N × 3 Specially Structured Flow Shop Scheduling To Minimize The Rental Cost Including Job Restrictions And Transportation Time

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Abstract

The present paper is an attempt to develop a heuristic algorithm for three machines specially structured flow shop scheduling in which processing time of jobs are under some well defined structural relationship to one another including job block criteria and transportation time. Many heuristic approaches have already been discussed in literature to minimize the makespan. But it is not necessary that minimization of makespan always lead to minimize rental cost of machines. The objective of the paper is to obtain a optimal / near optimal sequence of jobs in order to minimize the rental cost of machines under a specified rental policy. A numerical example is given to clarify the algorithm.

Keywords: Specially structured flow shop scheduling, Processing time, Rental cost, Utilization time.

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1. Introduction

Scheduling of jobs and activities is an important area of research for maintaining a firm's competitive market position. The problem is to find the order in which a given number of jobs must be processed on a specified set of machines so that one or more decision objectives are optimized. In 1954, Johnson [1] proposed an algorithm called Johnson's rule, to achieve the minimum makespan for two - machines flow shop problem. The work was developed by Ignall and Schwarge [2], Bagga [3], Gupta, J.N.D [4], Maggu and Das [6], Szware [5], Yoshida and Hitomi [7], Singh, T.P. [10], Gupta Deepak and Sharma Sameer [17] etc. Narain [14] studied a problem to obtain a sequence which gives minimum possible rental cost while minimizing total elapsed time under pre-defined rental policy.

Further the transportation times (loading time, moving time and unloading time etc) of the jobs from one machine to another are negligible therefore could be included in the jobs processing time. However in some application transportation time have major impact in the performance measure considered for the scheduling problem, so they need to be considered separately. The idea of jobs restrictions has a practical significance to create a balance between the cost of providing priority in service to the customers and cost of giving service with non-priority customers thereby making the problem more wider and applicable in a production concern.

Gupta, Sharma and Shashi [18] introduces the concept of specially structured flow shop scheduling to minimize the rental cost of machines, in which processing times are associated with probabilities. The present paper is an attempt to extend their study by introducing the concept of jobs restrictions including transportation time.

2. Practical Situation

Many applied and experimental situations occur in our day to day working in factories and industrial concern where we have to restrict the processing of some jobs. The practical situation may be taken in a production industry; manufacturing industry etc, where some jobs has to give priority over other. Various practical situations occur in real life when one has got the assignments but does not have one's own machine or does not have enough money or does not want to take risk of investing huge amount of money to purchase machines. Under such circumstances, the machines have to be taken on rent in order to complete the assignments. In his starting career, we find a medical practitioner does not buy expensive machines say X-ray machine, the Ultra Sound Machine, Rotating Triple Head Single Positron Emission Computed Tomography, Scanner, Patient Monitoring Equipment, and Laboratory Equipment etc., but instead takes on rent. Rental of medical equipment is an affordable and quick solution for hospitals, nursing homes, physicians, which are presently constrained by the availability of limited funds due to the recent global economic recession. Renting enables saving working capital, gives option for having the equipment, and allows up-gradation to new technology.

3. Notations

S	: Sequence of jobs 1, 2, 3,….,n
S_k	: Sequence obtained by applying Johnson's procedure, k = 1, 2, 3, ------
M_j	: Machine; j= 1,2,3.
a_{ij}	: Processing time of i^{th} job on machine M_j
$t_{ij}(S_k)$	: Completion time of i^{th} job of sequence S_k on machine M_j
$T_{i,1\to 2}$	: Transportation time of i^{th} job from 1st machine to 2nd machine.
$T_{i,2\to 3}$	: Transportation time of i^{th} job from 2nd machine to 3rd machine.
T_{ij} (S_k)	: Idle time of machine M_j for job I in the sequence S_k.
$U_j(S_k)$	: Utilization time for which machine M_j is required
$R(S_k)$	: Total rental cost for the sequence S_k of all machine
C_j	: Renal cost of j^{th} machine.

$CT(S_i)$: Total completion time of the jobs for sequence S_k

β : Equivalent job block.

4. Definition

Completion time of i^{th} job on machine M_j is denoted by t_{ij} and is defined as:

$t_{i2} = \max(t_{i-1,j}, t_{i,j-1} + t_{i\,1-2}) + a_{i2}$

$t_{i3} = \max(t_{i-1,j}, t_{i,j-1} + t_{i\,2-3}) + a_{i3}$

5. Rental Policy (P)

The machines will be taken on rent as and when they are required and are returned as and when they are no longer required.

6. Problem Formulation

Let some job i (i = 1,2,……..,n) are to be processed on three machines M_j (j = 1,2,3) under the specified rental policy P. Let a_{ij} be the processing time of i^{th} job on j^{th} machine and t_i and g_i be transportation time of i^{th} job from machine M1 to M2 and machine M2 to machine M3 respectively. Satisfying the structural relationship.

either $\quad \text{Min}(a_{i1} + t_{i1-2}) \geq \max(a_{i2} + t_{i1-2})$

Or $\quad \text{Min}(a_{i3} + g_{i2-3}) \geq \max(a_{i2} + g_{i2-3})$

Our aim is to find the sequence (S_k) of the jobs which minimize the rental cost of the machine.

The mathematical model of the problem in matrix form can be stated as:

Jobs	Machine M1	$T_{i1\rightarrow 2}$	Machine M2	$T_{i2\rightarrow 3}$	Machine M3
I	a_{i1}	t_i	a_{i2}	g_i	a_{i3}
1	a_{11}	t_1	a_{12}	g_1	a_{13}
2	a_{21}	t_2	a_{22}	g_2	a_{23}
3	a_{31}	t_3	a_{32}	g_3	a_{33}
-	-	-	-	-	-
n	a_{n1}	t_{n1}	a_{n2}	g_{n2}	a_{n3}

Tableau -1

Mathematically, the problem is stated as:

Minimize $R(S_k) = \sum A_{i1} \times C_1 + U_2(S_k) \times C_2 + U_3(S_k) \times C_3$

Subject to constraint: Rental Policy (P)

Our objective is to minimize rental cost of machines while minimizing the utilization time.

For a problem satisfying check the condition:

Either $a_{i1} + t_i \geq a_{j2} + t_i$ for all i, j.

or $a_{i2} + g_i \geq a_{j3} + g_i$

i.e. either min $(a_{i1} + t_i) \geq max\ (a_{j2} + t_i)$

or min $(a_{j3} + g_i) \geq max\ (a_{i2} + g_i)$

If the conditions are satisfied then go to step 2 else the data is not in the standard form.

7. Algorithm

The algorithm to minimize the rental cost is as follows:

Step 1: Convert the problem into two machine problem. Let g and H be factious machines having g_i and Hi as their processing times as

$G_i = a_{i1} + t_i + a_{i2} + g_i$

$H_i = t_i + a_{i2} + g_i + a_{i3}$

Step 2: Take equivalent job $\beta(k,m)$ and define the processing time as follows

i. $G_\beta = G_k + G_m - \min(G_m, H_k)$

ii. $H_\beta = H_k + H_m - \min(G_m, H_k)$

Step 3: Define a new reduces problem with the processing times G_i and H_i where job block (k,m) is replaced by single equivalent job β with processing time G_β and H_β as obtained in step 3.

Step 4: Apply Johnson's (1954) technique and obtain as optimal schedule of given jobs. Let the sequence be S_1.

Step 5: Obtain other sequence by putting 2^{nd}, 3^{rd}, ..., n^{th} jobs of sequence S_1 in the 1^{st} position and all other jobs of S_1 in same order let these sequences be S_2, S_3,....S_{n-1}.

Step 6: Compute $\sum A_{i1}, U_2(S_k), U_3(S_k)$ and

$R(S_k) = \sum A_{i1} \times C_1 + U_2(S_k) \times C_2 + U_3(S_k) \times C_3$

For all possible sequences S_k (k = 1,2,......n).

Step 7: Find min $R(S_k)$; k = 1,2,....,n. Let it be minimum for the sequence S_p, then sequence S_p will be the optimal sequence with rental cost $R(S_p)$.

8. Numerical Illustration

Consider 5 jobs, 3 machines flow shop problem in which processing timeswith transportation time are given in the table. The rental cost per unit time for machines M_1 , M_2 and M_3 are 4 units, 5 units and 2 units respectively under the rental policy P. Let β = (3,5) be job block.

Jobs	Machine M_1	$T_{i1 \to 2}$	Machine M_2	$T_{i2 \to 3}$	Machine M_3
1	20	3	15	2	35
2	25	2	20	2	40
3	15	3	10	4	45
4	10	5	30	2	50
5	30	2	25	3	55

Tableau :2

Solution: Check conditions:

max $(a_{i2} + g_i) \leq \min (a_{j3} + g_i)$ *satisfies*

As per step 1 : the processing time for two fictions machines G and H.

Jobs	G	H
I	G_i	H_i
1	40	55
2	49	64
3	32	62
4	47	87
5	60	85

Tableau : 3

As per step 2: Here β= (3,5)

$G_\beta = 32 + 60 – 60 = 32$ and $H_\beta = 62 + 85 – 60 = 87$.

As per step 3: The new reduced problem is

Jobs	Machine G	Machine H
1	40	55
2	49	64
β	32	87
4	47	87

Tableau : 4

As per step 4 : Obtaining the sequence with minimum makespan is

S_1= β - 1 – 4 – 2 i.e. 3 – 5– 1 – 4 – 2

Other feasible sequences which may corresponds to minimum rental cost are:

S_2 = 1 – 3 – 5 – 4 – 2,

S_3 = 4 – 3 – 5 – 1 – 2,

S_4 = 2 – 3 – 5 – 1 – 4.

From in – out tables for these sequences, we have.

For S_1 : $CT(S_1) = 257$; $U_2 (S_1) = 119$; $U_3(S_1) = 225$; $R(S_1) = 1545$.

For S_2 : $CT(S_2) = 265$; $U_2 (S_2) = 119$; $U_3(S_2) = 225$; $R(S_2) = 1545$.

For S_3 : $CT(S_3) = 272$; $U_2(S_3) = 107$; $U_3(S_3) = 225$; $R(S_3) = 1485$.

For S_4 : $CT(S_4) = 274$; $U_2(S_4) = 115$; $U_3(S_4) = 225$; $R(S_4) = 1525$.

Therefore, min $R(S_k) = 1485$ units and is for sequence S_3.

Hence the sequence $S_3 = 4 - 3 - 5 - 1 - 2$ is optimal sequence. With minimum rental cost 1485 units although the total elapsed time for S_3 is not minimum.

9. Conclusion

The algorithm proposed in this paper for to minimize the rental cost of machines gives an optimal sequence having minimum rental cost of machines irrespective of total elapsed time. The algorithm proposed by Johnson's(1954) to find an optimal sequence to minimize the makespan / total elapsed time is not always corresponds to minimum rental cost of the machines. Hence proposed algorithm is more efficient to minimize the rental cost of machines under a specified rental policy.

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