Minimizing Rental Cost For Specially Structured Two Stage Flow Shop Scheduling Processing Time Associated With Probabilities Including Weightage Of Jobs and Job Block Criteria

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Abstract
The purpose of this paper is to develop a new heuristic algorithm to find the optimal sequence to minimize the utilization time of machines and hence their rental cost for two stage specially structured flow shop scheduling under specified rental policy in which processing times are associated with their respective probabilities including weightage of jobs and job block criteria. The algorithm is supported by a computer programme followed by a numerical illustration.

Keywords: Specially structured flow shop scheduling, weightage of jobs, Rental cost, Processing time, idle time.

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1 Introduction

In a flow shop scheduling problem, the objective is to obtain a sequence of jobs which when processed on machine will optimize some well define criteria. Every job will go on these machines in a fixed order. The research into flow shop problems has drawn a great attention in the last decades with the aim to decrease the cost and to increase the effectiveness of industrial production. Johnson [1] gave a procedure to obtain the optimal sequence for n-jobs, two – three machines flow shop scheduling problem with an objective to minimize the makespan. The work was develop by Ignall and Scharge [2], Bagga [3], Smith et at [], Gupta, J.N.D [4], Maggu and Das [6], Yoshida and Hitomi [7], Singh, T.P. [10], Chander Sekharan [11], Anup [12], Gupta Deepak [13], Chander Mouli [] etc. by considering the various parameters.

Gupta Deepak, Sharma & Bala Shashi [18] introduced the concept of specially structured n × 2 flow shop scheduling under specified rental policy in which processing times were associated with probabilities.

Maggu and Das (1982) introduced the concept of job block criteria in the theory of scheduling. This concept is useful and significant in the sense to create a balance between the costs of providing priority in service to the customer and cost of giving services with non-priority customers. The decision maker may decide how much to charge extra to priority customers.

The paper is an attempt to extend the study made by Gupta Deepak, Sharma & Bala Shashi [18] by introducing the weightage in jobs & job block criteria. This making the problem wider and more practical in process/ production industry. We have obtained an algorithm which gives minimum utilization time and hence minimum rental cost.

2 Practical Situation

Many applied and experimental situations occur in our day to day working in factories and industrial concern where we have to restrict the processing of some jobs. The practical situation may be taken in a production industry; manufacturing industry etc, where some jobs has to give priority over other. Various practical situations occur in real life when one has got the assignments but does not have one's own machine or does not have enough money or does not want to take risk of investing huge amount of money to purchase machine. Under such circumstances, the machine has to be taken on rent in order to complete the assignments. In his starting career, we find a medical practitioner does not buy expensive machines say X-ray machine, the Ultra Sound Machine, Rotating Triple Head Single Positron Emission Computed Tomography, Scanner, Patient Monitoring Equipment, and Laboratory Equipment etc., but instead takes on rent. Rental of medical equipment is an affordable and quick solution for hospitals, nursing homes, physicians, which are presently constrained by the availability of limited funds due to the recent global economic recession. Renting enables saving working capital, gives option for having the equipment, and allows up-gradation to new technology.

3 Notations

S	: Sequence of jobs 1, 2, 3,….,n
M_j	: Machine; j= 1,2,3.
a_{ij}	: Processing time of i^{th} job on machine M_j
A_{ij}	: Expected processing time of i^{th} job on machine M_j
A'_{ij}	: Weightage flow time of i^{th} job on machine M_j
w_i	: weight of i^{th} job.
$t_{ij}(S_k)$	: Completion time of i^{th} job of sequence S_k on machine M_j
$U_j(S_k)$	: Utilization time for which machine M_j is required
$R(S_k)$	: Total rental cost for the sequence S_k of all machine
C_j	: Rental cost of j^{th} machine.
$CT(S_i)$	: Total completion time of the jobs for sequence S_k
β	: Equivalent job block.

4 Definition

Completion time of i^{th} job on machine M_j is denoted by t_{ij} and is defined as:

t_{ij} = $max\,(t_{i-1}, j\,,\, t_{i,,j-1}) + a_{ij} \times p_{ij}$ for $j \geq 2$

= $max\,(t_{i-1}, j\,,\, t_{i,,j-1}) + A_{ij}$ for $j \geq 2$

Where A_{ij} = Expected processing time of i^{th} job on j^{th} machine.

5 Rental Policy

The machines will be taken on rent as and when they are required and are returned as and when they are no longer required. i.e. the first machine will taken on rent in the starting of processing the jobs, 2nd machine will be taken on rent at time when 1st job is completed on the 1st machine.

6 Problem Formulation

Let some job i (i = 1,2,……..,n) are to be processed on two machines M_j (j = 1,2) under the specified rental policy P. Let a_{ij} be the processing time of i^{th} job on j^{th} machine with probabilities p_{ij}. Let w_i be the weight of i^{th} job. Let A''_{ij} be the weighted flow time of i^{th} job on j^{th} machine such that

either $A''_{i1} \geq A''_{i2}$

or $A''_{i1} \leq A''_{i2}$ for all values of i.

Our aim is to find the sequence (S_k) of the jobs which minimize the rental cost of the machine.

The mathematical model of the problem in matrix form can be stated as:

Jobs	Machine M1		Machine M2		Weight
i	a_{i1}	p_{i1}	a_{i2}	p_{i2}	w_i
1	A_{11}	p_{11}	a_{i2}	p_{i2}	w_1
2	A_{21}	p_{21}	a_{12}	p_{12}	w_2
3	a_{31}	p_{31}	a_{22}	p_{22}	w_2
-	-	-	-	-	-
n	a_{n1}	p_{n1}	a_{n2}	p_{n2}	w_n

Tableau -1

Mathematically, the problem is stated as:

Minimize $R(S_k) = \sum_{i=1}^{n} A_{i1} \times C_1 + U_j(S_k) \times C_2$

Subject to constraint: Rental Policy (P)

Our objective is to minimize rental cost of machines while minimizing the utilization time.

7 Assumptions

1. Jobs are independent to each other. Let n-jobs be processed through two machines A,B in the order AB.

2. Machine break down is not considered.
3. Pre-emption is not allowed i.e. once a job started on a machine the process on that machine cannot be stopped unless job in completed.
4. It is given to sequence K jobs $i_1, i_2, ... i_k$ as a block or group job in order ($i_1, i_2, ... i_k$) showing priority of job i_1 over i_2.
5. Either the weighted flow time of i^{th} job of machine M_1 is longer than the weighted flow time of i^{th} job on machine M_2 or the weighted flow time of i^{th} job on machine M_1 is shorter than the weighted flow time of i^{th} job on machine M_2 for all i.

   i.e. either $M''_{i1} \geq M''_{i2}$

   or $M''_{i1} \leq M''_{i2}$ for each i.

8 Algorithm

Step 1: Calculate the expected processing times, $A_{ij} = a_{ij} \times p_{ij} \forall i, j$.

Step 2: If $\min(A_{i1}, A_{i2}) = A_{i1}$

Then $G_i = A_{i1} + w_i$, $H_i = A_{i2}$

If $\min(A_{i1}, A_{i2}) = A_{i2}$

Then $G_i = A_{i1}$, $H_i = A_{i2} + w_i$

Step 3: Find the weighted flow time for two machines M1 and M2 as follow:

$A'_{i1} = G_i / w_i$ and $A'_{i2} = H_i / w_i \quad \forall i$

Step 4: Take equivalent job β =(k,m) and calculate the processing time $A'_{\beta1}$ and $A'_{\beta2}$ on the guide line of Maggu and Das (1982) as follows:

$A'_{\beta1} = A'_{k1} + A'_{m1} - \min(A'_{m1}, A'_{k2})$

$A'_{\beta2} = A'_{k2} + A'_{m2} - \min(A'_{m1}, A'_{k2})$

Step 5: Define a new reduces problem with the processing times A'_{i1} and A'_{i2} as defined in step 3 and jobs (k,m) are replaced by single equivalent job β with processing time $A'_{\beta1}$ and $A'_{\beta2}$ as defined in step 4.

Step 6: Obtain the job J_1 (say) having maximum processing time on 1st machine obtain the job J_n (say) having minimum processing time on 2nd machine.

Step 7: If $J_1 \neq J_n$ then put J_1 on the first position and J_n as the last position & go to step 11, Otherwise go to step 9.

Step 8: Take the difference of processing time of job J_1 on M_1 from job J_2 (say) having next maximum processing time on M_1. Call this difference as G_1.Also, Take the difference of processing time of job J_n on M_2 from job J_{n-1}(say) having next minimum processing time on M_2. Call the difference as G_2.

Step 9: If $G_1 \leq G_2$ put J_n on the last position and J_2 on the first position otherwise put J_1 on 1st position and J_{n-1} on the last position.

Step 10: Arrange the remaining (n-2) jobs between 1st job & last job in any order, thereby we get the sequences $S_1, S_2 \dots S_r$.

Step 11: Compute the total completion time $CT(S_k)$; k=1, 2…r. By computing in – out table for sequence S_k k=1, 2…r.

Step 12: Calculate utilization time U_2 of 2nd machine

$U_2 = CT(S_k) – A_{11}(S_k)$; k=1,2,…. r.

Step 13: Find rental cost $R(S_i) = \sum_{i=1}^{n} A_{i1}(S_k) \times C_1 + U_2 \times C_2$, where C_1 & C_2 are the rental cost per unit time of 1st & 2nd machine respectively.

9 **Computer Program:**

```
#include<iostream.h>
#include<stdio.h>
#include<conio.h>
#include<process.h>

int n;
float a_1[16],b_1[16],a11[16],b11[16];
float macha[16],machb[16],maxv,u2;
int j[16],j1[16],j2[16],j11[16],j12[16],j3[16];
float costa,costb,cost;
int group[16];//variables to store two job blocks
float minv,gbeta,hbeta;
int f=1;
int main()
{
        clrscr();
        int a[16],b[16];
        float p[16],q[16],g1,g2,w[16];
        cout<<"How many Jobs (<=15) : ";
        cin>>n;
        if(n<1 || n>15)
        {
                cout<<endl<<"Wrong input, No. of jobs should be less than 15..\n Exitting";
                getch();
                exit(0);
        }
        for(int i=1;i<=n;i++)
                {
                cout<<"\nEnter the processing time and its probability of "<<i<<" job for
machine A : ";
                cin>>a[i]>>p[i];
                cout<<"\nEnter the processing time and its probability of "<<i<<" job for
machine B : ";
                cin>>b[i]>>q[i];
                cout<<"\nEnter the weightage of "<<i<<"job:";
                cin>>w[i];
                //Calculate the expected processing times of the jobs for the machines:
                a_1[i] = a[i]*p[i];
                b_1[i] = b[i]*q[i];
                j[i]=i;
                }
                cout<<endl<<"Expected processing time of Machine A and B with
weightage:\n";
```

```
	cout<<"\n Enter the rental cost for Machine M1 & Machine M2 :";
	cin>>costa>>costb;
	cout<<endl<<"Expected processing time of machine A and B: \n";
	for(i=1;i<=n;i++)
	{
	cout<<"\n"<<j[i]<<"\t"<<a_1[i]<<"\t"<<b_1[i]<<"\t"<<w[i]<<"\t";
	cout<<endl;
	}

		float g11[16],h11[16];
	for(i=1;i<=n;i++)
	if(a_1[i]<=b_1[i])
	{g11[i]=a_1[i]+w[i];h11[i]=b_1[i];}
	else{g11[i]=a_1[i];h11[i]=b_1[i]+w[i];}
		float g22[16],h22[16];
	for(i=1;i<=n;i++)
	{g22[i]=g11[i]/w[i];h22[i]=h11[i]/w[i];
		cout<<"\n"<<j[i]<<"\t"<<g22[i]<<"\t"<<h22[i]<<"\t";
	}
	cout<<endl<<endl<<"displaying original scheduling table"<<endl;



	cout<<"\nEnter the two job blocks(two numbers from 1 to "<<n<<"):";
	cin>>group[0]>>group[1];
//calculate G_Beta and H_Beta
if(g22[group[1]]<h22[group[0]])
	{
	minv=g22[group[1]];
	}
else
	{
	minv=h22[group[0]];
	}
gbeta=g22[group[0]]+g22[group[1]]-minv;
hbeta=h22[group[0]]+h22[group[1]]-minv;
cout<<endl<<endl<<"G_Beta="<<gbeta;
cout<<endl<<"H_Beta="<<hbeta;
int j1[16];
float a1[16],b1[16];
for(i=1;i<=n;i++)
	{
	if(j[i]==group[0]||j[i]==group[1])
	{
	f--;
	}
else
	{
	j1[f]=j[i];
```

```
        }
        f++;
        }
        j1[n-1]=17;

for(i=1;i<=n-2;i++)
        {
        a1[i]=g22[j1[i]];
        b1[i]=h22[j1[i]];
        }
        a1[n-1]=gbeta;
        b1[n-1]=hbeta;
        cout<<endl<<endl<<"displaying original scheduling table"<<endl;
for(i=1;i<=n-1;i++)
        {
        cout<<j1[i]<<"\t"<<a1[i]<<"\t"<<b1[i]<<endl;
        }
        for(i=1;i<=n-1;i++)
        {
        if((a1[i]>=b1[i])^(a1[i]<=b1[i]))
          {
                a1[i]=a1[i],b1[i]=b1[i];
          }
        else
        {
        cout<<"\n The data is not in standard form";
        getch();
        exit(0);
        }
        }
int j11[16], j12[16],j2[16],j3[16];
void sort(float [],int []);// function declaration
for(i=1;i<=n-1;i++)
{
a11[i]=a1[i];
j3[i]=j1[i];
}
sort(a11,j3);//fuction call
cout<<"\nSorted processing times in ascending order of Machine A :\n";

for(i=1;i<=n-1;i++)
{
j11[i]=j3[i];
cout<<"\n"<<j11[i]<<"\t"<<a11[i];
}
for(i=1;i<=n-1;i++)
{
b11[i]=b1[i];
j2[i]=j1[i];
}
```

```
sort(b11,j2);// function call
cout <<"\nSorted processing times in ascending order of Machine B :\n";
for(i=1;i<=n-1;i++)
{
j12[i]=j2[i];
cout<<"\n"<<j12[i]<<"\t"<<b11[i];
}

if(j11[n-1]!=j12[1])
{
j3[1]=j11[n-1];j3[n-1]=j12[1];
for(int k=2;k<=n-2;k++)
{
if(j11[k-1]!=j12[1])
{
j3[k]=j11[k-1];
}
else
{
if(j11[n-2]!=j12[1])
{
j3[k]=j11[n-2];
}
}
}
}
else
{
g1=a11[j11[n-1]]-a11[j11[n-2]];
g2=b11[j2[12]]-b11[j12[1]];
if(g1<=g2)
{
j3[1]=j11[n-2];j3[n-1]=j12[1];
for(int g=2;g<=n-2;g++)
{
j3[g]=j11[g-1];
}
}
else
{
j3[1]=j11[n-1];j3[n-1]=j12[2];
for(int f=2;f<=n-2;f++)
{
j3[f]=j2[f+1];
}
}
}
int arr[16],m=1;
for(i=1;i<=n;i++)
        {
```

```
        if(j3[i]==17)
        {
         arr[m]=group[0];
        arr[m+1]=group[1];
        m=m+2;
        continue;
        }
else
{
        arr[m]=j3[i];
        m++;
        }
        }

macha[1]=a_1[arr[1]];machb[1]=macha[1]+b_1[arr[1]];

// displaying solution
cout<<"\n\n\t************************************************************";
cout<<"\n\t"<<"optimal sequence is";
for(i=1;i<=n;i++)
{
cout<<"\t"<<arr[i];
}
float time =0.0;
cout<<endl<<endl<<"In-Out Table is"<<endl<<endl;
cout<<"Jobs"<<"\t"<<"Machine M1"<<"\t"<<"Machine M2"<<endl;
cout<<arr[1]<<"\t"<<time<<"--"<<macha[1]<<"\t"<<"\t"<<macha[1]<<"--
"<<machb[1]<<"\t"<<endl;
for(i=2;i<=n;i++)
{
macha[i]=macha[i-1]+a_1[arr[i]];
if(machb[i-1]>macha[i])
{
maxv= machb[i-1];
}
else
{
maxv=macha[i];
}
machb[i]=maxv+b_1[arr[i]];
cout<<arr[i]<<"\t"<<macha[i-1]<<"--"<<macha[i]<<"\t"<<"\t"<<maxv<<"--
"<<machb[i]<<"\t"<<endl;
}
u2=machb[n]-macha[1];
cost=macha[n]*costa+u2*costb;
cout<<"\n\nThe total rental cost of machines is:"<<cost;
cout<<"\n\n\t************************************************************";
getch();
return 0;
}
```

```
void sort(float x[],int y[])// function decleration
{
float temp; int temp1;
//outer for loop to control no of passea
for(int k=1;k<n;k++)
{
//inner for loop for making comparison per pass
for(int m=1;m<n-k;m++)
{
        if(x[m]>x[m+1])
        {
        temp=x[m];temp1=y[m];
        x[m]=x[m+1];y[m]=y[m+1];
        x[m+1]=temp;y[m+1]=temp1;
        }
}
}
}
```

10 Numerical Illustration

Consider 5 jobs, 2 machines problem to minimize the rental cost. The processing times associated with their probabilities and weights of jobs are given. The renatal cost per unit time machines M_1 and M_2 are 8 and 6 units respective. Also jobs 2, 5 are to be processed as a group job (2, 5). Our objective is to obtain optimal schedule to minimize the utilization time and hence the total rental cost of the machines, under the rental policy p.

Jobs	Machine M1		Machine M2		Weight
i	a_{i1}	p_{i1}	a_{i2}	p_{i2}	w_i
1	80	.2	120	.3	4
2	30	.3	150	.1	3
3	40	.1	40	.2	2
4	100	.2	90	.3	1
5	60	.2	140	.1	1

Tableau :2

Solution: As per step 1: The expected processing time for machines M_1 and M_2 is as follow:

Jobs	Machine M1	Machine M2	Weight
I	a_{i1}	a_{i2}	w_i
1	16	36	4
2	9	15	3
3	4	8	2
4	20	27	1
5	12	14	1

Tableau : 3

As per step 2 & 3: Reduced problem with weighted flow time for two machine M_1 and M_2 is as follow:

Jobs i	Machine M1 A'_{i1}	Machine M2 A'_{i2}
1	5	9
2	4	5
3	3	4
4	21	27
5	13	14

Tableau : 4

As per step 4: Here β= (2, 5)

$A'_{\beta 1} = 4 + 13 - 5 = 12.$

$A'_{\beta 2} = 5 + 14 - 5 = 14 .$

As per step 5: New reduced problem is as follow

Jobs i	Machine M1 A'_{i1}	Machine M2 A'_{i2}
1	5	9
β	12	14
3	3	4
4	21	27

Tableau : 5

Here each $A'_{i1} \leq A'_{i2}$ for all i.
Also max A'_{i1} = 21 which is for job 4 i.e. $J_1 = 4$
Min A'_{i2} = 4 which is for job 3.
i.e. $j_n = 3$.
Since $J_1 \neq J_n$
put $J_1 = 4$ on the 1^{st} position
and J_n= 3 on the last position.
Therefore, the optimal sequence are:
$S_1 = 4 - 2 - 5 - 1 - 3$.
$S_2 = 4 - 1 - 2 - 5 - 3$.
The total elapsed time is same for all the sequences.
The in – out table for the sequence $S_2 = 4 - 1 - 2 - 5 - 3$ is

Jobs	Machine M_1	Machine M_2
i	In - Out	In - Out
4	0 – 20	20 – 47
1	20 – 36	47 – 73
2	36 – 45	73 – 88
5	45 – 57	88 – 102
3	57 – 61	102 – 110

Tableau : 6

Total elapsed time
= CT (S_2) = 110 units
Utilization time for M_2
= $U_2(S_2)$ = 110 – 20
= 90 units
Also $\sum A_{i1} = 61$
Total rental cost
= 61 × 8 + 110 × 6
= 488 + 660
= 1148 units.

11 Remarks

By Maggu & Dass (1982) method
The in – out table for the sequence S = 3 – 1 – 2 – 5 – 4 is

Jobs	Machine M_1	Machine M_2
3	0 – 4	4 – 12
1	4 – 20	12 – 48
2	20 – 29	48 – 63
5	29 – 41	63 – 77
4	41 – 61	77 – 104

Tableau : 7

Total elapsed time
= CT (S) = 104 units

Utilization time for M_2

$= U_2(S) = 104 – 4$

= 100 units

Also $\sum A_{i1} = 61$

Therefore total rental cost

$= 61 \times 8 + 100 \times 6$

= 488 + 600

= 1088 units.

12 Conclusion

The algorithm proposed in this paper for to minimize the rental cost of machines gives an optimal sequence having minimum rental cost of machines irrespective of total elapsed time. The algorithm proposed by Maggu & Dass (1982) to find an optimal sequence to minimize the makespan / total elapsed time is not always corresponds to minimum rental cost the machines. Hence proposed algorithm is more efficient to minimize the rental cost of machines under a specified rental policy.

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