ZERO-ONE DOUBLE SAMPLING PLAN FOR TRUNCATED LIFE TESTS BASED ON GENERALISED LOG-LOGISTIC DISTRIBUTION

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ABSTRACT

The design of zero-one double sampling plan is proposed for the truncated life tests assuming that the lifetime of a product follows generalized log-logistic distribution . The minimum sample sizes of the zero-one double sampling plan are determined to ensure that the median life is longer than the given life at the specified consumer's confidence level with minimum ASN . The operating characteristic values are analysed. The minimum median ratios are obtained so as to meet the producer's risk at the specified consumer's confidence level. Numerical illustrations are provided to explain the use of constructed tables. The efficiency of zero-one double sampling plan is analysed with single sampling plan.

KEYWORDS

Zero-one double sampling plan, Truncated life tests, Operating characteristic function, Average sample number, Consumer's risk, Producer's risk.

INTRODUCTION

Acceptance sampling plan is an important tool for ensuring quality on the basis of information yielded by the sample. It is inevitable when the testing is destructive and cost of inspection is very high. If the lifetime of a product is high then it might be time consuming to wait until the failure of the product. Therefore for economy it is usual to terminate a life test by a pre-assigned time.With these aspects truncated life test plans are initiated.

Several authors studied the acceptance sampling plans using various life tests distributions. Epstein (1954) first developed an single acceptance sampling procedure for truncated life tests in the exponential case. Goode and Kao (1961) developed the sampling plan on the Weibull distribution. Gupta and Groll (1961) developed the truncated life test sampling plan with the Gamma distribution. Rosaiah and Kantam (2005) introduced acceptance sampling plan based on the inverse Rayleigh distribution using mean life. Tsai and Wu (2006) developed the acceptance sampling plan for truncated life tests for generalised Rayleigh distribution. Srinivasa Rao, et.al (2012) designed single acceptance sampling plan for percentiles based on inverse Rayleigh distribution.

The purpose of this paper is to propose a zero-one double sampling plan for the truncated life test assuming that the life time of a product follows generalized log-logistic distribution. Since this distribution plays a vital role in hydrology to model stream flow and precipitation, estimating mortality rate from cancer following diagonosis and so on and also double sampling plan is economical than the corresponding single sampling plan.

The minimum sample sizes are calculated by incorporating minimum average sample number for various consumers confidence levels .The operating characteristic values of the designed sampling plans are analysed and the minimum median ratios of the life time are obtained to ensure the specified producer's risk. Applications of the designed plan is also explained.

GENERALISED LOG-LOGISTIC DISTRIBUTION

Assume that the lifetime of a product under consideration follows generalized log-logistic distribution. The probability density function and cumulative distribution function of generalized log-logistic distribution are

$$f(t) = \left[\frac{(\beta/\sigma)(t/\sigma)^{\beta-1}}{\{1+(t/\sigma)^{\beta}\}^{2}} \right]^{\theta}, t > 0, \sigma > 0, \alpha > 0, \beta > 0$$

and

$$F(t,\sigma,\beta,\theta) = \left[\frac{(t/\sigma)^{\beta}}{1+(t/\sigma)^{\beta}} \right]^{\theta}, t \geq 0, \sigma > 0, \beta > 0, \theta > 0$$

where σ is the scale parameter, β and θ are the shape parameters.

The median of generalized log-logistic distribution is $m = \sigma \left(0.5^{\frac{1}{\theta}} \Big/ 1 - 0.5^{\frac{1}{\theta}} \right)^{\frac{1}{\beta}}$

Here the median is proportional to σ, the scale parameter. If θ=1generalised log-logistic distribution becomes log-logistic distribution and its median is equal to the scale parameter σ.

DESIGN OF THE PROPOSED ZERO-ONE DOUBLE SAMPLING PLAN

Assume that the quality of a product can be represented by its median lifetime ,m. The submitted lot will be accepted if the data supports the null-hypothesis H_0: $m \geq m_0$ against the alternative hypothesis,

H_1:$m<m_0$,where m_0 is the specified median life. The significance level for the test is 1-P* , where P* is the consumer's confidence level.The operating procedure of zero-one double sampling plan for the truncated life test is

Select a first random sample of size n_1 from the submitted lot and put on experiment for time t_0 within the experimental time.If there are 0 failures,accept the lot. If there are 2 or more failures,reject the lot.

otherwise select a second random sample of size n_2, if the number of failures in the second sample is 0,accept the lot otherwise reject the lot.

It is more convenient to make a termination time as a multiple of the specified median life m_0, by assigning $t_0 = am_0$ for the specified multiplier **a**. For a given P*, the proposed sampling plan may be characterized by the parameters (n_1,n_2,β,θ,a) .

The probability of lot acceptance for a zero-one double sampling plan is

$$P_a = (1-p)^{n_1}\left[1 + n_1 p(1-p)^{n_2-1}\right]$$

where p, the probability that an item fails before t_o, is given by

$$p = \left[\frac{(t_o/\sigma)^{\beta}}{1+(t_o/\sigma)^{\beta}}\right]^{\theta} = \left[\frac{(a\gamma)^{\beta}}{(m/m_0)^{\beta} + (a\gamma)^{\beta}}\right]^{\theta}$$

with $\gamma = \left(0.5^{\frac{1}{\theta}} \Big/ \left(1 - 0.5^{\frac{1}{\theta}}\right)\right)^{\frac{1}{\beta}}$

and replacing m by m_0, p reduces to $p = \left[(a\gamma)^{\beta} / 1 + (a\gamma)^{\beta}\right]^{\theta}$

The minimum sample sizes ensuring $m \geq m_0$ at the consumer's confidence level P*may be found as the solution of the inequality

$$P_a \leq 1\text{-}P^* \qquad (1)$$

Equation (1) provides multiple solutions for the sample sizes n_1 and n_2.In order to find the optimal sample sizes the minimum of ASN is incorporated along with specification (1).The determination of the minimum sample sizes for zero-one double sampling plan reduces to

Minimize $ASN = n_1 + n_1 n_2 p(1-p)^{n_1-1}$

subject to $$(1-p)^{n_1}\left[1+n_1 p(1-p)^{n_2-1}\right]\leq 1-P^* \qquad (2)$$

where n_1 and n_2 are integers with $n_2 \leq n_1$.

Table 1 Minimum sample sizes for zero-one double sampling plan

Log-logistic distribution								
β	P*	**a**						
		0.5	0.7	0.9	1.1	1.5	1.9	2.5
2	0.75	8,7	5,3	3,3	3,1	2,1	2,1	2,1
	0.9	12,9	7,5	5,3	4,2	3,1	2,2	2,1
	0.95	15,10	8,8	6,4	5,2	3,2	3,1	2,2
	0.99	21,19	12,9	8,7	6,5	4,4	4,1	3,1
3	0.75	15,12	6,6	4,2	3,1	2,1	1,1	1,1
	0.9	22,18	9,7	5,4	4,2	2,2	2,1	2,1
	0.95	27,24	11,9	6,5	4,3	3,1	2,1	2,1
	0.99	40,33	16,13	9,6	6,4	4,2	3,1	2,2
4	0.75	28,26	8,8	4,3	2,2	2,1	1,1	1,1
	0.9	42,38	12,11	6,3	3,3	2,1	2,1	1,1
	0.95	52,49	15,13	7,4	4,3	2,2	2,1	1,1
	0.99	77,72	22,17	10,5	6,3	3,2	2,2	2,1
Generalised log-logistic distribution								
(β,θ)	P*	**a**						
		0.5	0.7	0.9	1.1	1.5	1.9	2.5
(2,2)	0.75	4,3	3,2	2,2	2,1	2,1	1,1	1,1
	0.9	6,4	4,2	3,2	2,2	2,1	2,1	2,1
	0.95	7,6	5,2	3,3	3,2	2,2	2,1	2,1
	0.99	10,9	6,6	5,2	4,2	3,2	3,1	2,2
(2,3)	0.75	3,2	2,2	2,1	2,1	1,1	1,1	1,1
	0.9	4,4	3,2	2,2	2,1	2,1	2,1	1,1
	0.95	5,4	4,2	3,1	2,2	2,1	2,1	2,1
	0.99	7,6	5,3	4,2	3,2	3,1	2,2	2,1
(3,2)	0.75	7,5	3,3	2,2	2,1	1,1	1,1	1,1
	0.9	10,8	5,3	3,2	2,2	2,1	2,1	1,1
	0.95	12,12	6,3	4,2	3,1	2,1	2,1	2,1
	0.99	18,14	8,6	5,3	4,2	3,1	2,1	2,1
(3,3)	0.75	5,3	3,1	2,1	2,1	1,1	1,1	1,1
	0.9	7,5	4,2	2,2	2,1	2,1	1,1	1,1
	0.95	9,5	4,3	3,2	2,2	2,1	2,1	1,1
	0.99	12,11	6,4	4,2	3,2	2,2	2,1	2,1

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Table 1 is constructed to present the minimum sample sizes for the first and second sample with specified P*(=0.75,0.90,0.95,0.99),**a**(=0.3,0.5,0.7,0.9,1.1,1.5,1.9) and shape parameter β (=2,3,4) under log-logistic distribution and generalized log-logistic distribution using equation (2)

Numerical values of Table 1 reveal that

(i) increase in **a** decreases the sample sizes for any P* with fixed **β.**
(ii) increase in θ increases the sample sizes for any P* with fixed **a** and **β.**
(iii) increase in **β** increases sample sizes for any P* with fixed **a.**

The trend of the first sample size as a function of test time duration when under the log-logistic distribution and the generalized log-logistic distributions with different pair of β and θ when P*=0.99 are given in Fig 1 and Fig 2 respectively.

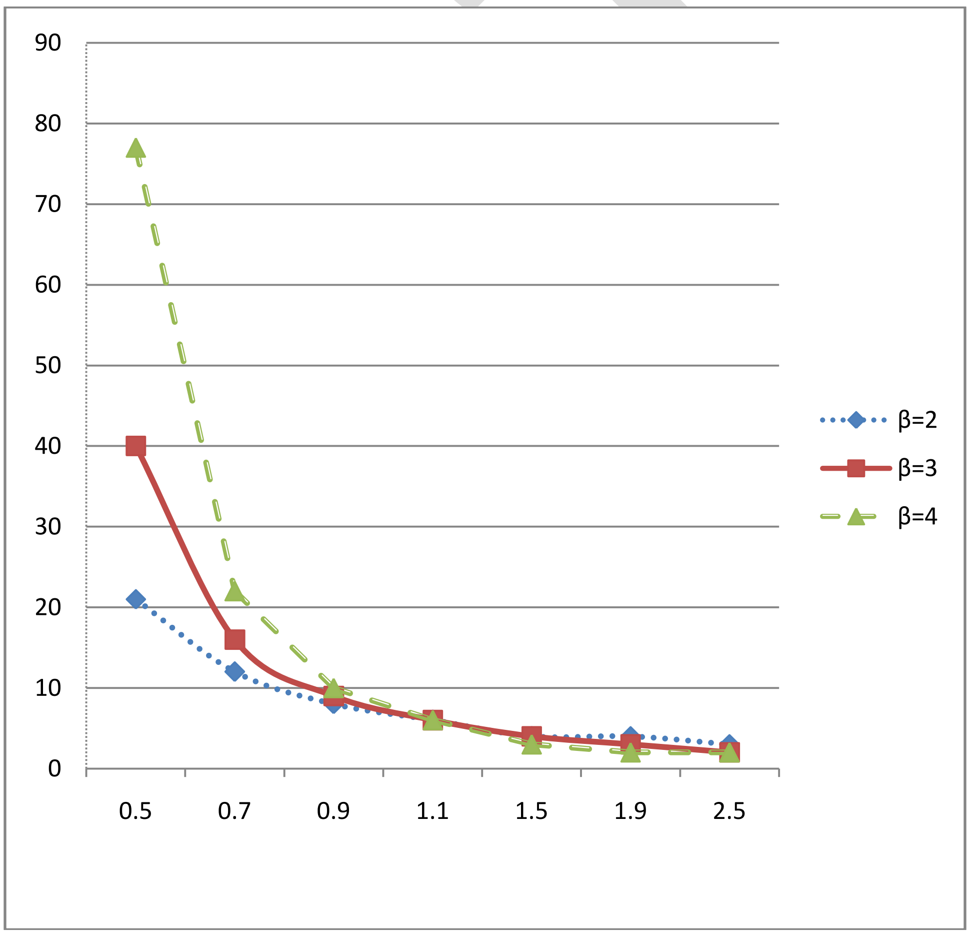

Fig.1The first sample size Vs. experiment time for log-logistic distributions when P*=0.99

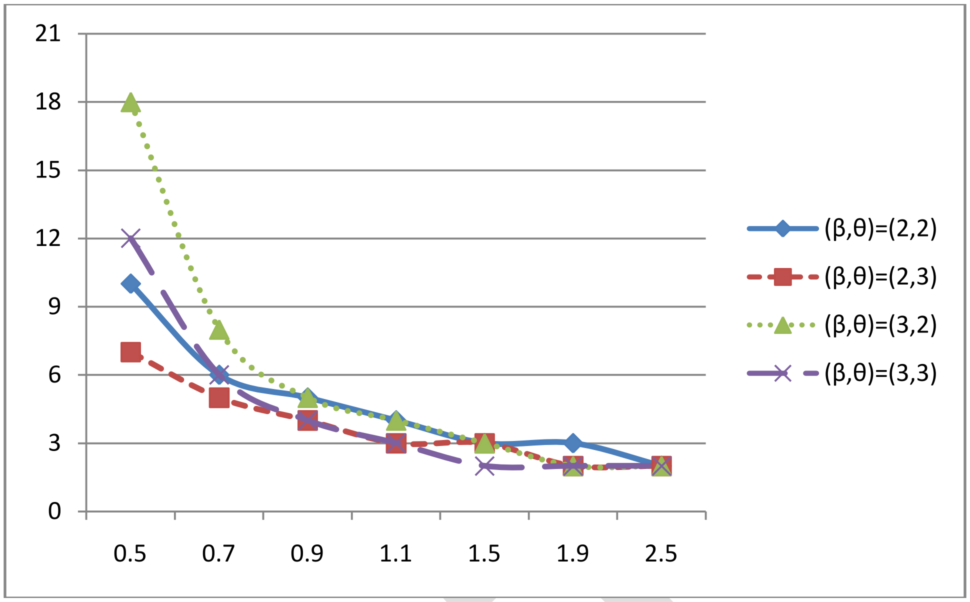

Fig. 2 The first sample size Vs. experiment time for generalised log-logistic distributions when P*=0.99

OPERATING CHARACTERISTIC VALUES

OC values depict the performance of the sampling plan in discriminating the quality of the submitted product. Tables 2 and 3 give the operating characteristic values for log-logistic distribution when β=4 and generalised log-logistic distribution when (β,θ)= (2,2) respectively.

Numerical values of Table 2 and Table 3 reveal that

(i) increase in m/m_0 increases the OC value for fixed **a** and P*

(ii) increase in **a** decreases the OC value for fixed P* and m/m_0

(iii) increase in P* decreases the OC value for fixed **a** and m/m_0

MINIMUM MEDIAN RATIO

The minimum median ratio m/m_0 at the specified producer's risk and consumer's confidence level,may be obtained by solving $P_a \geq 1\text{-}\alpha$ and is presented in Table 4 by using the sample sizes presented in Table 1.

Minimum median ratios under log-logistic and generalised log-logistic distribution of zero-one double sampling plan shows that

(i) increase in P* increases the minimum median ratios for fixed **a** and **β**.

(ii) increase in **a** increases the minimum median ratios for fixed P* and **β**.

(iii) increase in **β** decreases the minimum median ratios for fixed P* and **a**.

Further for Generalised log-logistic distribution increase in **θ** decreases the minimum median ratios for fixed P*,**a** and **β**.

Table 2 Operating Characteristic values for zero-one double sampling plan using log-logistic distribution when β=4

P*	a	n_1	n_2	m/m_0					
				2	4	6	8	10	12
0.75	0.5	28	26	0.9852	0.9999	0.9999	1	1	1
	0.7	8	8	0.9823	0.9999	0.9999	1	1	1
	0.9	4	3	0.9753	0.9999	1	1	1	1
	1.1	2	2	0.9683	0.9998	0.9999	0.9999	1	1
	1.5	2	1	0.8545	0.9989	0.9999	0.9999	0.9999	1
	1.9	1	1	0.7985	0.9977	0.9999	0.9999	0.9999	1
	2.5	1	1	0.4967	0.9825	0.9991	0.9999	0.9999	0.9999
0.9	0.5	42	38	0.9691	0.9999	0.9999	0.9999	1	1
	0.7	12	11	0.9642	0.9998	0.9999	0.9999	1	1
	0.9	6	3	0.9571	0.9998	0.9999	0.9999	1	1
	1.1	3	3	0.9313	0.9996	0.9999	0.9999	1	1
	1.5	2	1	0.8545	0.9989	0.9999	0.9999	0.9999	1
	1.9	2	1	0.5764	0.9932	0.9997	0.9999	0.9999	0.9999
	2.5	1	1	0.4967	0.9825	0.9991	0.9999	0.9999	0.9999
0.95	0.5	52	49	0.9535	0.9998	0.9999	0.9999	1	1
	0.7	15	13	0.9481	0.9997	0.9999	0.9999	1	1
	0.9	7	4	0.9383	0.9997	0.9999	0.9999	1	1
	1.1	4	3	0.9028	0.9994	0.9999	0.9999	1	1
	1.5	2	2	0.7878	0.9982	0.9999	0.9999	0.9999	1
	1.9	2	1	0.5764	0.9932	0.9997	0.9999	0.9999	0.9999
	2.5	1	1	0.4967	0.9825	0.9991	0.9999	0.9999	0.9999
0.99	0.5	77	72	0.9089	0.9995	0.9999	0.9999	1	1
	0.7	22	17	0.9053	0.9995	0.9999	0.9999	1	1
	0.9	10	5	0.8935	0.9993	0.9999	0.9999	1	1
	1.1	6	3	0.8411	0.9989	0.9999	0.9999	0.9999	1
	1.5	3	2	0.6785	0.9967	0.9999	0.9999	0.9999	0.9999
	1.9	2	2	0.4541	0.9889	0.9995	0.9999	0.9999	0.9999
	2.5	2	1	0.2042	0.9521	0.9975	0.9997	0.9999	0.9999

Table 3 Operating Characteristic values for zero-one double sampling plan using Generalised log-logistic distribution when (β,θ)=(2,2)

P*	a	n_1	n_2	m/m_0					
				2	4	6	8	10	12
0.75	0.5	4	3	0.9482	0.9959	0.9992	0.9997	0.9999	0.9999
	0.7	3	2	0.9133	0.9925	0.9984	0.9995	0.9998	0.9999
	0.9	2	2	0.8853	0.9891	0.9976	0.9992	0.9997	0.9998
	1.1	2	1	0.8632	0.9859	0.9969	0.999	0.9996	0.9998
	1.5	2	1	0.7045	0.9581	0.9901	0.9966	0.9986	0.9993
	1.9	1	1	0.7751	0.9661	0.9917	0.9971	0.9988	0.9994
	2.5	1	1	0.6282	0.9211	0.9781	0.9921	0.9965	0.9983
0.9	0.5	6	4	0.8996	0.9915	0.9982	0.9994	0.9998	0.9999
	0.7	4	2	0.8784	0.9885	0.9976	0.9992	0.9997	0.9998
	0.9	3	2	0.8167	0.9809	0.9958	0.9986	0.9994	0.9997
	1.1	2	2	0.7997	0.9773	0.9949	0.9983	0.9993	0.9997
	1.5	2	1	0.7045	0.9581	0.9901	0.9966	0.9986	0.9993
	1.9	2	1	0.5384	0.9108	0.9766	0.9918	0.9964	0.9982
	2.5	2	1	0.3381	0.8076	0.9408	0.9777	0.9901	0.9951
0.95	0.5	7	6	0.8531	0.9866	0.9971	0.9991	0.9996	0.9998
	0.7	5	2	0.8339	0.9839	0.9965	0.9989	0.9995	0.9998
	0.9	3	3	0.7761	0.9752	0.9945	0.9982	0.9992	0.9996
	1.1	3	2	0.6946	0.9611	0.9611	0.9971	0.9987	0.9994
	1.5	2	2	0.5983	0.9348	0.9839	0.9945	0.9976	0.9988
	1.9	2	1	0.5384	0.9108	0.9766	0.9918	0.9964	0.9982
	2.5	2	1	0.3381	0.8076	0.9408	0.9777	0.9901	0.9951
0.99	0.5	10	9	0.7429	0.9728	0.9941	0.9981	0.9992	0.9996
	0.7	6	6	0.6836	0.9624	0.9915	0.9972	0.9988	0.9994
	0.9	5	2	0.6762	0.9603	0.9909	0.9971	0.9987	0.9994
	1.1	4	2	0.5952	0.9424	0.9864	0.9954	0.9981	0.9991
	1.5	3	2	0.4433	0.8924	0.9722	0.9903	0.9958	0.9979
	1.9	3	1	0.3519	0.8431	0.9561	0.9841	0.9931	0.9965
	2.5	2	2	0.2248	0.7261	0.9091	0.9645	0.9839	0.9918

Table 4 Minimum median ratios for zero-one double sampling plan

Log-logistic distribution								
β	P*	a						
		0.5	0.7	0.9	1.1	1.5	1.9	2.5
2	0.75	6.1756	6.7756	6.9786	7.9987	8.9234	11.2261	14.9991
	0.9	7.7887	7.9987	8.9234	9.9876	11.2126	11.2261	14.9991
	0.95	8.4234	8.9234	9.9876	10.3261	11.1261	14.2261	14.9981
	0.99	9.9234	10.5231	11.1881	11.9261	12.8261	15.9261	15.9267
3	0.75	3.2756	3.4756	3.8786	4.2987	4.9234	5.1756	6.4756
	0.9	3.6987	3.7956	4.0999	4.5987	4.9234	6.2756	8.1756
	0.95	3.9987	4.1956	4.4999	4.5987	5.9234	6.3756	8.1956
	0.99	4.5987	4.6956	4.9999	5.2987	6.3234	7.1999	8.2856
4	0.75	2.3756	2.4756	2.6786	2.7587	3.8234	4.7006	5.0996
	0.9	2.6099	2.7756	2.9786	3.0087	3.8234	4.7006	5.0996
	0.95	2.8099	2.8756	2.9996	3.0087	4.1234	4.6806	5.0991
	0.99	3.1289	3.1756	3.2996	3.6087	3.7234	4.6906	6.1991
Generalised log-logistic distribution								
(β,θ)	P*	a						
		0.5	0.7	0.9	1.1	1.5	1.9	2.5
(2,2)	0.75	2.1543	2.8111	3.2112	3.8999	5.2234	5.6721	7.0234
	0.9	2.4231	2.9768	3.5673	3.8999	5.2234	6.7451	8.8712
	0.95	2.4766	3.2341	3.5799	4.4523	5.2234	6.7451	8.8998
	0.99	2.7812	3.3678	4.1231	4.8934	5.9898	7.5612	8.8999
(2,3)	0.75	1.6499	2.1129	2.7012	3.3234	3.7812	4.7712	6.3612
	0.9	1.7776	2.3611	2.8512	3.2999	4.4866	5.7456	6.3659
	0.95	1.8299	2.4612	2.9612	3.2999	4.4999	5.7451	7.4998
	0.99	2.0412	2.5991	3.1324	3.6712	4.9412	5.7901	7.5211
(3,2)	0.75	1.4991	1.7999	2.1213	2.5234	2.9912	3.8612	4.9876
	0.9	1.5776	1.9161	2.2412	2.5999	3.4981	4.4961	4.9899
	0.95	1.6211	2.0021	2.3612	2.7612	3.4712	4.4531	5.7997
	0.99	1.7234	2.1451	2.4879	2.8898	3.7978	4.3995	5.7996
(3,3)	0.75	1.1912	1.5785	1.8873	2.3232	2.7912	3.5678	4.6976
	0.9	1.2776	1.6082	1.9192	2.3221	3.1569	3.5569	4.6099
	0.95	1.3221	1.6512	1.9989	2.3299	3.1099	3.9969	4.7489
	0.99	1.3699	1.7112	2.0989	2.4253	3.1566	3.9989	5.2367

ILLUSTRATION OF THE TABLES

Assume that the lifetime distribution of the test items follows log-logistic distribution with shape parameter β=2 and we want to establish that the median lifetime is atleast 1000 hours with probability

0.90.The life test is planned to be terminated at 900 hours.Then, from Table1,the minimum sample sizes required for the assumed specification from Table 1 are $n_1 = 5$ and $n_2 = 3$. This sampling plan is put into operation as follows. First 5 randomly selected items are put on test for 300 hours and the lot will be accepted if no failure occurs during the experimental time .The lot is rejected if more than one failure occurs during the experimental time.When exactly one failure i observed,the second random sample of size 3 will be drawn and put on test for 300 hours.The lot will be accepted if there are no failures from the second sample and will be rejected ,otherwise.

COMPARITIVE STUDY

Operating characteristic value with same amount of inspection are considered as the base for comparision.The OC values according to the ratio m/m_0 (=2,4,6,8) for zero-one double sampling plan and single sampling plan with the same ASN value of 9 are obtained as

plan \ ratio		2	4	6	8
SSP	C=0	0.6179	0.9581	0.9782	0.9970
	C=1	0.5154	0.9234	0.9434	0.9127
DSP(0,1)		0.5511	0.9367	0.9851	0.9951

The table values indicate that zero-one double sampling plan has higher OC values than the single sampling plans for the same inspection level.This makes the plans ability of discrimination.

CONCLUSION

Designing of zero-one double sampling plan with truncated life test when the lifetime of the product follows generalized log-logistic distribution is proposed ,which is useful in system reliability analysis with economy. The determination of optimal parameters by using two points on the operating characteristic curve approach is under consideration.

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