11.1005	11.1019
20	7.4837	8.4309	8.7056	8.7857	8.7972	8.8011
50	3.9741	6.4712	7.6604	8.0746	8.1374	8.1586
100	1.5231	3.9473	6.4093	7.7472	7.9846	8.0669

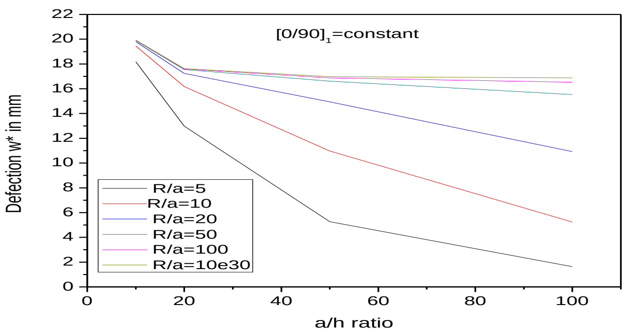


Fig 7.Deflection versus a/h ratio for different radius of curvature ratio.

Effect of Loading

In this part we take Radius of curvature (R/a) ratio =100 & fibre orientation $[0/90]_1$ as a constant. The fig 8. & Table shows that as a variation of a shell thickness (a/h) ratio along the different Loading conditions then deflection decreases gradually.

Effect of shell thickness (a/h) ratio for different Loading Conditions

a/h ratio	UDL	SDL
10	18.1826	11.7841
20	12.9854	8.3521
50	5.2603	3.4437
100	1.6373	1.1270

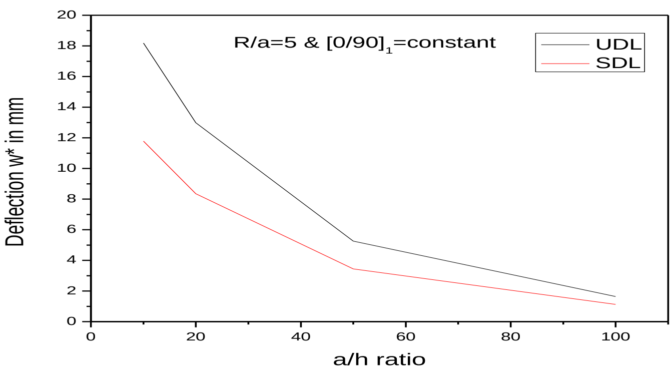


Fig 8.Deflection versus a/h ratio for different Loading Conditions

Results of Cylindrical shells

Effect of a/h ratio (Shell thickness ratio):

In this part we take fibre orientation $[0/90]_1$ as a constant.

The fig.9 & Table shows that as variation of shell thickness ratio along the increase of different Radius of curvature (R/a) ratio then deflection goes on decreases.

Effect of shell thickness (a/h) ratio for different radius of curvature (R/a) ratio

a/h ratio	5	10	20	50	100	10e30
10	19.5052	19.8322	19.9073	19.9230	19.9230	19.9210
20	16.0814	17.2116	17.5172	17.6033	17.6151	17.6184
50	10.5727	14.7448	16.3528	16.8700	16.9475	16.9747
100	4.8816	10.5123	14.6620	16.4785	16.7776	16.8828

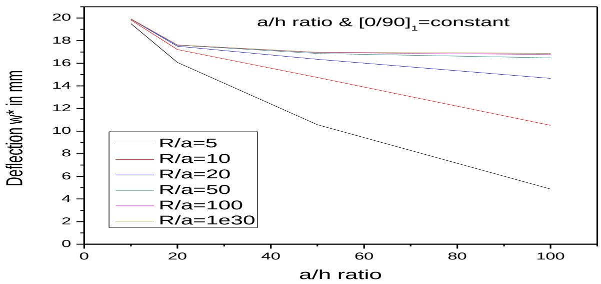


Fig 9. Deflection versus a/h ratio for different radius of curvature (R/a) ratio

Results of Doubly Curved shells

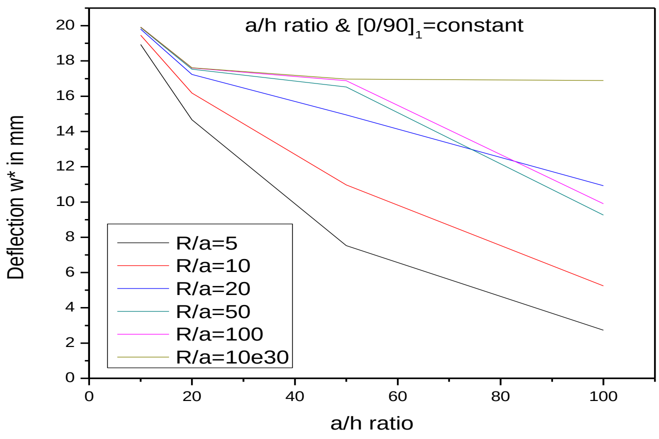
Effect of a/h ratio(Shell thickness ratio):

In this part we take fibre orientation $[0/90]_1$ as a constant.

The fig & Table shows that as variation of shell thickness ratio along the increase of different Radius of curvature (R/a) ratio then deflection goes on decreases.

Effect of shell thickness (a/h) ratio for different radius of curvature (R/a) ratio.

a/h ratio	5	10	20	50	100	10e30
10	18.9319	19.4567	19.8029	19.8953	19.9162	19.9210
20	14.6637	16.1811	17.2360	17.5391	17.6028	17.6184
50	7.5295	10.9761	14.9436	16.5209	16.8831	16.9747
100	2.7357	5.2456	10.9295	9.2648	9.9003	16.8828



CONCLUSIONS

The Finite element model for the bending analysis of doubly curved, laminated composite shells are presented in this paper. The present formulation is based on the stress resultant type Koiter's shell theory. Transverse shear deformation is also considered according to Mindlin's hypotheses. The numerical results obtained from the present analysis have been compared with the deflection for the case when a/h=10,20,50&100 and R/a=5,10,20,50,100 &10e30. While the results reported here are concentrated on the spherical, doubly curved & cylindrical shells with simply supported edges conditions considering four-noded isoparametric elements, the theory presented is valid for different shells.

SCOPE FOR FUTURE WORK

1. Piezo-electric layer can be applied to the surface of composite shell.Joining the sensors and actuators through a control circuit will result in reducing the deflection and hence stresses produced in smart structures. This control setup tries to minimize the deflection in the structure. The underlying principle behind this is that every deflection causes a simultaneous change in voltage. Sensing this change if we apply a somewhat opposite voltage through the control circuitry this deflection would be minimized as a contradictory effect is produced.
2. This project can be formulated by using Triangular shell element.
3. To analyse the free vibration analysis of simply supported composite shell.
4. This project can be formulated by using Ansys software also.
5. To analyse Stress strain Displacement relations.

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