An effective Manpower Planning approach in an organization through two grade system

P. Pandiyan[1], G. Subash Chandra Bose[2], R. Vinoth[1*] and K. Kannadasan[1]
[1]Department of Statistics, Annamalai University, Tamil Nadu, India.
[2]Department of Statistics, Manonmaniam Sundranar University, Tirunelveli, Tamil Nadu, India.
*maalinga@ gmail.com

Abstract

Growing attention to long-range planning is required of management inmost organizations. The expected time to reach the threshold level, in the time of recruitment with the assumptions that the times between decisions epochs are independent and identically distributed (i.i.d) random variable. The expected time to attain the threshold is calculated by Generalized Rayleigh distribution using shock model approach and cumulative damage process. Numerical examples are given to illustrate various aspects of the model considered for the expected time.

Keywords: expected time, grade, organization, threshold

Introduction

Maintenance activities are the backbone of a successful and profitable company. Assuming that, organization has two grades of personnel and that the loss of manpower in one category is compensated by utilization of manpower available in the order to sustain the activities intact. The two grades are assumed to have Generalized Rayleigh distribution. Obviously the breakdown occurs only when the total depletion crosses the maximum of the two threshold levels. One can see for more detail in Esary et al., (1973), Pandiyan et al., (2010), discussed about the expected time to cross threshold level period.

These assumptions are somewhat artificial, but are made because of the lack of detailed real-world information on one hand and in order to illustrate the proceedings on the other hand. The organization comprises two grades of personnel. Mobility or transfer of manpower from one grade to the other is permitted. The time to recruitment is equal to the maximum of the time taken for each one of the two grades to cross the threshold which follows Generalized Rayleigh distribution. The processes which give rise to policy revisions and the threshold random variables are statistically independent. The policy decisions are taken with inter arrival times which are i.i.d. random variables depending upon the market environment, production, etc.

Notations

X_i : a continuous random variable denoting the amount of loss of manpower caused to the system on the i^{th} occasion of policy announcement (Shock), 1, 2….k and $X_i's$ are i.i.d

Y_1, Y_2 : continuous random variable denoting the threshold levels for the two grades which follows Generalized Rayleigh distribution.

U_i : a random variable denoting the inter-arrival times between contact with c.d.f. $F_i(.)$, $i = 1,2,3 \dots k$.

$g(.)$: The probability density function of X_i; $g^*(.)$: Laplace transform of $g(.)$

$g_k(.)$: the k- fold convolution of $g(.)$ i.e., p.d.f. of $\sum_{j=1}^{k} X_i$

$f(.)$: p.d.f. of random variable denoting between successive policy announcement with the corresponding c.d.f. $F(.)$.
$F_k(.)$: k-fold convolution of $F(.)$; $S(.)$: Survival function.
$V_k(t)$: Probability of exactly k policy announcements; $L(t)$: $1 - S(t)$.

Model Description

The two-parameters generalized Rayleigh distribution is a particular member of the generalized Weibull distribution, originally proposed by Mudholkar and Srivastava (1993).
In general, assuming that the threshold is Y

$$P(X_i < Y) = \int_0^\infty g_k(x)\,\bar{H}(x)dx \tag{1}$$

Now the threshold Y is such that it has two components namely Y_1 component and Y_2 component. Transfer of component from Y_1 to Y_2 is also possible. We have the breakdown of the system is at $Y = max(Y_1, Y_2)$.

$$P[max(Y_1, Y_2)] = P[(Y_1 < y) \cap (Y_2 < y)]$$
$$= P[Y_1 < y]P[Y_2 < y]$$

Now that, Y_1 and Y_2 follow Generalized Rayleigh distribution with parameter λ_1, λ_2.

$$P\left(\sum_{i=1}^{k} X_i < Y\right) = \int_0^\infty g_k(x)\left[e^{-2\lambda_2 x} + e^{-2\lambda_1 x} - e^{-2x(\lambda_1+\lambda_2)}\right]dx$$
$$= [g^*(2\lambda_1)]^k + [g^*(2\lambda_2)]^k - \left[g^*\left(2(\lambda_1 + \lambda_2)\right)\right]^k \tag{2}$$

Now the survival function $S(t)$ is

$$S(t) = P(T > t) = \text{Probability that the total damage survives beyond t}$$

$$= \sum_{k=0}^{\infty} P\ \{\text{there are exactly k epochs in } (0,t] * P\ (\text{the total cumulative } (0,t]\}$$

$$S(t) = P(T > t) = \sum_{k=0}^{\infty} V_k(t)P\left(X_i < max(Y_1, Y_2)\right)$$

It is also known from renewal process that

$$P(\text{exactly k policy decesions in } (0,t]) = F_k(t) - F_{K+1}(t) \quad \text{with} \quad F_0(t) = 1$$
$$= \sum_{k=0}^{\infty} V_k(t)P(X_i < Y)$$

$$= \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)]\,[g^*(2\lambda_1)]^k + \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)]\,[g^*(2\lambda_2)]^k - \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)]\,\left[g^*\big(2(\lambda_1+\lambda_2)\big)\right]^k$$

Now, $L(T) = 1 - S(t)$

Taking Laplace transform of L(T), we get

$$= \left\{ [1 - (1 - g^*(2\lambda_1))] \sum_{k=1}^{\infty} F_k(t)\,[g^*(2\lambda_1)]^{k-1} + [1 - (1 - g^*(2\lambda_2))] \sum_{k=1}^{\infty} F_k(t)\,[g^*(2\lambda_2)]^{k-1} - \left[1 - \left(1 - g^*\big(2(\lambda_1+\lambda_2)\big)\right)\right] \sum_{k=1}^{\infty} F_k(t)\,\left[g^*\big(2(\lambda_1+\lambda_2)\big)\right]^{k-1} \right\} \quad (3)$$

Let the random variable U denoting inter arrival time which follows exponential with parameter c. Now $f^*(s) = \left(\frac{c}{c+s}\right)$, substituting in the above equation (3) we get

$$l^*(s) = \left\{ \frac{(1 - g^*(2\lambda_1))f^*(s)}{(1 - g^*(2\lambda_1)f^*(s))} + \frac{(1 - g^*(2\lambda_2))f^*(s)}{(1 - g^*(2\lambda_2)f^*(s))} - \frac{\left(1 - g^*\big(2(\lambda_1+\lambda_2)\big)\right)f^*(s)}{\left(1 - g^*\big(2(\lambda_1+\lambda_2)\big)f^*(s)\right)} \right\}$$

$$= \left\{ \frac{c[1 - g^*(2\lambda_1)]}{[c + s - g^*(2\lambda_1)c]} + \frac{c[1 - g^*(2\lambda_2)]}{[c + s - g^*(2\lambda_2)c]} - \frac{c\left[1 - g^*\big(2(\lambda_1+\lambda_2)\big)\right]}{\left[c + s - g^*\big(2(\lambda_1+\lambda_2)\big)c\right]} \right\} \quad (4)$$

$$E(T) = -\frac{d}{ds} l^*(s), given\ s = 0$$

$$E(T^2) = \frac{d^2}{ds^2} l^*(s) \quad given \quad s = 0$$

From which variance can be obtained

$$E(T) = \frac{1}{c[1 - g^*(2\lambda_1)]} + \frac{1}{c[1 - g^*(2\lambda_2)]} - \frac{1}{c\left[1 - g^*\big(2(\lambda_1+\lambda_2)\big)\right]} \quad on\ simplification$$

$$E(T^2) = \frac{2}{c^2[1 - g^*(2\lambda_1)]^2} + \frac{2}{c^2[1 - g^*(2\lambda_2)]^2} - \frac{2}{c^2\left[1 - g^*\big(2(\lambda_1+\lambda_2)\big)\right]^2} \quad on\ simplification$$

$$V(T) = E(T^2) - [E(T)]^2$$

$$g^*(.)\sim exp(\lambda), g^*(2\lambda_1) = \frac{\mu}{\mu+\lambda_1}, g^*(2\lambda_2) = \frac{\mu}{\mu+\lambda_2}, g^*\big(2(\lambda_1+\lambda_2)\big) = \frac{\mu}{\mu + 2(\lambda_1+\lambda_2)}$$

Results

$$E(T) = \frac{1}{c}\left\{ \frac{\mu + 2\lambda_1}{2\lambda_1} + \frac{\mu + 2\lambda_2}{2\lambda_2} - \frac{\mu + 2(\lambda_1+\lambda_2)}{2(\lambda_1+\lambda_2)} \right\} \quad on\ simplification \quad (5)$$

$$V(T) = \frac{2(\mu + 2\lambda_1)^2}{4\lambda_1^2 c^2} + \frac{2(\mu + 2\lambda_2)^2}{4\lambda_2^2 c^2} - \frac{3\big(\mu + 2(\lambda_1 + \lambda_2)\big)^2}{4c^2(\lambda_1 + \lambda_2)^2} - 2\left[\frac{\mu + 2\lambda_1(\mu + 2\lambda_2)}{4c^2\lambda_1\lambda_2}\right] + 2\left[\frac{(\mu + 2\lambda_2)\big(\mu + 2(\lambda_1 + \lambda_2)\big)}{4c^2\lambda_2(\lambda_1 + \lambda_2)}\right] + 2\left[\frac{(\mu + 2\lambda_1)\big(\mu + 2(\lambda_1 + \lambda_2)\big)}{4c^2\lambda_1(\lambda_1 + \lambda_2)}\right] \quad on\ simplification \tag{6}$$

Numerical Illustration

On the basis of the numerical illustration the following conclusions regarding expected time and variance consequent to the changes in the different parameters can be observed in Figures 1 to 6 that follow.

Figure 1

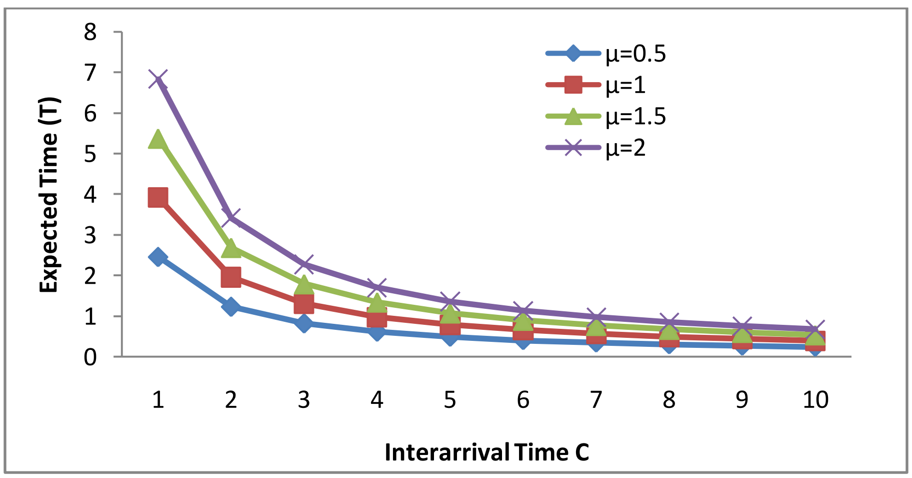


Figure 2

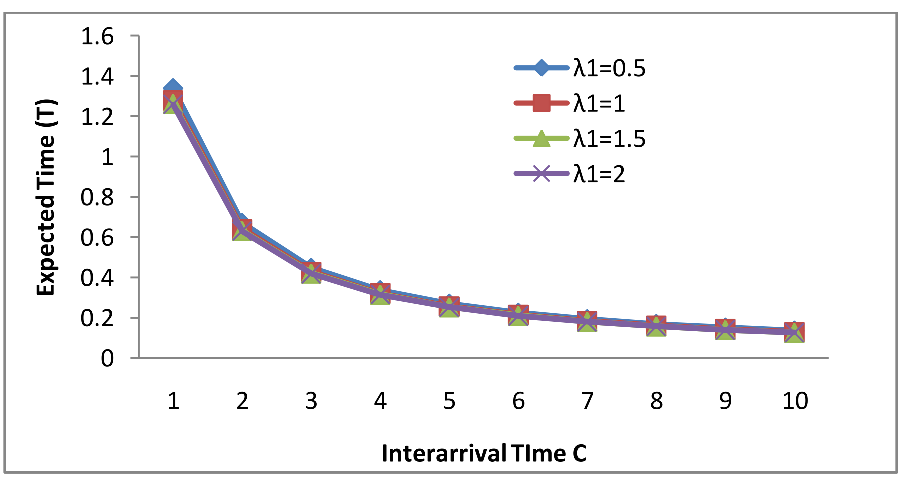


Figure 3

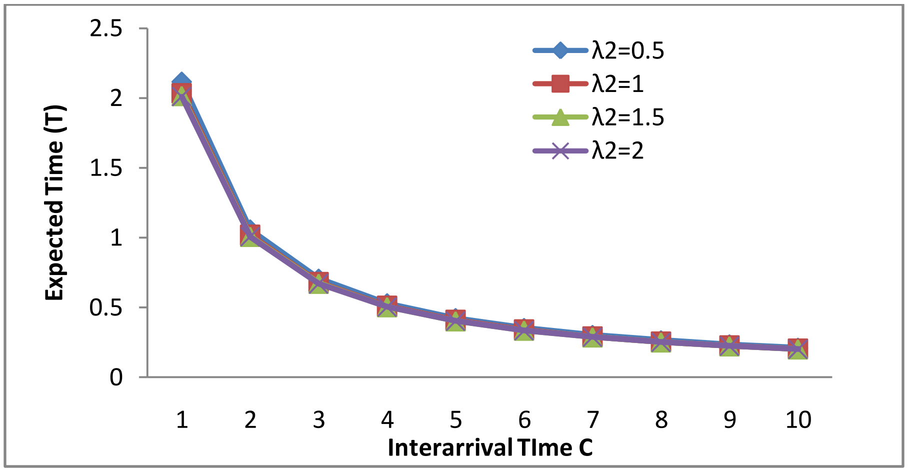


Figure 4

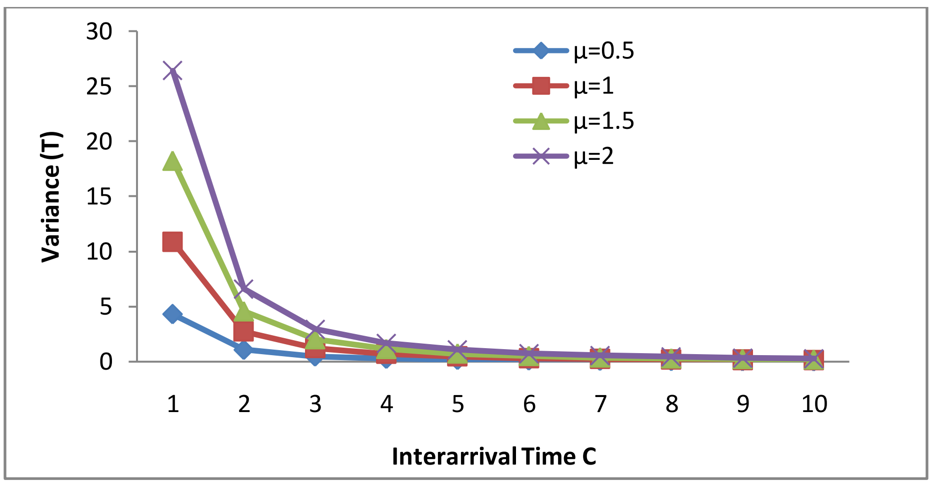


Figure 5

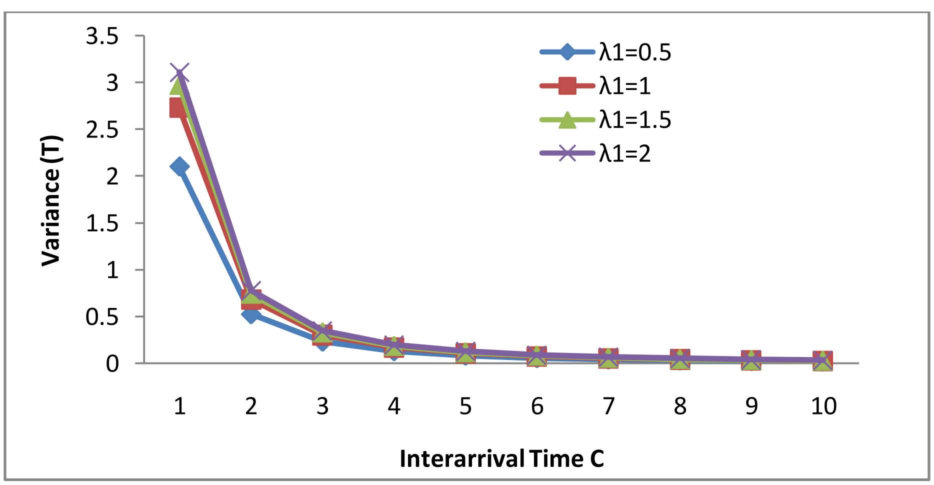
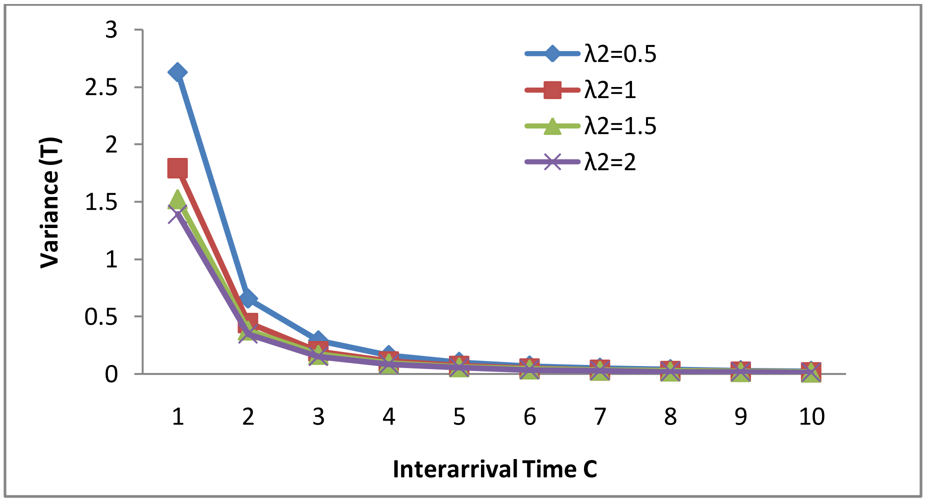


Figure 6



Conclusions:

When μ is kept fixed with other parameters λ_1, λ_2, the inter-arrival time $'c'$, which follows exponential distribution, is an increasing parameter. Therefore, the value of the expected time $E\ (T)$ to cross the threshold is decreasing, for all cases of the parameter value $\mu = 0.5,1,1.5, 2$. When the value of the parameter μ increases, the expected time is found increasing, this is observed in Figure 1. The same case is found in Variance V (T) which is observed in Figure 4.

When λ_1 is kept fixed with other parameters μ, λ_2, the inter-arrival time $'c'$ increases, the value of the expected time E (T) to cross the threshold is found to be decreasing, in all the cases of the parameter value $\lambda_1 = 0.5,1,1.5, 2$. When the value of the parameter λ_1 increases, the expected time is found increasing. This is indicated in Figure 2. The same case is observed in the Variance V (T) which is observed in Figure 5.

When λ_2 is kept fixed with other parameters μ, λ_1, the inter-arrival time $'c'$ increases, the value of the expected time E (T) to cross the threshold is found to be decreasing, in all the cases of the parameter value $\lambda_2 = 0.5,1,1.5, 2$. When the value of the parameter λ_2 increases, the expected time is found increasing. This is indicated in Figure 3. The same case is observed for the Variance V (T) which is observed in Figure 6.

References

1. Esary, J.D., A.W. Marshall and F. Proschan. (1973), Shock models and wear processes. Ann. Probability, 1(4), 627-649.
2. Mudholkar, G.S. and Srivastava, D.K. (1993), Exponentiated Weibull family for analyzing bathtub failure data. IEEE Trans. Reliability 42, 299–302.
3. Pandiyan, P., K. Kannadasan, R. Vinoth and R. Saminathan (2010), Stochastic Model for Mean and Variance to Recruit Manpower in Organization. Journal of Advanced in Applied Mathematical Analysis.vol.5, pp.77-82.