# Specially Structured Two Stage Flow Shop Scheduling To Minimize The Rental Cost, Processing Time Each Associated With Probabilities Including Weightage of Jobs

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**Abstract**
The paper deals with a specially structured n-jobs 2-machine flow shop scheduling problem under specified rental policy in which processing times are associated with their respective probabilities. Further jobs are attached with weights to indicate their relative importance. The objective of the paper is to obtain a optimal / near optimal sequence of jobs in order to minimize the rental cost of machines. A computer program followed by a numerical illustration is given to justify the algorithm.
**Keywords:** Specially structured flow shop scheduling. Rental policy, Processing time, Weightage of jobs.

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## 1. Introduction

Scheduling problems involve jobs that must scheduled on machine subject to certain constraints to optimize some objective function. The goal to specify a schedule that specifies when and on which machine each job is to be executed. All the scheduling models beginning from johnson's work in 1954 onward upto the year 1980 do not take into consideration the weight (in the sense of relative importance) of a job.
Miyazaki (and co-authors) in 1980 and Maggu (and co-authors) in 1982 have studied flowshop scheduling problems to the objective criterion as the minization of weighted mean flow time of jobs. The work was developed by Ignall a Scharge(1965), Bagga (1969), J.N.D. Gupta (1975), Szware (1977) Yoshida & Hitomi (1979) Singh T.P.(1985), Gupta Deepak (2005) etc. by considering various parameters. In the sense of providing relative importance in the process Chander Mouli (2005) associated weight with the jobs. Gupta Deepak (2012) studied specially structured nx2 flowshop scheduling under specified rental policy in which processing times were associated with probabilities. This paper is an attempt to extend the study made by Gupta Deepak (2012) by introducing the concept of weightage with jobs. Hence the problem

becomes wider and more applicable in process/ production industries. We have obtained an algorithm which gives minimum utilization time and hence minimum rental cost.

## 2. Practical Situation

Many applied and experimental situations occur in our day to day working in factories and industrial concern etc. The practical situation may be taken in a paper mill, sugar factory and oil refinery etc. Where various quality of papers, sugar and oil are produced with relative importance i.e. weight in jobs hence weightage of jobs is significant. An industrialist, to establish an industry or factory does not have enough money or does not want to take risk of investing huge money to purchase machines. So he prefer to take the machines on rent. Renting enables saving working capital given option for having the equipment, and allows up gradation to new technology.

## 3. Notations

$S$ : Sequence of jobs 1, 2, 3,….,n
$M_j$ : Machine j, j= 1,2.
$a_{ij}$ : Processing time of $i^{th}$ job on machine $M_j$
$p_{ij}$ : Probability associated to the processing time $a_{ij}$
$A_{ij}$ : Expected processing time of $i^{th}$ job on machine $M_j$
$A'_{ij}$ : Expected flow time of $i^{th}$ job on machine $M_j$
$w_i$ : weigth of $i^{th}$ job.
$A''_{ij}$ : Weighted flow time of $i^{th}$ job on machine $M_j$
$t_{ij}(S_k)$ : Completion time of $i^{th}$ job of sequence $S_k$ on machine $M_j$
$U_j(S_k)$ : Utilization time for which machine $M_j$ is required
$R(S_k)$ : Total rental cost for the sequence $S_k$ of all machine
$C_j$ : Renal cost of $j^{th}$ machine.

## 4. Definition

Completion time of $i^{th}$ job on machine $M_j$ is denoted by $t_{ij}$ and is defined as:
$t_{ij} = max\ (t_{i-1,,j}\ ,\ t_{i,,j-1}) + A_{ij}\ ;\ \ j \geq 2.$
where $A_{i,,j}$= Expected processing time of $i^{th}$ job on $j^{th}$ machine.

## 5. Rental Policy

The machines will be taken on rent as and when they are required and are returned as and when they are no longer required. i.e. the first machine will be taken on rent in the starting of the processing the jobs, 2nd machine will be taken on rent at time when 1st job is completed on the 1st machine.

## 6. Problem Formulation

Let some n jobs say $i$ ($i$ = 1,2,……..,n) are to be processed on two machines $M_j$ ( $j$ = 1,2) under the specified rental policy P such that no passing is allowed. Let $a_{ij}$ be the processing time of $i^{th}$ job on $j^{th}$ machine with $p_{ij}$ be their respective probabilities such that $\sum p_{i1}=1=\sum p_{i2}$; *Let* $w_i$ be the weight of $i^{th}$ *job*. Our objective is to find the sequence $\{S_k\}$ of jobs which minimize the utilization time and hence rental cost of the machines.
The mathematical model of the problem in matrix form can be stated as:

| Jobs | Machine $M_1$ | | Machine $M_2$ | | Weight |
|---|---|---|---|---|---|
| $i$ | $a_{i1}$ | $p_{i1}$ | $a_{i2}$ | $p_{i2}$ | $w_i$ |
| 1 | $a_{11}$ | $p_{11}$ | $a_{i2}$ | $p_{i2}$ | $w_1$ |
| 2 | $a_{21}$ | $p_{21}$ | $a_{12}$ | $p_{12}$ | $w_2$ |
| 3 | $a_{31}$ | $p_{31}$ | $a_{22}$ | $p_{22}$ | $w_3$ |
| - | - | - | - | - | - |
| $n$ | $a_{n1}$ | $p_{n1}$ | $a_{n2}$ | $p_{n2}$ | $w_n$ |

Tableau -1

Mathematically, the problem is stated as:

Minimize $R(S_k) = U_1(S_k) \times C_1 + U_2(S_k) \times C_2$

Subject to constraint: Rental Policy (P) i.e. our objective is to minimize rental cost of machines while minimizing the utilization time.

## 7. Assumptions

1. Jobs are independent to each other let n jobs be processed through two machines $M_1$ and $M_2$ in the order $M_1M_2$.
2. Pre-emption is not allowed. Once a job started on a machine the process on that machine cannot be stopped unless job is completed.
3. Either the weighted flow time of $i^{th}$ job on machine $M_1$is longer than the weighted flow time of $i^{th}$ job on machine $M_2$ or the weighted flow time of $i^{th}$ job on machine $M_1$ is shorter a than the weighted flow time of $i^{th}$ job on machine $M_2$ for all $i$.

   i.e. Either $A''_{i1} \geqq A''_{i2}$ or $A''_{i1} \leqq A''_{i2}$ for all $i$
4. Machine break down is not considered.

## 8. Algorithm

**Step 1:** Compute $A_{ij} = a_{ij} \times p_{ij}$ for j=1,2.

**Step 2:** Compute $A'_{i1}$ and $A'_{i2}$ as follows:

i. If min $(A_{ij}) = A_{i1}$ then ; j=1.2.

$A'_{i1} = A_{i1} + w_i$ and $A'_{i2} = A_{i2}$

ii. If min $(A_{ij}) = A_{i2}$ then ; j=1.2.

$A'_{i1} = A_{i1}$ and $A'_{i2} = A_{i2} + w_i$

**Step 3:** Find $A''_{ij} = A''_{ij} / w_i$ $i = 1,2,\ldots n$. and $j = 1,2$.

**Step 4:** Check the condition

either $A''_{i1} \geq A''_{i2}$

or $A''_{i1} \leq A''_{i2}$ for each $i$

**Step 5:** Obtain the job $J_1$ (say) having maximum processing time on 1st machine and job $J_n$ (say) having minimum processing time on 2nd machine.

**Step 6:** If $J_1 \neq J_n$ then put $J_1$ on the first position and $J_n$ as the last position and go to step 9 otherwise go to step 7.

**Step 7:** Take $G_1$ as the difference of processing time of job $J_1$ on $M_1$ from job $J_2$ (say) having next maximum processing time on $M_1$. Take $G_2$ as the difference of processing time of job $J_n$ on $M_2$ from job $J_{n-1}$ (say) having next minimum processing time on $M_2$.

**Step 8:** If $G_1 \leq G_2$ put $J_n$ on the last position and $J_2$ on the first position otherwise put $J_1$ on $1^{st}$ position and $J_{n-1}$ on the last position.

**Step 9:** Arrange the remaining (n-2) jobs between $1^{st}$ job & last job in any order, thereby we get the sequences $S_1, S_2 \ldots S_r$.

**Step 10:** Compute the total completion time $CT(S_k)$ k=1, 2…r.

**Step 11:** Calculate utilization time $U_2$ of $2^{nd}$ machine $U_2 = CT(S_k) - A_{i1}(S_k)$; $k$=1,2,…. r.

**Step 12:** Find rental cost $R(S_i) = \sum_{i=1}^{n} A_{i1}(S_k) \times C_1 + U_2 \times C_2$, where $C_1$ & $C_2$ are the rental cost per unit time of $1^{st}$ & $2^{nd}$ machine respectively.

**Computer Program:**

```
#include<iostream.h>
#include<stdio.h>
#include<conio.h>
#include<process.h>
int n;
float
a1[16],b1[16],a11[16],b11[16],w[16],g11[16],g22[16],l11[16],l22[16];
float macha[16],machb[16],maxv,u2;
int j[16],j1[16],j2[16],j3[16];
float costa,costb,cost;
int main()
{
clrscr();
int a[16],b[16];
float p[16],q[16],g1,g2;
cout<<"How many Jobs (<=15) : ";cin>>n;
if(n<1 || n>15)
{
cout<<endl<<"Wrong input, No. of jobs should be less than 15..\n
Exitting";
getch();exit(0);
}
for(int i=1;i<=n;i++)
{
```

```
cout<<"\nEnter the processing time and its probability of "<<i<<" job for machine A : ";
cin>>a[i]>>p[i];
cout<<"\nEnter the processing time and its probability of "<<i<<" job for machine B : ";
cin>>b[i]>>q[i];
cout<<"\nEnter the weight of "<<i<<" job  : ";
cin>>w[i];

//Calculate the expected processing times of the jobs for the machines:
g11[i] = a[i]*p[i];g22[i] = b[i]*q[i];j[i]=i;
}
cout<<"\n Enter the rental cost for Machine M1 & Machine M2 :";
cin>>costa>>costb;
cout<<endl<<"Expected processing time of machine A and B: \n";
for(i=1;i<=n;i++)
{
cout<<"\n"<<j[i]<<"\t"<<g11[i]<<"\t"<<g22[i]<<"\t"<<w[i]<<"\t";
cout<<endl;
}
float l11[16],l22[16];
for (i=1;i<=n;i++)
if(g11[i]<=g22[i])
{l11[i]=g11[i]+w[i];l22[i]=g22[i];}
else
{l11[i]=g11[i];l22[i]=g22[i]+w[i];}
cout<<endl<<"Expected processing time of machine A and B: \n";
float a1[16],b1[16];
for(i=1;i<=n;i++)
{
a1[i]=l11[i]/w[i];b1[i]=l22[i]/w[i];
cout<<"\n"<<j[i]<<"\t"<<a1[i]<<"\t"<<b1[i]<<"\t";
}
cout<<endl<<endl<<"displaying original table"<<endl;


for(i=1;i<=n;i++)
{
if((a1[i]<=b1[i])^(a1[i]>=b1[i]))
{ a1[i]=a1[i],b1[i]=b1[i]; }
 else
{ cout<<"\n The data is not in standard form"; getch();exit(0); }}
```

```
 void sort(float [],int);
// function declaration
for(i=1;i<=n;i++)
 { a11[i]=a1[i]; }
sort(a11,n);//fuction call
cout<<"\nSorted processing times in ascending order of Machine A :";
 for(i=1;i<=n;i++)
 { j1[i]=j[i]; cout<<"\n"<<j1[i]<<"\t"<<a11[i]; }
for(i=0;i<=n;i++)
{ b11[i]=b1[i];j[i]=i; }
 sort(b11,n);
// function call
cout <<"\nSorted processing times in ascending order of Machine B :";
 for(i=1;i<=n;i++)
{
j2[i]=j[i];
cout<<"\n"<<"j2[i]"<<j2[i]<<"\t"<<b11[i];
}
 if(j1[n]!=j2[1])

 {
  for(int k=2;k<=n;k++)

{
cout<<"j1[]"<<j1[k-1];
if(j1[k-1]!=j2[1])
 {
  j3[k]=j1[k-1];
   }
  else
  j3[k]=j1[k+1];

   }
  j3[1]=j1[n];
  j3[n]=j2[1];

 }
 else { g1=a11[j1[n]]-a11[j1[n-1]];
 g2=b11[j2[2]]-b11[j2[1]];
if(g1<=g2) { j3[1]=j1[n-1];
j3[n]=j2[1];
for(int g=2;g<=n-1;g++) {
```

```
j3[g]=j1[g-1]; }}
 else { j3[1]=j1[n];j3[n]=j2[2];
for(int f=2;f<=n-1;f++)
 { j3[f]=j2[f+1]; }}}
 macha[1]=g11[j3[1]];machb[1]=macha[1]+g22[j3[1]];
 // displaying solution

cout<<"\n\n\t**************************************************
************";
  cout<<"\n\t"<<"optimal sequence is";
  for(i=1;i<=n;i++) { cout<<"\t"<<j3[i]; }
   float time =0.0; cout<<endl<<endl<<"In-Out Table is"<<endl<<endl;
    cout<<"Jobs"<<"\t"<<"Machine M1"<<"\t"<<"Machine M2"<<endl;
     cout<<j3[1]<<"\t"<<time<<"--
"<<macha[1]<<"\t"<<"\t"<<macha[1]<<"--"<<machb[1]<<"\t"<<endl;
      for(i=2;i<=n;i++)
       { macha[i]=macha[i-1]+g11[j3[i]];
       if(machb[i-1]>macha[i])
       { maxv= machb[i-1];
       } else { maxv=macha[i]; }
        machb[i]=maxv+g22[j3[i]];
         cout<<j3[i]<<"\t"<<macha[i-1]<<"--
"<<macha[i]<<"\t"<<"\t"<<maxv<<"--"<<machb[i]<<"\t"<<endl; }
         u2=machb[n]-macha[1];
         cost=macha[n]*costa+u2*costb;
          cout<<"/n/nThe total rental cost of machines is:"<<cost;

cout<<"\n\n\t**************************************************
************";
            getch();
            return 0;
            }
            void sort(float x[],int n)
            // function decleration
             { float temp; int temp1; //outer for loop to control no of passea
for(int k=1;k<=n;k++)
 { //inner for loop for making comparison per pass
for(int m=1;m<=n-k;m++)
 { if(x[m]>x[m+1])
 { temp=x[m];temp1=j[m];
x[m]=x[m+1];j[m]=j[m+1];
x[m+1]=temp;j[m+1]=temp1; } } }getch();
```

}

## 9. Numerical Illustration

Consider 5 jobs, 2 machines flow shop problem with weight of jobs, processing times are associated with their respective probabilities are given in the following table. The rental cost per unit time for machines $M_1$ and $M_2$ are 6 units and 8 units respectively. Our objective is to obtain optimal schedule to minimize the total production time / total elapsed subject to minimization of the total of machine, under the rental policy P.

| Jobs | Machine $M_1$ | | Machine $M_2$ | | Weight of jobs |
|---|---|---|---|---|---|
| I | $a_{i1}$ | $p_{i1}$ | $a_{i2}$ | $p_{i2}$ | $w_i$ |
| 1 | 26 | 0.2 | 16 | 0.3 | 2 |
| 2 | 20 | 0.3 | 25 | 0.2 | 3 |
| 3 | 29 | 0.1 | 12 | 0.2 | 2 |
| 4 | 35 | 0.2 | 40 | 0.1 | 4 |
| 5 | 21 | 0.2 | 18 | 0.2 | 2 |

Tableau :2

**Solution :**

As per step 1: The expected processing time for machines $M_1$ and $M_2$ are

| Jobs | Machine $M_1$ | Machine $M_2$ | Weight of jobs |
|---|---|---|---|
| i | $A_{i1}$ | $A_{i2}$ | Wi |
| 1 | 5.2 | 4.8 | 2 |
| 2 | 6.0 | 5.0 | 3 |
| 3 | 2.9 | 2.4 | 2 |
| 4 | 7.0 | 4.0 | 4 |
| 5 | 4.2 | 3.6 | 2 |

Tableau : 3

As per step 2 and 3: The new reduced problem with weighted flow time for two machine $M_1$ and $M_2$ is :

| Jobs | Machine $M_1$ | Machine $M_2$ |
|---|---|---|
| i | $A_{i1}$ | $A_{i2}$ |
| 1 | 2.6 | 3.4 |
| 2 | 2.0 | 2.66 |
| 3 | 1.45 | 2.2 |
| 4 | 1.75 | 2.0 |
| 5 | 2.1 | 2.8 |

Tableau : 4

Here, we observe that $A_{i1} \leq A_{j2}$ for all values of $i$ ,
Max $A_{i1}$=2.6, which is for the $1^{st}$ job, .i.e. $J_1$=1.
Min $A_{i2}$=2.0, which is for the $4^{th}$ job, .i.e. $J_n$=4.
Also $J_1 \neq J_n$. On placing $J_1$ on first place and $J_n$ on last one, the optimal sequences are
$S_1$= 1 – 2 – 5 – 3 – 4.
$S_2$= 1 − 3 − 5 – 2 – 4,
$S_3$= 1 − 5 − 2 – 3 – 4,

$S_4$= 1 − 3 − 2 – 5 – 4, ----------, So on. There are six possible optimal sequences. The In-Out table for any of these six optimal sequences say $S_1$=1 – 5 – 2 – 3 – 4. is

| Jobs | Machine $M_1$ | Machine $M_2$ |
|---|---|---|
| i | In-Out | In-Out |
| 1 | 0.0 – 5.2 | 5.2 – 10.0 |
| 5 | 5.2 – 9.4 | 10.0 – 13.6 |
| 2 | 9.4 – 15.4 | 15.4 – 20.4 |
| 3 | 15.4 – 18.3 | 20.4 – 22.8 |
| 4 | 18.3 – 25.3 | 25.3 – 29.3 |

Tableau : 5

Here, total time elapsed CT($S_1$) = 29.3 units and

Utilization time of machine $M_2$ = $U_2(S_1)$ =24.1 units. Also $\sum_{i=1}^{n} A_{i1}$ =25.3 units.

Therefore the total rental cost for each of the sequence ($S_k$), k = 1, 2, --- 6 is
R($S_k$) = 25.3 × 6 + 24.1 × 8= 344.6 units.

## 10. Remarks

If we solve the above problem by Maggu & Das methods we get the optimal sequence as S= 3 – 4 – 2 – 5 – 1. The in-out flow table is

| Jobs | Machine $M_1$ | Machine $M_2$ |
|---|---|---|
| i | In - Out | In - Out |
| 3 | 0 – 2.9 | 2.9 – 5.3 |
| 4 | 2.9 – 9.9 | 9.9 – 13.9 |
| 2 | 9.9 – 15.9 | 15.9 – 20.9 |
| 5 | 15.9 – 20.1 | 20.9 – 24.5 |
| 1 | 20.1 – 25.3 | 25.3 – 30.1 |

Tableau : 6

Therefore, the total elapsed time =CT(S) =30.1 units and

Utilization time for $M_2$ = $U_2(S)$ = 27.2 units. Also $\sum_{i=1}^{n} A_{i1}$ =25.3 units.

Therefore Rental Cost is R(S) = 25.3 × 6 + 27.2 × 8 = 369.4 units.

## 11.Conclusion

The algorithm proposed here for specially structured two stage flow shop scheduling problem with weightage of jobs is more efficient as compared to the algorithm proposed by Maggu & Dass (1982) to find an optimal sequence to minimize the utilization time of the machines and hence their rental cost. The study may further be extended by introducing the concept of independent set up time, Transportation time (Cost), Weightage jobs etc.

## References

[1] Johnson S M (1954), “Optimal two and three stage production schedule with set up times included”, Naval Research Logistic, Vol.1, No.1, pp. 61-64.
[2] Ignall E and Schrage L (1965), “Application of the branch and bound technique to some flow shop scheduling problems”, Operation Research, Vol.13, pp.400-412.
[3] Bagga P C (1969), “Sequencing in a rental situation”, Journal of Canadian Operation Research Society, Vol.7, pp.152-153.
[4] Gupta J N D (1975), “Optimal Schedule for specially structured flow shop”, Naval Research Logistic, Vol.22, No.2, pp. 255-269.
[5] Szware W (1977), “Special cases of the flow shop problems”, *Naval Research Logistic*, Vol.22, No.3, pp. 483-492.
[6] Maggu P L and Das G (1977), “Equivalent jobs for job block in job scheduling”, Opsearch, Vol. 14, No.4, pp. 277-281.
[7] Yoshida and Hitomi (1979), “Optimal two stage production scheduling with set up times separated”, AIIE Transactions, Vol.11, No. 3,pp. 261-269.
[8] Gupta J N D (1988), “Two stage hybrid flow shop scheduling problem”, J. Ope. Res. Soc., Vol. 39, No. 4, pp. 359 – 364.
[9] Chandrasekharan R (1992), “Two-Stage Flowshop Scheduling Problem with Bicriteria ” O.R. Soc. ,Vol. 43, No. 9, pp.871-84.
[10] Singh T P (1985), “On n×2 shop problem involving job block. Transportation times and Break-down Machine times”, PAMS, Vol. XXI , pp.1-2.
[11] Chander S, K Rajendra & Deepak C (1992), “An Efficient Heuristic Approach to the scheduling of jobs in a flow shop”, European Journal of Operation Research, Vol. 61, pp.318-325.
[12] Anup (2002), “On two machine flow shop problem in which processing time assumes probabilities and there exists equivalent for an ordered job block”, JISSOR, Vol. XXIII No. 1-4, pp. 41-44.
[13] Singh T P, K Rajindra & Gupta Deepak (2005), “Optimal three stage production schedule the processing time and set times associated with probabilities including job block criteria”, Proceedings of National Conference FACM-2005,pp. 463-492.
[14] Narian L & Bagga P C (2005), “Two machine flow shop problem with availability constraint on each machine”, JISSOR, Vol. XXIV 1-4, pp.17-24.
[15] Singh, T P, Gupta Deepak (2005), “Minimizing rental cost in two stage flow shop , the processing time associated with probabilies including job block”, Reflections de ERA, Vol 1, No.2, pp.107-120.
[16] Pandian & Rajendran (2010), “Solving Constraint flow shop scheduling problems with three machines”, Int. J. Contemp. Math. Sciences, Vol.5, No. 19, pp.921-929.
[17] Gupta Deepak & Sharma Sameer (2011), “Minimizing rental cost under specified rental policy in two stage flow shop , the processing time associated with probabilities including breakdown interval and Job-block criteria”, European Journal of Business and Management, Vol. 2, No. 3, pp.85-103.
[18] Gupta Deepak, Sharma Sameer and Shashi Bala (2012), “Specially Structured Two Stage Flow Shop Scheduling To Minimize the Rental Cost”, International Journal of Emerging trends in Engineering and Development, Vol. 1, Issue 2, pp.206-215.