A Common Fixed Point Theorem in Complex Valued b-Metric Spaces

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Abstract. In this paper we prove a common fixed point theorem for four self-mappings in a complete complex valued b-metric space.

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1.Introduction

Banach contraction principle in [1] was the starting point for many researchers during last decades in the field of nonlinear analysis. In 1989, Bakhtin [2] introduced the concept of b-metric space as a generalization of metric spaces. The concept of complex valued b-metric spaces was introduced in 2013 by Rao et al. [3], which was more general than the well-known complex valued metric spaces that were introduced in 2011 by Azam et al. [4]. The main purpose of this paper is to present common fixed point results of four self-mappings satisfying a rational inequality on complex valued b-metric spaces.

Definition 1 (see [5]). Let X be a nonempty set and let $s \geq 1$ be a given real number. A function $d: X \times X \rightarrow [0, \infty)$ is called a b-metric if for all $x, y, z \in X$ the following conditions are satisfied:

(i) $d(x,y) = 0$ if and only if $x = y$,
(ii) $d(x,y) = d(y,x)$ and
(iii) $d(x,y) \leq s[d(x,z) + d(z,y)]$.

The pair (X, d) is called a b-metric space. The number $s \geq 1$ is called the coefficient of (X, d).

Example 2 (see [6]). Let (X, d) be a metric space and $\rho(x,y) = \left(d(x,y)\right)^p$, where

$p > 1$ is a real number. Then (X, ρ) is a b-metric space with $s = 2^{p-1}$.

Let $\mathbb{C}$ be the set of all complex numbers and $z_1, z_2 \in \mathbb{C}$. Define a partial order relation $\precsim$ on $\mathbb{C}$ as follows:

$$z_1 \precsim z_2 \text{ if and only if } Re(z_1) \leq Re(z_2) \text{ and } Im(z_1) \leq Im(z_2).$$

Thus $z_1 \precsim z_2$ if one of the followings holds:

(1) $Re(z_1) = Re(z_2)$ and $Im(z_1) = Im(z_2)$,
(2) $Re(z_1) < Re(z_2)$ and $Im(z_1) = Im(z_2)$,
(3) $Re(z_1) = Re(z_2)$ and $Im(z_1) < Im(z_2)$ and
(4) $Re(z_1) < Re(z_2)$ and $Im(z_1) < Im(z_2)$.

We write $z_1 \lnsim z_2$ if $z_1 \precsim z_2$ and $z_1 \neq z_2$ i.e., one of (2), (3) and (4) is satisfied and we will write $z_1 \prec z_2$ if only (4) is satisfied.

Remark 1: We can easily check the followings:

(i) $a, b \in \mathbb{R}, a \leq b \Rightarrow az \precsim bz \ \forall z \in \mathbb{C}$.
(ii) $0 \precsim z_1 \lnsim z_2 \Rightarrow |z_1| < |z_2|$.
(iii) $z_1 \precsim z_2$ and $z_2 \prec z_3 \Rightarrow z_1 \prec z_3$.

Definition 3 (see [4]). Let X be a nonempty set. A function $d: X \times X \rightarrow \mathbb{C}$ is called a complex valued metric on X if for all $x, y, z \in X$ the following conditions are satisfied:

(i) $0 \precsim d(x, y)$ and $d(x, y) = 0$ if and only if $x = y$,
(ii) $d(x, y) = d(y, x)$ and
(iii) $d(x, y) \precsim d(x, z) + d(z, y)$

The pair (X, d) is called a complex valued metric space.

Example 4 (see [7]). Let $X = \mathbb{C}$. Define the mapping $d: X \times X \rightarrow \mathbb{C}$ by

$$d(x, y) = i|x - y|, \text{ for all } x, y \in X.$$

Then (X, d) is a complex valued metric space.

Example 4.1. Let $X = \mathbb{C}$. Define the mapping $d: X \times X \rightarrow \mathbb{C}$ by

$$d(x, y) = e^{ik}|x - y|, \text{ where } k \in [0, {}^{\pi}/_{2}], \forall x, y \in X.$$

Then (X, d) is a complex valued metric space.

Definition 5 (see[3]). Let X be a nonempty set and let $s \geq 1$ be given real number. A function $d: X \times X \rightarrow \mathbb{C}$ is called a complex valued b-metric on X if for all $x, y, z \in X$ the following conditions are satisfied:

(i) $0 \precsim d(x, y)$ and $d(x, y) = 0$ if and only if $x = y$
(ii) $d(x, y) = d(y, x)$

(iii) $d(x,y) \precsim s[d(x,z) + d(z,y)]$.

The pair (X,d) is called a complex valued b-metric space.

Example 6(see[3]). Let $X = [0,1]$. Define the mapping $d: X \times X \rightarrow \mathbb{C}$ by

$$d(x,y) = |x-y|^2 + i\,|x-y|^2, \text{ for all } x, y \in X.$$

Then (X,d) is a complex valued b-metric space with $s = 2$.

Definition 7(see[3]). Let (X,d) be a complex valued b-metric space. Consider the following.

(i) A point $x \in X$ is called an interior point of a set $A \subseteq X$ whenever there exists $0 \prec r$, such that $B(x,r) = \{ y \in X : d(x,y) \prec r\} \subseteq A$.

(ii) A point $x \in X$ is called a limit point of a set A whenever, for every $0 \prec r \in \mathbb{C}$, $B(x,r) \cap (A - \{x\}) \neq \phi$.

(iii) A subset A of X is called open whenever each point of A is an interior point of A.

(iv) A subset A of X is called closed whenever each limit point of A belongs to A.

(v) A subbasis for a Hausdorff topology τ on X is a family $F = \{ B(x,r) : x \in X \text{ and } 0 \prec r\}$.

Definition 8 (see [3]). Let (X,d) be a complex valued b-metric space and a sequance $\{x_n\}$ in X and $x \in X$. Consider the following.

(i) If for every c, with $0 \prec c$, there is $N \in \mathbb{N}$ such that, for all $n > N$, $d(x_n, x) \prec c$, then $\{x_n\}$ is said to be convergent, $\{x_n\}$ converges to x, and x is the limit point of $\{x_n\}$. We denote this by $\lim_{n\to\infty} x_n = x$ or $\{x_n\} \to x$ as $n \to \infty$.

(ii) If for every $\in \mathbb{C}$, with $0 \prec c$, there is $N \in \mathbb{N}$ such that, for all $n > N$, $d(x_n, x_{n+m}) \prec c$, where $m \in \mathbb{N}$, then is $\{x_n\}$ said to be Cauchy sequence.

(iii) If every Cauchy sequence in X is convergent, then (X,d) is said to be a complete complex valued b-metric space.

Definition 9 (see[8]): Let (X,d) be a complex valued metric space. The self-maps S and T are said to be commuting if $STx = TSx$ for all $x \in X$.

Definition 10 (see[9]): Let (X,d) be a complex valued metric space. The self-maps S and T are said to be compatible if $\lim_{n\to\infty} d(STx_n, TSx_n) = 0$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$ for some $t \in X$.

Definition 11 (see[10]): Let (X, d) be a complex valued metric space. The self-maps S and T are said to be weakly compatible if $STx = TSx$ whenever $Sx = Tx$, i.e., they commute at their coincidence points.

Lemma 12 (see [3]). Let (X, d) be a complex valued b-metric space and let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ converges to x if and only if $|d(x_n, x)| \to 0$ as $n \to \infty$.

Lemma 13 (see [3]). Let (X, d) be a complex valued b-metric space and let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ is a Cauchy sequence if and only if $|d(x_n, x_{n+m})| \to 0$ as $\to \infty$, where $m \in \mathbb{N}$.

2. Main Result

Theorem 1. Let (X, d) be a complete complex valued b-metric space with coefficient $s \geq 1$. Let S, T, f and g be self-mappings of X, satisfying the conditions:

(i) The pairs $\{ S, f \}$ and $\{ T, g \}$ are weakly compatible,

(ii) $TX \subseteq fX$ and $SX \subseteq gX$,

(iii) fX or gX is a complete subspace of X and

(iv) $d(Sx, Ty) \precsim \lambda_1 d(fx, gy) + \lambda_2 d(fx, Sx) + \lambda_3 d(gy, Ty) + \lambda_4 [d(gy, Sx) + d(fx, Ty)], \forall\, x, y \in X,$

where $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ are non-negative reals with $\lambda_1 + s\lambda_2 + \lambda_3 + 2s\lambda_4 < 1$.

Then S, T, f and g have a unique common fixed point in X.

Proof. Let $x_0 \in X$ be arbitrary.

Using condition (ii), we define a sequence $\{y_n\}$ in X, as

$$y_{2n+1} = gx_{2n+1} = Sx_{2n}, \qquad y_{2n+2} = fx_{2n+2} = Tx_{2n+1}, \quad n = 0, 1, 2, \ldots\ldots\ldots..$$

Then $d(y_{2n+1}, y_{2n+2}) = d(Sx_{2n}, Tx_{2n+1})$

$$\precsim \lambda_1 d(fx_{2n}, gx_{2n+1}) + \lambda_2 d(fx_{2n}, Sx_{2n}) + \lambda_3 d(gx_{2n+1}, Tx_{2n+1}) + \lambda_4 [d(gx_{2n+1}, Sx_{2n}) + d(fx_{2n}, Tx_{2n+1})]$$

$$= \lambda_1 d(y_{2n}, y_{2n+1}) + \lambda_2 d(y_{2n}, y_{2n+1}) + \lambda_3 d(y_{2n+1}, y_{2n+2}) + \lambda_4 [d(y_{2n+1}, y_{2n+1}) + d(y_{2n}, y_{2n+2})]$$

$$\precsim \lambda_1 d(y_{2n}, y_{2n+1}) + \lambda_2 d(y_{2n}, y_{2n+1}) + \lambda_3 d(y_{2n+1}, y_{2n+2}) + \lambda_4 s[d(y_{2n}, y_{2n+1}) + d(y_{2n+1}, y_{2n+2})]$$

Therefore $d(y_{2n+1}, y_{2n+2}) \precsim \frac{\lambda_1+\lambda_2+s\lambda_4}{1-\lambda_3-s\lambda_4} d(y_{2n}, y_{2n+1})$

i.e. $d(y_{2n+1}, y_{2n+2}) \precsim \delta_1 d(y_{2n}, y_{2n+1})$, where $\delta_1 = \frac{\lambda_1+\lambda_2+s\lambda_4}{1-\lambda_3-s\lambda_4}$,

here $0 \le \delta_1 < 1$, since $0 \le \lambda_1 + s\lambda_2 + \lambda_3 + 2s\lambda_4 < 1$ and $s \ge 1$.

Similarly

$$\begin{aligned} d(y_{2n+2}, y_{2n+3}) &= d(Sx_{2n+2}, Tx_{2n+1}) \\ &\precsim \lambda_1 d(fx_{2n+2}, gx_{2n+1}) + \lambda_2 d(fx_{2n+2}, Sx_{2n+2}) + \lambda_3 d(gx_{2n+1}, Tx_{2n+1}) + \\ &\quad \lambda_4[d(gx_{2n+1}, Sx_{2n+2}) + d(fx_{2n+2}, Tx_{2n+1})] \\ &= \lambda_1 d(y_{2n+2}, y_{2n+1}) + \lambda_2 d(y_{2n+2}, y_{2n+3}) + \lambda_3 d(y_{2n+1}, y_{2n+2}) + \\ &\quad \lambda_4[d(y_{2n+1}, y_{2n+3}) + d(y_{2n+2}, y_{2n+2})] \\ &\precsim \lambda_1 d(y_{2n+2}, y_{2n+1}) + \lambda_2 d(y_{2n+2}, y_{2n+3}) + \lambda_3 d(y_{2n+1}, y_{2n+2}) + \\ &\quad \lambda_4 s[d(y_{2n+1}, y_{2n+2}) + d(y_{2n+2}, y_{2n+3})] \end{aligned}$$

Therefore $d(y_{2n+2}, y_{2n+3}) \precsim \frac{\lambda_1+\lambda_3+s\lambda_4}{1-\lambda_2-s\lambda_4} d(y_{2n+1}, y_{2n+2})$

i.e. $d(y_{2n+2}, y_{2n+3}) \precsim \delta_2 d(y_{2n+1}, y_{2n+2})$, where $\delta_2 = \frac{\lambda_1+\lambda_3+s\lambda_4}{1-\lambda_2-s\lambda_4}$,

Here $0 \le \delta_2 < 1$, since $0 \le \lambda_1 + s\lambda_2 + \lambda_3 + 2s\lambda_4 < 1$ and $s \ge 1$.

Take $\delta = max\{\delta_1, \delta_2\}$. Then $0 \le \delta < 1$.

Then $d(y_{2n+1}, y_{2n+2}) \precsim \delta d(y_{2n}, y_{2n+1})$ and $d(y_{2n+2}, y_{2n+3}) \precsim \delta d(y_{2n+1}, y_{2n+2})$,

where $n = 0,1,2, \ldots\ldots$

Now $d(y_{n+2}, y_{n+1}) \precsim \delta d(y_{n+1}, y_n) \precsim \delta^2 d(y_n, y_{n-1}) \ldots\ldots \precsim \delta^{n+1} d(y_1, y_0)$

So for $m > n$,

$$\begin{aligned} d(y_m, y_n) &\precsim s d(y_n, y_{n+1}) + s d(y_{n+1}, y_m) \\ &\precsim s d(y_n, y_{n+1}) + s^2 d(y_{n+1}, y_{n+2}) + s^2 d(y_{n+2}, y_m) \\ &\precsim s d(y_n, y_{n+1}) + s^2 d(y_{n+1}, y_{n+2}) + s^3 d(y_{n+2}, y_{n+3}) + \cdots \ldots + \\ &\quad s^{m-n-1} d(y_{m-2}, y_{m-1}) + s^{m-n-1} d(y_{m-1}, y_m) \end{aligned}$$

$$\precsim sd(y_n, y_{n+1}) + s^2 d(y_{n+1}, y_{n+2}) + s^3 d(y_{n+2}, y_{n+3}) + \cdots \ldots \ldots \ldots \ldots \ldots + s^{m-n-1} d(y_{m-2}, y_{m-1}) + s^{m-n} d(y_{m-1}, y_m), \quad [\text{since} \quad s \geq 1]$$

$$\precsim [s\delta^n + s^2\delta^{n+1} + s^3\delta^{n+2} + \cdots \ldots \ldots \ldots \ldots + s^{m-n}\delta^{m-1}] d(y_1, y_0)$$

$$\precsim \frac{s\delta^n}{1 - s\delta} d(y_1, y_0)$$

Therefore $|d(y_m, y_n)| \leq \frac{s\delta^n}{1-s\delta} |d(y_1, y_0)|$

$\rightarrow 0$ as $n \rightarrow \infty$.

Thus $d(y_m, y_n) \rightarrow 0$ as $n \rightarrow \infty$.

Therefore $\{y_n\}$ is a Cauchy sequence in X.

Since X is complete, there exists $z \in X$, such that $y_n \rightarrow z$ as $n \rightarrow \infty$.

Thus $\lim_{n\rightarrow\infty} Sx_{2n} = \lim_{n\rightarrow\infty} gx_{2n+1} = \lim_{n\rightarrow\infty} Tx_{2n+1} = \lim_{n\rightarrow\infty} fx_{2n+2} = z$.

Now if fX is complete subspace of X, there exists $u \in X$, such that, $fu = z$.

Now from the condition (iv),

$$d(Su, z) \precsim sd(Su, Tx_{2n+1}) + sd(Tx_{2n+1}, z)$$

$$\precsim s[\lambda_1 d(fu, gx_{2n+1}) + \lambda_2 d(fu, Su) + \lambda_3 d(gx_{2n+1}, Tx_{2n+1}) + \lambda_4\{d(gx_{2n+1}, Su) + d(fu, Tx_{2n+1})\}] + sd(Tx_{2n+1}, z)$$

$$= s[\lambda_1 d(z, y_{2n+1}) + \lambda_2 d(z, Su) + \lambda_3 d(y_{2n+1}, y_{2n+2}) + \lambda_4\{d(y_{2n+1}, Su) + d(z, y_{2n+2})\}] + sd(y_{2n+2}, z)$$

Thus

$$|d(Su, z)| \leq s[\lambda_1 |d(z, y_{2n+1})| + \lambda_2 |d(z, Su)| + \lambda_3 |d(y_{2n+1}, y_{2n+2})| + \lambda_4\{d|(y_{2n+1}, Su)| + |d(z, y_{2n+2})|\}] + s|d(y_{2n+2}, z)|$$

Letting $n \rightarrow \infty$, we get

$$|d(Su, z)| \leq s[\lambda_2 |d(z, Su)| + \lambda_4 |d(z, Su)|$$

Therefore $(1 - s\lambda_2 - s\lambda_4)|d(Su, z)| \leq 0$

Now since $\lambda_1 + s\lambda_2 + \lambda_3 + 2s\lambda_4 < 1$, then $0 \leq s\lambda_2 + s\lambda_4 < 1$.

Therefore $|d(Su, z)| = 0$, i.e. $d(Su, z) = 0$ and hence $Su = z$.

Thus $fu = Su = z$.

Since $SX \subseteq gX$, there exists $v \in X$, such that $Su = gv$.

Thus $fu = Su = gv = z$.

Again from condition (iv),

$$d(z, Tv) = d(Su, Tv)$$

$$\precsim \lambda_1 d(fu, gv) + \lambda_2 d(fu, Su) + \lambda_3 d(gv, Tv) + \lambda_4 [d(gv, Su) + d(fu, Tv)]$$

$$= \lambda_3 d(z, Tv) + \lambda_4 d(z, Tv)$$

Therefore $(1 - \lambda_3 - \lambda_4)|d(z, Tv)| \leq 0$

Since $0 \leq \lambda_1 + s\lambda_2 + \lambda_3 + 2s\lambda_4 < 1,\ 0 \leq \lambda_3 + \lambda_4 < 1$.

Therefore $|d(z, Tv)| = 0$, i.e. $d(z, Tv) = 0$ and hence $z = Tv$.

Thus $fu = Su = z = gv = Tv$.

Since f and S are weakly compatible,

$$fz = fSu = Sfu = Sz.$$

Now $d(Sz, z) = d(Sz, Tv)$

$$\precsim \lambda_1 d(fz, gv) + \lambda_2 d(fz, Sz) + \lambda_3 d(gv, Tv) + \lambda_4 [d(gv, Sz) + d(fz, Tv)]$$

$$= \lambda_1 d(Sz, z) + 2\lambda_4 d(z, Sz)$$

Therefore $(1 - \lambda_1 - 2\lambda_4)|d(z, Sz)| \leq 0$.

Since $0 \leq \lambda_1 + s\lambda_2 + \lambda_3 + 2s\lambda_4 < 1$ and $s \geq 1$, then $0 \leq \lambda_1 + 2\lambda_4 < 1$.

Therefore $|d(z, Sz)| = 0, i.e. d(z, Sz) = 0$ and hence $Sz = z$.

$i.e.\ fz = Sz = z$.

Similarly, since g, T are weakly compatible,

$$gz = gTv = Tgv = Tz.$$

Also $d(z, Tz) = d(Sz, Tz)$

$$\precsim \lambda_1 d(fz, gz) + \lambda_2 d(fz, Sz) + \lambda_3 d(gz, Tz) + \lambda_4 [d(gz, Sz) + d(fz, Tz)]$$

$$= \lambda_1 d(z, Tz) + 2\lambda_4 d(z, Tz)$$

Therefore $(1 - \lambda_1 - 2\lambda_4)|d(z, Tz)| \leq 0.$

Therefore $|d(z, Tz)| = 0$, $i.e.$ $d(z, Tz) = 0$ and hence $Tz = z$.

Thus $gz = Tz = z$.

Thus $fz = Sz = z = gz = Tz$.

Thus z is a common fixed point of S, T, f and g.

For uniqueness, let $z^* \epsilon X$, such that, $fz^* = Sz^* = z^* = gz^* = Tz^*$.

Now from condition (iv),

$$d(z, z^*) = d(Sz, Tz^*)$$

$$\precsim \lambda_1 d(fz, gz^*) + \lambda_2 d(fz, Sz) + \lambda_3 d(gz^*, Tz^*) + \lambda_4 [d(gz^*, Sz) + d(fz, Tz^*)]$$

$$= \lambda_1 d(z, z^*) + 2\lambda_4 d(z, z^*)$$

Therefore $(1 - \lambda_1 - 2\lambda_4)|d(z, z^*)| \leq 0.$

Therefore $|d(z, z^*)| = 0$, $i.e. d(z, z^*) = 0$ and hence $z = z^*$.

Thus z is the unique fixed common fixed point of S, T, f and g.

If gX is a complete subspace of X, then similarly we can prove the theorem.

Corollary 1.1. Let (X, d) be a complete complex valued b-metric space with coefficient $s \geq 1$. Let S, T be self-mappings of X satisfying:

$$d(Sx, Ty) \precsim \lambda_1 d(x, y) + \lambda_2 d(x, Sx) + \lambda_3 d(y, Ty) + \lambda_4 [d(y, Sx) + d(x, Ty)], \forall\, x, y \,\epsilon\, X,$$

where $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ are non-negative reals with $\lambda_1 + s\lambda_2 + \lambda_3 + 2s\lambda_4 < 1$.

Then S and T have a unique common fixed point in X.

Proof. If we put $fx = x, gx = x$ in the above theorem, we can easily get the result.

Corollary 1.2. . Let (X, d) be a complete complex valued b-metric space with coefficient $s \geq 1$. Let T, f and g be self-mappings of X, satisfying the conditions:

(i) The pairs $\{T, f\}$ and $\{T, g\}$ are weakly compatible,
(ii) $TX \subseteq fX$ and $TX \subseteq gX$,
(iii) fX orgX is a complete subspace of X and

(iv) $d(Tx, Ty) \precsim \lambda_1 d(fx, gy) + \lambda_2 d(fx, Tx) + \lambda_3 d(gy, Ty) + \lambda_4 [d(gy, Tx) + d(fx, Ty)], \forall\, x, y \,\epsilon\, X,$

where $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ are non-negative reals with $\lambda_1 + s\lambda_2 + \lambda_3 + 2s\lambda_4 < 1$.

Then T, f and g have a unique common fixed point in X.

Proof. If we set $S = T$ in the above theorem, we get the result.

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