PREDICTING THE GRADE SYSTEM ON MAX (Y_1, Y_2) IN AN ORGANIZATION THROUGH PARTICULAR DISTRIBUTION

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ABSTRACT

This paper describes the development of atime to recruitment in any organization. Manpower planning is essentially the process of getting the right number of qualified people into the right job at the right time. It is a system of matching the supply of people, existing employees and those to be hired for with openings the organization expects over a time period.The expectedtime to reach the recruitment status of the organization and its variance is found through three parameter generalized Pareto distribution. The analytical results are numerically illustrated by assuming the distribution for the practical use of the model.

Keyword: Grades; Threshold; Three parameter generalized Pareto distribution; Recruitment.

INTRODUCTION

Inmanpower planning we are concerned with the description and prediction of the behaviour of large numbers of people.The three parameter generalized Pareto distribution, which is a special case of both exponential and Wakeby distribution, has good potential for the analysis of flood peaks because of its inherent properties. Point after which radical changes are likely to occur is called threshold level. The generalized Pareto distribution has been widely used to model rare events in several fields. The generalized Pareto distribution, introduced by Pickands (1975), is a limit distribution for the excess over a (large) threshold for data coming from generalized extreme value distributions, as well as a generalization of the Pareto distribution.One can see for more detail in Esary et al., (1973), Pandiyan, et al., (2010), discussed about the expected time to cross threshold level period.

These assumptions are somewhat artificial, but are made because of the lack of detailed real-world information on one hand and in order to illustrate the proceedings on the other hand. The organization comprises two grades of personnel. Mobility or transfer of manpower from one grade to the other is permitted. The time to recruitment is equal to the maximum of the time taken for each one of the two grades to cross the threshold which follows Three parameter generalized Pareto distribution. The processes which give rise to policy revisions and the threshold random variables are statistically independent. The policy decisions are taken with inter arrival times which are i.i.d. random variables depending upon the market environment, production, etc.

NOTATION

X_i : a continuous random variable denoting the amount of loss of manpower caused to the system on the i^{th} occasion of policy announcement (Shock), 1, 2….k and $X_i's$ are i.i.d

Y_1, Y_2 : continuous random variable denoting the threshold levels for the two grades which follows three parameter generalized pareto distribution.

U_i : a random variable denoting the inter-arrival times between contact with c.d.f. $F_i(.)$, $i = 1,2,3 \dots k$.

$g(.)$: The probability density function of X_i; $g^*(.)$: Laplace transform of $g(.)$

$g_k(.)$: the k- fold convolution of $g(.)$i.e., p.d.f. of $\sum_{j=1}^{k} X_i$

$f(.)$: p.d.f. of random variable denoting between successive policyannouncement with the corresponding c.d.f. $F(.)$.

$F_k(.)$: k-fold convolution of $F(.)$; $S(.)$: Survival function.

$V_k(t)$: Probability of exactly k policy announcements; $L(t)$: $1 - S(t)$.

MODEL DESCRIPTION

Manpower requirements models are designed to tell the human resource planner how many of what types of people are needed to produce given levels of output.Any component exposed to shocks which cause damage to the component is likely to fail when the total cumulated damage exceed a level called threshold. In general, assuming that the threshold Y, follows three-parameter generalized Pareto distributiondiscussed by Pickands (1975).

$$\overline{H}(x) = 2e^{\left(\frac{d-x}{b}\right)} - e^{\left(\frac{2d-2x}{b}\right)} \tag{1}$$

There may be no practical way to inspect an individual item to determine its threshold Y. Three-parameter generalized Pareto distribution with parameter *b* and *d*, can be proved that

$$P(X_i < Y) = \int_0^\infty g^*(x)\overline{H}(x)dx \tag{2}$$

Now the threshold Y is such that it has two components namely Y_1 and Y_2. Transfer of component from Y_1 to Y_2 is also possible. We have the breakdown of the system at $\mathrm{Y} = \max(\mathrm{Y}_1, \mathrm{Y}_2)$.

$\mathrm{P}[\max(\mathrm{Y}_1, \mathrm{Y}_2)] = \mathrm{P}[(\mathrm{Y}_1 < y) \cap (\mathrm{Y}_2 < y)] \ = \mathrm{P}[\mathrm{Y}_1 < y]\mathrm{P}[\mathrm{Y}_2 < y]$

Now that, Y_1 and Y_2 follow three parameter generalized Pareto distribution with parameterb, d.

$$P\left(\sum_{i=1}^{k} X_i < Y\right) = 2\int_0^\infty g^*(x)e^{-\left(\frac{x-d}{b}\right)} - \int_0^\infty g^*(x)e^{\left(\frac{2x-2d}{b}\right)}dx$$

$$= 2\left[g^*\left(\frac{1-d}{b}\right)\right]^k - \left[g^*\left[2\left(\frac{1-d}{b}\right)\right]\right]^k \tag{3}$$

Survival analysis is a class of statisticalmethods for studying the occurrence andtiming of events.The survival function$\mathrm{S(t)}$ which is the probability that an individual survives for a time t

$\mathrm{S(t)} = \mathrm{P(T} > t) =$ Probability that the total damage survives beyond *t*

$$= \sum_{k=0}^{\infty} \mathrm{P}\{there\ are\ exactly\ k\ epochs\ in\ (0,t] * P\ (the\ total\ cumulative\ \ (0,t]\}$$

$$S(t) = P(T > t) \ = \sum_{k=0}^{\infty} V_k(t) P\big(X_i < max(Y_1, Y_2)\big)$$

It may happen that successive shocksbecome increasingly effective in causingdamage, even though they are independent.This means that$V_k(t)$, the distribution functionof the kth damage is decreasing in k=1,2,... foreach t.It is also known from renewal process that

$\mathrm{P}(\text{exactly k policy decesions in } (0,\mathrm{t}]) = \mathrm{F_k(t) - F_{K+1}(t)}$ with $\mathrm{F_0(t) = 1}$

$$= \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)] \left[g^* \left(\frac{1-d}{b}\right)\right]^k + \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)] \left[g^* \left(\frac{1-d}{b}\right)\right]^k - \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)] \left[g^* \, 2\left(\frac{1-d}{b}\right) + \left(\frac{1-d}{b}\right)\right]^k$$

$L(t) = 1 - S(t),$ TakinglaplacetransformofL(t), Weget

$$L(t) = 1 - \left\{ 2\sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)] \left[g^* \left(\frac{1-d}{b}\right)\right]^k - \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)] \left[g^* \left(\frac{2-2d}{b}\right)\right]^k \right\}$$

On simplification we get,

$$L(t) = 2\left[1 - g^* \left(\frac{1-d}{b}\right)\right] \sum_{k=1}^{\infty} F_k(t) \left[g^* \left(\frac{1-d}{b}\right)\right]^{k-1} - \left[1 - g^* \left(\frac{2-2d}{b}\right)\right] \sum_{k=1}^{\infty} F_k(t) \left[g^* \left(\frac{2-2d}{b}\right)\right]^{k-1}$$

$$l^*(t) = \frac{2\left[1 - g^*\left(\frac{1-d}{b}\right)\right] f^*(s)}{\left[1 - g^*\left(\frac{1-d}{b}\right) f^*(s)\right]} - \frac{\left[1 - g^*\left(\frac{2-2d}{b}\right)\right] f^*(s)}{\left[1 - g^*\left(\frac{2-2d}{b}\right) f^*(s)\right]} \tag{4}$$

Let the random variable U denoting inter arrival time which follows exponential with parameter c. Now $\mathrm{f^*(s)} = \left(\frac{\mathrm{c}}{\mathrm{c+s}}\right)$, substituting in the above equation (4) we get

$$= \frac{2\left[1 - g^*\left(\frac{1-d}{b}\right)\right]\left(\frac{c}{c+s}\right)}{\left[1 - g^*\left(\frac{1-d}{b}\right)\left(\frac{c}{c+s}\right)\right]} - \frac{\left[1 - g^*\left(\frac{2-2d}{b}\right)\right]\left(\frac{c}{c+s}\right)}{\left[1 - g^*\left(\frac{2-2d}{b}\right)\left(\frac{c}{c+s}\right)\right]} \tag{5}$$

$E(T) = -\frac{d}{ds} l^*(s) given s = 0, \qquad E(T^2) = -\frac{d^2}{ds^2} given s = 0$

From which variance $V(T) = E(T^2) - [E(T)]^2$ can be obtained

$$E(T) = \frac{2}{c\left[1 - g^*\left(\frac{1-d}{b}\right)\right]} - \frac{1}{c\left[1 - g^*\left(\frac{2-2d}{b}\right)\right]} on simpilification$$

$$E(T^2) = \frac{4}{c^2\left[1 - g^*\left(\frac{1-d}{b}\right)\right]^2} - \frac{2}{c^2\left[1 - g^*\left(\frac{2-2d}{b}\right)\right]^2} on simpilification$$

$g^*(.) \sim Mittag Leffler Distribution \frac{1}{1+\lambda^\alpha}$

$$g^*(.) \sim exp\ (\mu), g^*(\lambda) \sim exp\left(\frac{\mu}{\mu+\lambda}\right) and\, g^*(\lambda\theta) \sim exp\left(\frac{\mu}{\mu+\lambda\theta}\right)$$

$$E(T) = \frac{2}{c\left[1 - \left(\frac{\mu}{\mu+\frac{1}{b}}\right) - \left(\frac{\mu}{\mu+\frac{d}{b}}\right)\right]} - \frac{1}{c\left[1 - \left(\frac{\mu}{\mu+\frac{2}{b}}\right) - \left(\frac{\mu}{\mu+\frac{2d}{b}}\right)\right]}$$

$$E(T) = \frac{2[b^2\mu^2 + db\mu + b\mu + d]}{c[b^2\mu^2 + 2b\mu + d]} - \frac{[b^2\mu^2 + 2db\mu + 2b\mu + 4d]}{c[b^2\mu^2 + 4b\mu + 4d]} \tag{6}$$

$$E(T^2) = \frac{4[b^2\mu^2 + db\mu + b\mu + d]^2}{c^2[b^2\mu^2 + 2b\mu + d]^2} - \frac{2[b^2\mu^2 + 2db\mu + 2b\mu + 4d]^2}{c^2[b^2\mu^2 + 4b\mu + 4d]^2} \tag{7}$$

RESULTS

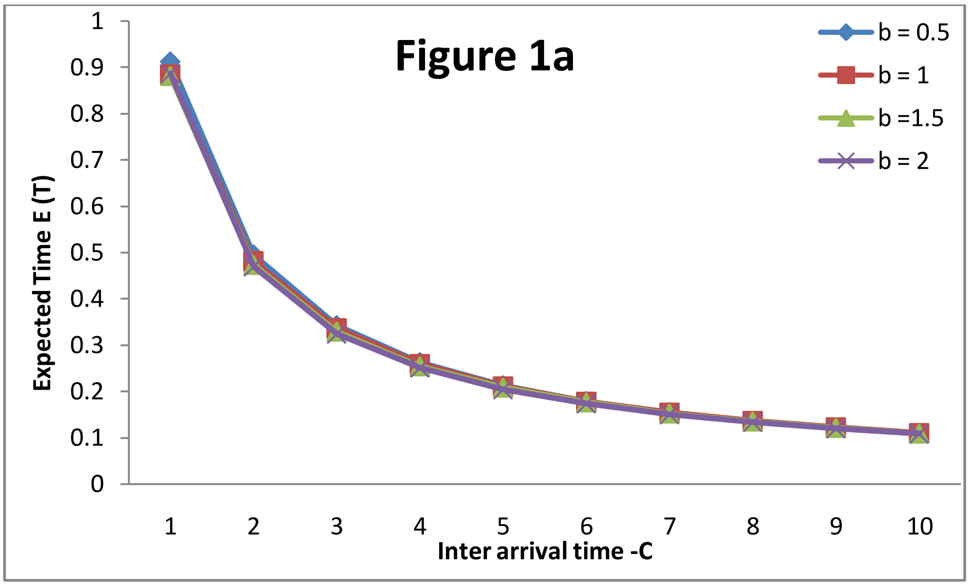
$$E(T) = \frac{2[b^2\mu^2 + db\mu + b\mu + d]}{c[b^2\mu^2 + 2b\mu + d]} - \frac{[b^2\mu^2 + 2db\mu + 2b\mu + 4d]}{c[b^2\mu^2 + 4b\mu + 4d]} \tag{8}$$

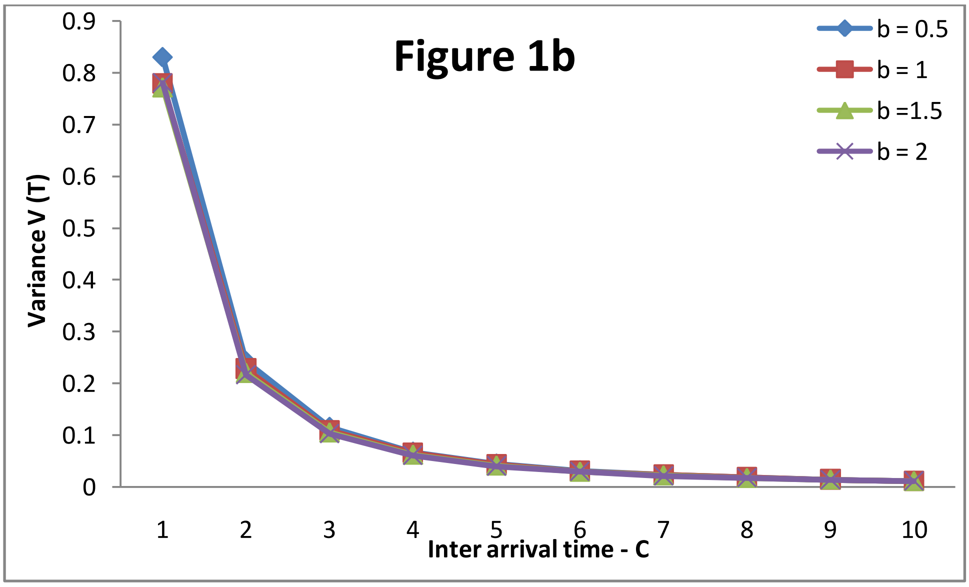
$$V(T) = \frac{4[b^2\mu^2 + db\mu + b\mu + d][b^2\mu^2 + 2db\mu + 2b\mu + 4d]}{c^2[b^2\mu^2 + 2b\mu + d][b^2\mu^2 + 4b\mu + 4d]}$$

$$- \frac{3[b^2\mu^2 + 2db\mu + 2b\mu + 4d]^2}{c^2[b^2\mu^2 + 4b\mu + 4d]^2} \tag{9}$$

NUMERICAL ILLUSTRATION

On the basis of the numerical illustration from the equation 8 and 9 the following conclusions regarding expected time and variance consequent to the changes in the different parameters can be observed in Figures 1 to 3 that follow.





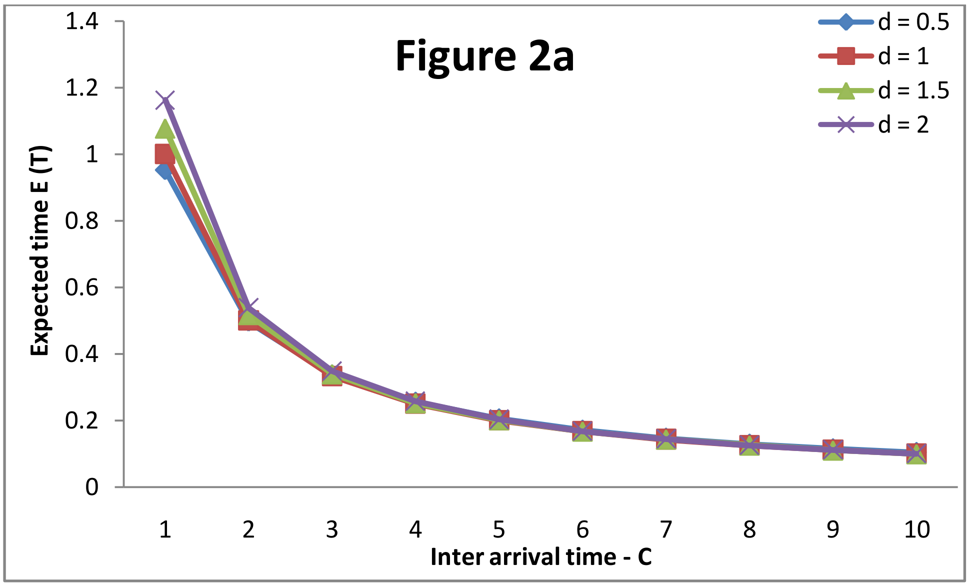


Figure 2a
Expected time E (T)
Inter arrival time - C
d = 0.5
d = 1
d = 1.5
d = 2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1
2
3
4
5
6
7
8
9
10

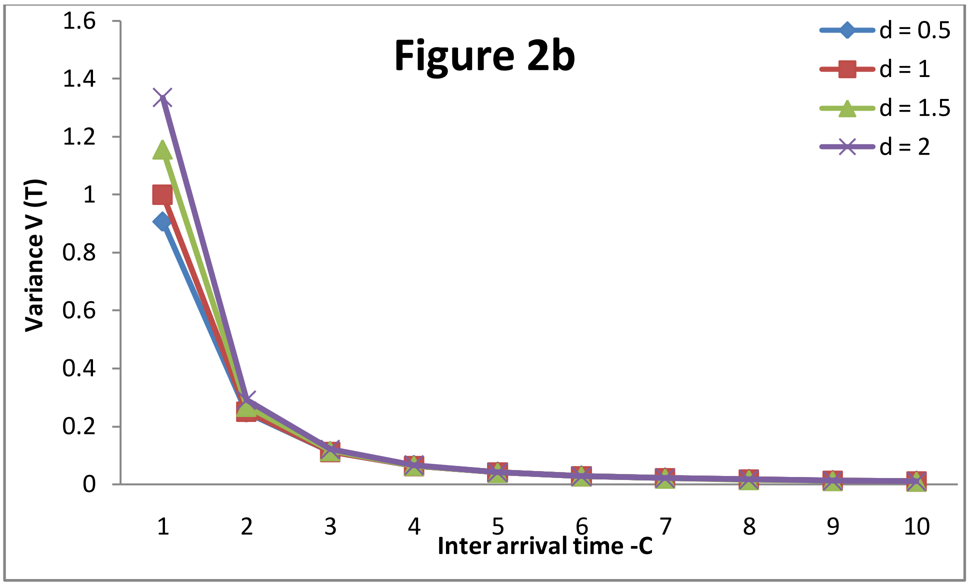
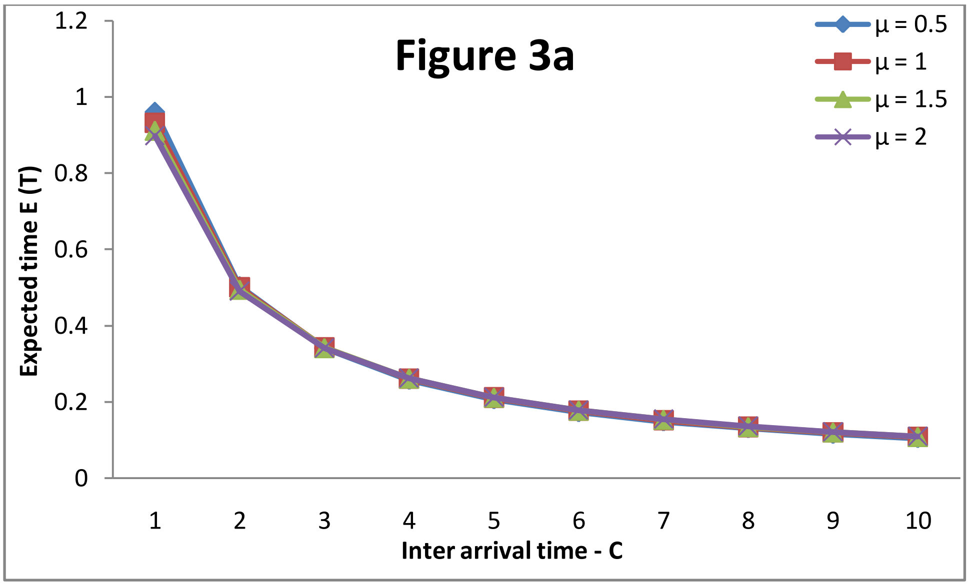
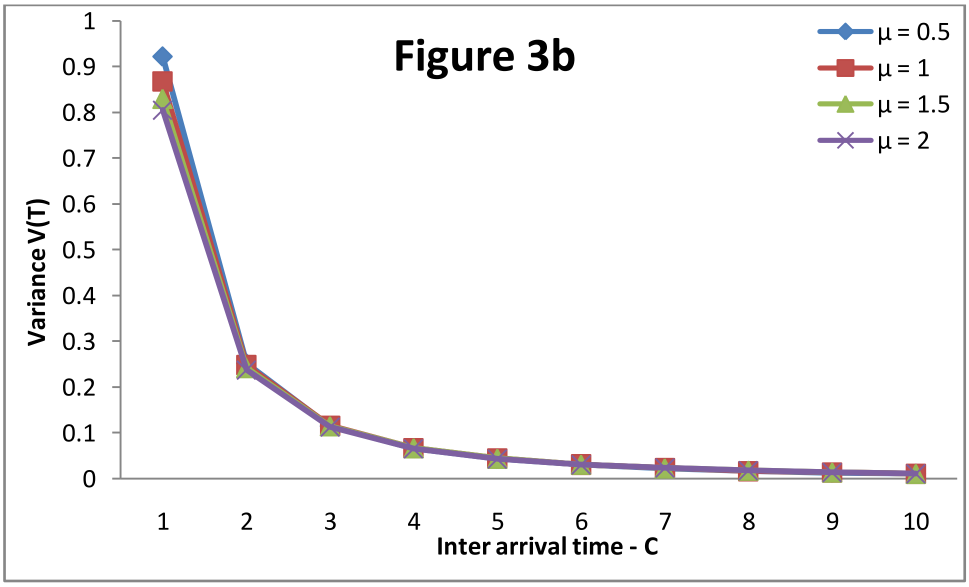


Figure 2b
Variance V (T)
Inter arrival time -C
d = 0.5
d = 1
d = 1.5
d = 2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1
2
3
4
5
6
7
8
9
10





CONCLUSIONS

When μ, d is kept fixed with other parameters b the inter-arrival time $'c'$, which follows exponential distribution, is an increasing parameter. Therefore, the value of the expected time E (T) to cross the threshold is decreasing, for all cases of the parameter value$b = 0.5, 1, 1.5, 2$. When the value of the parameter b increases, the expected time is found increasing, this is observed in Figure 1a. The same case is found in Variance V (T) which is observed in Figure 1b.

When μ, b is kept fixed with other parameters$d$ the inter-arrival time $'c'$ increases, the value of the expected time E (T) to cross the threshold is found to be decreasing, in all the cases of the parameter value$d = 0.5, 1, 1.5, 2$. When the value of the parameter d increases, the expected time is found increasing. This is indicated in Figure 2a. The same case is observed in the Variance V (T) which is observed in Figure 2b.

When b, d is kept fixed with other parameters$\mu$the inter-arrival time $'c'$ increases, the value of the expected time E (T) to cross the threshold is found to be decreasing, in all the cases of the parameter value$\mu = 0.5, 1, 1.5, 2$. When the value of the parameter μ increases, the expected time is found increasing. This is indicated in Figure 3a. The same case is observed for the Variance V (T) which is observed in Figure 3b.

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