Double Diffusive Convection of a Rotating Fluid Over a Vertical Plate Embedded in Darcy-Forchheimer Porous Medium with Non-Uniform Heat Sources

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Abstract

This paper considers the effect of non-uniform heat sources on convective heat and mass transfer over a vertical plate in a rotating system embedded in a non-Darcy porous medium. The governing boundary layer equations for momentum, heat and mass transfer, which are non-linear partial differential equations, are transformed into non-linear ordinary differential equations by using the similarity transformations and then solved by employing Runge-Kutta method with shooting technique. Numerical solutions are obtained for the primary and secondary velocity profiles, temperature and concentrations profiles for different parametric values and then results are reported graphically as well as skin friction coefficients, Nusselt number and Sherwood number for various physical parameters are presented in tabular form. Compare the present results with previously published work.

Keywords: Heat and Mass Transfer, Rotating System, Darcy-Forchheimer Porous Medium, Non-Uniform Heat Sources

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Introduction

Convective heat and mass transfer from various physical configurations embedded in fluid saturated porous medium has been investigated by many research investigators [3]-[6]. It is an important fact that convective heat and mass transfer in fluid saturated porous medium finds various applications in a variety of engineering processes and geophysical applications such as chemical catalytic reactors and compact heat exchangers, in the design of pebble-bed nuclear reactors, petroleum reservoirs, and use of fibrous materials in the thermal insulation of buildings, heat transfer from storage of agricultural products which generate heat as a result of metabolism, heat salt leaching in soils, oil extraction, underground disposal of nuclear waste, and others.

Raptis and Perdikis [7] investigated effects of mass transfer and free convection currents on the flow past an infinite porous plate in a rotating fluid. Vaidyanathan et.al. [8] studied the effect of magnetic field dependent viscosity on ferroconvection in a rotating sparsely distributed porous medium. Dileep et.al. [9] investigated the effects of hall current on MHD couette flow in a channel partially filled with a porous medium in a rotating system. The study of thermal convection in rotating porous media is motivated both theoretically and by its practical applications in engineering some of the important areas of applications in engineering include the food processing, chemical process, solidification and centrifugal casting of metals and rotating machinery. Thermal convection in rotating system also provides a system to study hydrodynamic instabilities, pattern formation and spatiotemporal chaos in nonlinear dynamical systems. Rotating heat exchangers are used by the chemical and automobile industries, in estimating the flight path of rotating wheels, and spin stabilized missiles, in the design of turbines and turbo machines. In addition, such flows are importance to the hydrologists to study and to aero dynamists to control drag in aero dynamical problems. Many researchers [10-12] investigated the double diffusive convective flow in rotating system and presented excellent results.

The Darcy law is used broadly for the problem of flow in porous media. It is valid only slow flow in porous media. It is an important to note that Darcy law is inadequate for high flow velocities and porous material of large pore radii, the inertia effects become significant. For this flow situation, the relation between pressure drop and the velocity is non-linear. In general, we must consider the effect of the fluid inertia, as well as viscous diffusion which may well become
significant for materials with very high porosities such as fibrous and foams. The Darcy-Forchheimer model was employed by many investigators [13-16]

The effect of heat source/sink on heat transfer is of immense importance in several physical problems. In literature there are many others using the importance of temperature dependent heat source/sink on the heat transfer of various fluids. The natural convection from a point heat source embedded in a non-Darcian porous medium is investigated by Jin-Sheng Leu et.al [17]. Ganapathy and Purushothaman [18] have studied thermal convection from an instantaneous point heat source in a porous medium. Rajesh Sharma [19] analyzed the effects of viscous dissipation and heat source on unsteady boundary layer flow and heat transfer past a stretching surface embedded in a porous medium using element free Galerkin method. However, they ignored space dependent heat source/sink effect which is of immense important in the heat transfer analysis. Some authors have studied and presented the significance of space dependent heat source/sink in addition to the temperature dependent heat source/sink. Some authors [20]-[22] have studied the effects of non-uniform heat source on convective heat and mass transfer in fluid saturated porous medium.

The aim of present paper is to investigate the convective heat and mass transfer flow of a rotating fluid over a vertical plate embedded in a porous medium. The Forchheimer non-Darcy which includes the inertia effects as well as non uniform heat source is included in this study. The governing nonlinear partial differential equations are transformed into similarity ordinary differential equations using a set of dimensionless variables and then are solved numerically by employing the Runge-Kutta fourth order method with shooting technique. Graphs are plotted for the dimensionless primary and secondary velocity, the temperature and the concentration profiles for different values of various parameters. The skin-friction coefficients, Nusselt number and Sherwood number are also calculated and displayed in tabular form. The results are compared with previous existing results.

**Formulation of the problem**

The treated problem is a two-dimensional steady incompressible flow on convective heat and mass transfer over a vertical plate in a rotating system which is embedded in homogeneous fluid saturate non-Darcy porous medium. The physical configuration of the problem is shown in
figure-1. We assumed here that the plate is semi-infinite and all physical quantities are depends on x and y. The flow is moving with a uniform velocity $U_o$ along the x-direction, which is taken along the plate in the upward direction and y-axis is taken normal to it. The uniform temperature $T_w$ and concentration $C_w$ are assumed at the plate, which are higher than ambient fluid temperature $T_{\infty}$ and concentration $C_{\infty}$. We also assumed that magnetic field, viscous and Ohmic dissipations are present. The appropriate conservation equations with aforesaid assumption for the boundary-layer regime can be shown to take the form with Boussinesq approximations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g' \beta (T - T_{\infty}) + g' \beta' (C - C_{\infty}) + 2 \Omega w + \frac{\nu}{K'} (U_0 - u)$$

$$+ \frac{c_b}{\sqrt{K'}} (U_0 - u)^2 + \frac{\sigma B^2}{\rho} (U_0 - u)$$

$$u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} = \nu \frac{\partial^2 w}{\partial y^2} + 2 \Omega (U_0 - u) - \frac{\nu}{K'} w - \frac{c_b}{\sqrt{K'}} (w)^2 - \frac{\sigma B^2}{\rho} w$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{D_m k_T}{c_p} \frac{\partial^2 C}{\partial y^2} + \nu \left[ \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial w}{\partial y} \right)^2 \right] + \frac{\sigma B^2}{\rho c_p} \left[ (U_0 - u)^2 + w^2 \right] + \frac{q''}{\rho c_p}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = \frac{D_m}{T_m} \frac{\partial C}{\partial y^2} + \frac{D_m k_T}{T_m} \frac{\partial^2 T}{\partial y^2}$$

The boundary conditions for this problem are:

$$u = 0, v = v_0 (x), w = 0, T = T_0, C = C_w \text{ at } y = 0$$

$$u = U_0, w = 0, T = T_{\infty}, C = C_{\infty} \text{ as } y \to \infty$$

By using the work of Sattar [2], a transformation is assumed as,

$$u_1 = U_0 - u \Rightarrow u = U_0 - u_1$$

Eqs. (1) - (5) with boundary conditions (6), are transformed respectively into
\[
\frac{\partial u_1}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  

(7)

\[
(U_0 - u_1) \frac{\partial u_1}{\partial x} + v \frac{\partial u_1}{\partial y} = w \frac{\partial^2 u_1}{\partial y^2} - g' \beta (T - T_\infty) - g' \beta^* (C - C_\infty) - 2 \Omega w - \frac{\nu}{K'} u_1 - \frac{c_b}{\sqrt{K'}} (u_1)^2 - \frac{\sigma B^2}{\rho} u_1
\]  

(8)

\[
(U_0 - u_1) \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} = w \frac{\partial^2 w}{\partial y^2} + 2 \Omega u_1 - \frac{\nu}{K'} w - \frac{c_b}{\sqrt{K'}} (w)^2 - \frac{\sigma B^2}{\rho} w
\]  

(9)

\[
(U_0 - u_1) \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = k \frac{\partial^2 T}{\partial y^2} + \frac{D_m k_T}{c_p c_p} \frac{\partial^2 T}{\partial y^2} + \frac{\nu}{c_p} \left( \left( \frac{\partial u_1}{\partial y} \right)^2 + \left( \frac{\partial w}{\partial y} \right)^2 \right) + \frac{\sigma B^2}{\rho c_p} \left[ u_1^2 + w^2 \right] + \frac{q''}{\rho c_p}
\]  

(10)

\[
(U_0 - u_1) \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = \frac{D_m}{T_m} \frac{\partial^2 C}{\partial y^2} + \frac{D_m k_c}{T_m} \frac{\partial^2 T}{\partial y^2}
\]  

(11)

and

\[
\begin{align*}
& u_1 = U_0, \quad v = v_0(x), \quad w = 0, \quad T = T_w, \quad C = C_w \quad \text{at} \quad y = 0, \\
& u_1 = 0, \quad w = 0, \quad T \to T_\infty, \quad C \to C_\infty \quad \text{as} \quad y \to \infty
\end{align*}
\]  

(12)

where \( u, v, w \) are the velocity components in the direction \( x, y, z \) respectively, \( \beta \) is the coefficient of volumetric thermal expansion, \( \beta^* \) is the volumetric mass expansion, \( g' \) is the acceleration due to gravity, \( \rho \) is the density, \( \nu \) is the kinematics viscosity, \( K' \) is the permeability of the porous medium, \( k \) is the thermal conductivity of the medium, \( T, T_w, T_\infty, D_m, c_p, c_s, T_m \) and \( k_T \) are the temperature of the fluid inside the thermal boundary layer, the plate temperature and the fluid temperature in the free stream, coefficient of mass diffusivity, specific heat at constant pressure, concentration susceptibility, mean fluid temperature, and thermal diffusion ratio, respectively. \( C, C_w, C_\infty \) and \( C_b \) are the concentration of the fluid inside the concentration boundary layer, the plate concentration, the fluid concentration in the free stream and drag coefficient which is independent of viscosity respectively and other symbols have their usual meaning.

The non-uniform heat source \( q'' \) is assumed as

\[
q'' = \frac{k u_w(x)}{xD} \left[ a^* (T_w - T_\infty) f' + (T - T_w) b^* \right]
\]  

(13)
where \( a^* \) and \( b^* \) are the co-efficient of space and temperature dependent heat source, respectively. It is worth to mention here that \( a^*>0 \) and \( b^*>0 \) represents the internal heat generation and that \( a^*<0 \) and \( b^*<0 \) represents the internal heat absorption.

To render dimensionless solutions and facilitate numerical analysis, we define the following dimensionless variables:

\[
\eta = y \sqrt{\frac{U_0}{2\nu x}}, \quad g(\eta) = \frac{w}{U_0}, \quad \theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty}, \quad \phi(\eta) = \frac{C-C_\infty}{\bar{x}(C_0-C_\infty)}, \quad \psi(\eta) = \sqrt{2\nu x U_0} f(\eta)
\]

\[
v = -\frac{\partial \psi}{\partial x} = -\sqrt{\frac{\nu U_0}{2x}} \left[ \eta f'(\eta) - f(\eta) \right], \quad u_1 = \frac{\partial \psi}{\partial y} = U_0 f'(\eta) \Rightarrow \frac{u}{U_0} = 1 - f'(\eta)
\]  \hspace{1cm} (14)

Now the plate of concentration is assumed as

\[
C_w(x) = C_\infty + \bar{x}(C_0 - C_\infty), \text{ where } C_0 \text{ is the mean concentration and } \bar{x} = \frac{x U_0}{\nu}.
\]

Also we have, \( f_w = v_0(x) \sqrt{\frac{2x}{U_0^3}} \)  \hspace{1cm} (15)

where \( f_w \) is the suction parameter or transpiration parameter [1].

Making use of equations (13) – (15) the governing equations (7) – (11) with boundary conditions (12) are transformed into

\[
f'''' + (\eta - f)f'' - Gr\theta - Gm\phi - Rg - (K + M)f' - F(f')^2 = 0
\]  \hspace{1cm} (16)

\[
g'' + (\eta - f)g' + Rf' - (K + M)g - F(w)^2 = 0
\]  \hspace{1cm} (17)

\[
\theta'' + Pr(\eta - f)\theta' + D:\phi'' + Pr Ec\left[(f'')^2 + (g')^2\right] + Pr Ec M\left[(f')^2 + (g)^2\right] + af'' + b\theta = 0
\]  \hspace{1cm} (18)

\[
\phi'' + Sc(\eta - f)\phi' + 2Sc(f' - 1)\phi + S_0\theta'' = 0
\]  \hspace{1cm} (19)

\[
f = f_w, \quad f' = 1, \quad g = 0, \quad \theta = 1, \quad \phi = 1 \quad \text{at} \quad \eta = 0
\]

\[
f' = 0, \quad g = 0, \quad \theta = 0, \quad \phi = 0 \quad \text{as} \quad \eta \rightarrow \infty
\]  \hspace{1cm} (20)
where

\[
Gr = \frac{g'\beta(T_w - T_\infty)2x^2}{\nu^2},
Gm = \frac{g'\beta'(C_w - C_\infty)2x^2}{\nu^2},
K = \frac{2\mu x}{K'U_0},
Ec = \frac{U_0^2}{c_p(T_w - T_\infty)},
Sc = \frac{\nu}{D_m}
\]

\[
D_f = \frac{D_m k_r \rho(C_w - C_\infty)}{c_s K'(T_w - T_\infty)},
S_0 = \frac{k_f(T_w - T_\infty)}{T_m(C_w - C_\infty)},
F = \frac{2x c_b}{\sqrt{K'U_0}}
\]

Here Gr is the Grashof number, Gm is the modified Grashof number, K is the Permeability parameter, M is the Magnetic parameter and R is the rotational parameter, Pr is the Prandtl number, Df is the Dufour number, Ec is the Eckert number, Sc is the Schmidt number S0 is the Soret number, a is the space-dependent parameter, b is the temperature-dependent parameter F is the local inertia parameter or Forchheimer number.

The physical quantities of interest are the skin friction co-efficient \(C_f\), which is defined as

\[
C_f = \mu \left( \frac{\partial u}{\partial y} \right)_{y=0}
\]

and

\[
\mu \left( \frac{\partial w}{\partial y} \right)_{y=0}
\]

which are transformed into non-dimensional form:

\[
\tau_x = \left( \frac{\partial^2 f}{\partial \eta^2} \right)_{\eta=0}
\]

and

\[
\tau_z = \left( \frac{\partial g}{\partial \eta} \right)_{\eta=0}
\]

The Nusselt number is denoted by \(Nu = -\frac{1}{\Delta T} \left( \frac{\partial T}{\partial y} \right)_{y=0}\) is transformed into non-dimensional form:

\[
\left( \frac{\partial \theta}{\partial \eta} \right)_{\eta=0}
\]

The Sherwood number is denoted by \(Sh = -\frac{1}{\Delta C} \left( \frac{\partial C}{\partial y} \right)_{y=0}\) is transformed into non-dimensional form:

\[
\left( \frac{\partial \phi}{\partial \eta} \right)_{\eta=0}
\]
Numerical solution

The system of non-dimensional nonlinear ordinary differential equations (16) – (19) with the corresponding boundary conditions (20) have been solved numerically by utilizing a Mathematica software with shooting technique with fourth order Runge-Kutta integration scheme. Various non dimensional parameters $a$, $b$ and $F$ were considered in different phases. In all the computations the step size $\Delta \eta = 0.001$ was selected that satisfied a convergence criterion of $10^{-5}$ in almost all of different phases mentioned above. Without loss generality the maximum value of $\eta_\infty$ is assumed as 4. To assess the accuracy of the present numerical method, we have compared our results with those of Mahmud Alam et. al [1] in the absence of Forchhiemer number and non-uniform heat source in the fig-14. Fig-14 shows that the results of primary velocity, temperature and concentration which are compared with the results of Mahmud Alam et.al [1]. Therefore, the developed code can be used with great confidence to study the problem considered in this paper.

Results & Discussions

Numerical results for the effects of non-Darcy convective heat and mass transfer over a vertical plate in a rotating system in the presence of non-uniform heat source. The results are presented graphically from Figs. 2-13 for the flow field, temperature and concentration for different parametric values and conclusions are drawn that flow field, temperature and concentration of physical interest have significant effects. Comparisons with previously published works are performed and excellent agreement between the results is obtained. The governing equations are converted into non-dimensional ordinary differential equations and then integrated by using Runge - Kutta Fourth order method with shooting technique. The default values of the above mentioned parameters are considered as $Gr=4$, $Gm=2$, $R=0.2$, $K=0.5$, $M=0.5$, $Pr=0.71$, $Df=0.2$, $Ec=0.01$, $Sc=0.6$, $So=0.2$, $f_w=0.5$, $a=0.1$, $b=0.1$ and $F=0.1$ unless otherwise specified.

The primary velocity on different parameters is shown in figures 2-4. Fig-2 shows that space dependent heat source $(a)$ on primary velocity. It is observed that primary velocity increases with increasing values of space dependent heat source. Fig-3 illustrates that primary velocity for different values of temperature dependent heat source $(b)$. From this figure it is clear
that as $b$ increases, primary velocity increases. Fig-4 depicts the primary velocity for different values of inertia parameter ($F$) and has increasing effect with increase of $F$.

The secondary velocity is shown in figures 5-7 for different parametric values of space dependent heat source, temperature dependent heat source and inertia parameter respectively. From these figures it is observed that the secondary velocity field decreases with the increase of space dependent heat source $a$, temperature dependent heat source $b$ and inertia parameter $F$. The non-dimensional temperature is shown in figures 8-10 for different parametric values $a$, $b$ and $F$ respectively. It can be seen that increasing the space dependent heat source, temperature dependent heat source and inertia parameter results an enhancement in temperature profile.

The non-dimensional concentration profile is shown in figures 11-13. This shows that increase the space dependent heat source, temperature dependent heat source and inertia parameter results depreciation in the concentration profile. From table-1 we observed that the skin friction components $\tau_x$ and $\tau_z$ and Nusselt number $Nu$ are increases with increasing values of space dependent heat source, temperature dependent heat source and inertia parameter increases, but Sherwood number $Sh$ increases.

**Conclusion**

1. Increasing the space dependent heat source, temperature dependent heat source and inertia parameter results an enhancement in primary velocity and temperature profile as well as depreciation in secondary velocity and concentration profile.

2. As increase the space dependent heat source, temperature dependent heat source and inertia parameter, the skin friction components, Nusselt number increases, but Sherwood number are increases.

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References


Fig-1: Physical configuration of the problem and coordinate system
Fig-2: variation of velocity with space dependent heat source $(a)$

Fig-3: Variation of velocity with temperature dependent heat source $(b)$

Fig-4: Variation of velocity with inertia parameter $(F)$
Fig-5: Variation of secondary velocity (g) with $a$

Fig-6: Variation of secondary velocity (g) with $b$

Fig-7: Variation of secondary velocity (g) with $F$
Fig-8: Variation of temperature with space dependent heat source ($a$)

Fig-9: Variation of temperature with temperature dependent heat source ($b$)

Fig-10: Variation of temperature with inertia parameter ($F$)
Fig-11: Variation of concentration with space dependent heat source

Fig-12: Variation of concentration with temperature dependent heat source

Fig-13: Variation of concentration with inertia parameter
Fig-14: Comparison with results of Mahmud Alam et.al. [1]

Table-1: The effects of space dependent, temperature dependent heat sources and inertia parameter effects on skin-friction co-efficient, Nusselt number and Sherwood number.

<table>
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<tr>
<th>a</th>
<th>b</th>
<th>F</th>
<th>$\tau_x$</th>
<th>$\tau_z$</th>
<th>Nu</th>
<th>Sh</th>
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