# Optimal ordering policy for Deteriorating Items with varying demand and price discounts:

*** T. Mohan Reddy, # K. Srinivasa Rao, + C.T. Suryanarayana Chari**

* Department of mathematics, St.Joseph's Degree College, Kurnool # Professor, Department of O R & S Q C, Rayalaseema University, + Dept of Mathematics, Sileverjubliee College, Kurnool A.P. Pin - 518001

## Abstract

The present paper deals with an EOQ model of an Inventory problem with varying component demand rate. Viz.,

ii) For a certain period the demand rate is exponentially increasing depending on the stock available.

ii) After the some time period the demand rate is constant.

Here we considered the price discounts and the effect of deterioration is also considered. The sensitivity of the model is presented with two component demand i.e initially exponential demand and constant demand rate. This model also potrays deterioration of items along with price breaks.

## Key words

EOQ, Exponentially Increasing demand, shortages, price breaks, varying demand

## 1.1 Introduction

In Recent past several researchers attempted with different aspects to obtained optimal ordering policy for deteriorating Items. In practice several factors will play major role in obtaining optimal ordering quantity or EOQ. In real word the retailers will get price discounts for large quantity purchased. This will prompt the prudent stockist to store more Items in the stock when compared to normal situation, i.e., without price discounts. Several Researchers attempted Inventory models for price breaks for example Abad[1] has attempted to determined selling price and lot size for retailer when he is given all units discounts, further Arcelus et al[2] considered the forward buying policies of an Inventory model with deteriorating items and temporary price discounts. Fazal et al[3] also considered the classical quantity discount model from the variety of EOQ models basically presented by Harris[4]. Hwang et al [5] considered an EOQ Model with quantity discounts for both purchasing price and Freight cost. Matsuyama et al[6] portrayed EOQ models with price discounts. Rubin et al[7] also attempted to obtain EOQ with price discounts. A comparison with JIT (Vs) EOQ with price discounts presented by chaudhuri et al[8].

In the recent literature of Inventory models, several researchers considered deterioration of Items. Deterioration place a major role for stocking of goods even in supply chain management, in this area an attempt is made by Rau et al[9] by considering deterioration of Items in supply chain management. Inventory model for determining items by allowing shortages is considered by Dye et al[10], chung et al[11] also attempted in determining an EOQ Model with deteriorating Items under varying demand. The delay in payments for deteriorating Items in the context of EOQ models is discussed by skouri et al [12]. The deterioration of Inventory Items are classified into three categories by Ghare and Schrader [13]. These three categories are direct spoilage, physical deplection and detiration. Direct spoilage means breakage of Items which cannot be used further. whereas deterioration refers to the slow but gradual loss of qualitative properties of the items in a phased manner.

## 1.2. Notations

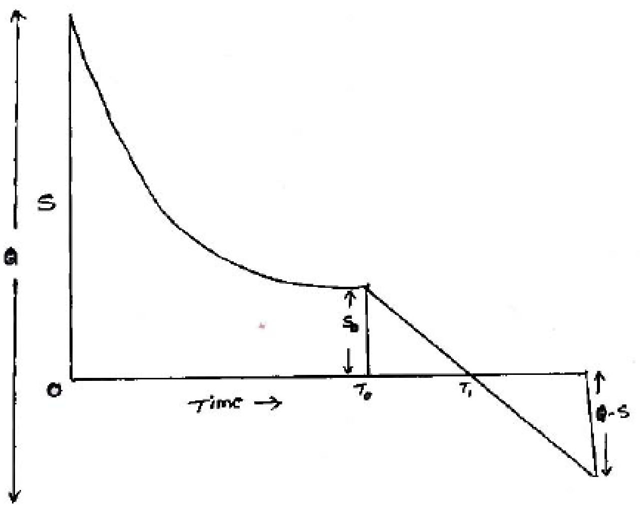
| | |
|---|---|
| q(t) | - Inventory level at time 't' |
| s=q(0) orders | = Stock level at the beginning of each ccycle after fulfilling back |
| $S_0$ = q (T0) | = Stock level below which the demand rate is constant. It is an external variable beyond the control of the decision maker |
| Q | = Stock level at the begininning the amount of shortages. |
| D | = Constant demand rate |
| $T_0$ | = Time unitl stock level reaches $S_0$ |
| $T_1$ | = Time unitl shortage begins |
| T | = Length of the cycle time |
| θ | = Deterioration rate, fraction of the on hand inventory |
| $C_2$ | = Shortage cost per unit per unit time |
| $C_3$ | = The ordering cost of Inventory |
| Pi | = The unit cost of item in $f_{i-1} < S < f_i$, where i = 1 to N and $\theta = f_0 < f_1 < f_2 \ldots f_N$ |
| $P_N$ | = The unit cost of item in $f_N < S$ |
| h | = Inventory holding cost per item per unit time |
| U | = The unit selling price of deteriorated items |

## 1.3. Assumptions

The following assumptions are considered

i) Shortages are allowed and are fully back logged
ii) The replenishment is Instantaneous.
iii) Deteriorated items are sold at a low price at the end of the stocks
iv) Two - Component exponential demand rate is considered i.e. demand depends on the inventory as well as on the fixed customers
v) price breaks are considered here i.e. the unit purchase price is discounted with the increasing ordering quantity.

## 1.4. Formulation of the model




The inventory level will be decreased at a rate of $ae^{\alpha t}$ during the period $(0,T_0)$, here $T_0$ is determined by $q(T_0) = S_0$ and corresponding value of $S_0$ is also be determined. During the period $(T_0, T_1)$ the inventory level will be depleted at a constant rate 'D'

The inventory level will be depleted at a rate of during the period $(0, T_0)$ is $ae^{\alpha t}$ During the period $(T_0, T_1)$ the Inventory level will be depleted at a constant rate 'D'.

The inventory falls to zero level at time $t = T_1$, shortages are than allowed for replenishment up to time $t = T$. Therefore, for a deteriortion rate the instantaneous inventory level will satisty the following differential equation.

The instantanceus inventary level will sabrsty the following eqluation

$$\frac{dq}{dt} + \theta q = -ae^{\alpha t} \;, \quad 0 \le t \le T_0 \rightarrow (1)$$

with the boundary conditions are

$$q(0) = S \rightarrow (2)$$

$$q(T_0) = S_0 \rightarrow (3)$$

$$\frac{dq}{dt}+\theta q=-D\,,\quad T_0\le t\le T_1\rightarrow(4)$$

With the boundary conditions

$$q(T_0)=S_0\rightarrow(5)$$

$$q(T_1)=0\rightarrow(6)$$

$$\frac{dq}{dt}=-D\text{, Where } T_1\le t\le T\rightarrow(7)$$

With the boundary Conditions are

$$q(T_1)=0\rightarrow(8)$$

$$q(T)=-(Q-S)\rightarrow(9)$$

For solution eqluation (1) with the boundary conditions (2) is

$$\frac{dq}{dt}+\theta q=-ae^{\alpha t}\rightarrow(1)\text{ where } 0\le t\le T_1$$

The solution of th e equation (1) is

$$Y.I.F=\int Q.I.F+K_1$$

$$Q.e^{\theta t}=\int -ae^{\alpha t}e^{\theta t}+K_1$$

$$q(t)=\frac{-a}{\alpha+\theta}e^{\alpha t}+K_1e^{-\theta t}\rightarrow(10)$$

with boundary condition ss $q(0)=S,$ and $t=0$

$$S=\frac{-a}{\alpha+\theta}+K_1$$

$$S+\frac{a}{\alpha+\theta}=K_1\text{ - (11)}$$

(11) sub in (10)

$$Q(t)=\frac{-a}{\alpha+\theta}e^{\alpha t}+\left(s+\frac{a}{\alpha+\theta}\right)e^{-\theta t}\rightarrow(12)$$

The solution of (12) with boundary conditiont = $T_0$ and $q(t_1)$ = 0

$$\frac{-a}{\alpha+\theta}e^{\alpha t_0}+\left(s+\frac{a}{\alpha+\theta}\right)e^{-\theta t_0}=0$$

$$\left(s+\frac{a}{\alpha+\theta}\right)e^{-\theta t_0}=\frac{a}{\alpha+\theta}e^{\alpha T_0}$$

$$\left(\frac{s(\alpha+\theta)}{a}+1\right)=e^{\alpha T_0}e^{\theta T_0}$$

$$\left(\frac{s(\alpha+\theta)}{a}+1\right)=e^{(\alpha+\theta)T_0}$$

$$\log\left(\frac{s(\alpha+\theta)}{a}+1\right)=(\alpha+\theta)T_0$$

$$\frac{1}{\alpha+\theta}\log\left(\frac{s(\alpha+\theta)}{a}+1\right)=T_0$$

$$T_0 = \frac{1}{\alpha+\theta}\log\left(\frac{s(\alpha+\theta)}{a}+1\right) \rightarrow (13)$$

For solving equlation (4), with the boundary condition $T_0 \leq t \leq T_1$

$$\frac{dq}{dt} + \theta q = -D$$

The general solution is

$$Y.I.F = \int Q.I.F dt + K_2$$

$$q(t).e^{\theta t} = \int -D.e^{\theta t} dt + K_2$$

$$q(t)e^{\theta t} = -D\int e^{\theta t} dt + K_2$$

$$q(t) = e^{-\theta t}\left(-D\frac{e^{\theta t}}{\theta} + K_2\right) \rightarrow (14)$$

By boundary condition $q(T_0) = S_0$

$$S_0 = e^{-\theta T_0}\left(\frac{-D}{\theta}e^{\theta T_0} + K_2\right) \rightarrow (15)$$

$$S_0 = \frac{-D}{\theta} + K_2 e^{-\theta T_0}$$

$$S_0 + \frac{D}{\theta} = K_2 e^{-\theta T_0}$$

$$K_2 = \left(S_0 + \frac{D}{\theta}\right)e^{\theta T_0} \rightarrow (16)$$

(16) Substituting (14) weget

$$S_0 = e^{-\theta T_0}\left(\frac{-D}{\theta}e^{\theta T_0} + \left(S_0 + \frac{D}{\theta}\right)e^{\theta T_0}\right)$$

$$q(t) = \frac{-D}{\theta} + \left(S_0 + \frac{D}{\theta}\right)e^{-\theta(t-T_0)} \rightarrow (17) \quad (T_0 \leq t \leq T_1)$$

The boundary condition from (6) is

$$q(T_1) = 0$$

It given $T_1 = T_0 + \frac{1}{\theta}\log\left(\frac{D+\theta S_0}{D}\right) \rightarrow (18)$

(13) sub in (18), weget

$$T_1 = \frac{1}{\alpha+\theta}\log\left(\frac{s(\alpha+\theta)}{a}+1\right) + \frac{1}{\theta}\log\left(\frac{D+\theta S_0}{D}\right) \rightarrow (19)$$

The solution of equation (7), with the boundary condition $q(T_1) = 0$ is

$$q(T) = -D(t-T_1) \rightarrow (20) \qquad (T_1 \leq t \leq T)$$

The boundary condition $q(T) = -(Q-S)$ Gives

$$T = T_1 + \frac{Q-S}{D} \rightarrow (21)$$

(19) sub in (21) weget

$$T=\frac{1}{\alpha+\theta}\log\left(\frac{S(\alpha+\theta)}{a}+1\right)+\frac{1}{\theta}\log\left(\frac{D+\theta S_0}{D}+\frac{Q-S}{D}\right)\rightarrow(22)$$

Here the total variable cost is consist of fixed cost, holding cost, shortage cost, purchased cost minus the selling price of the deteriorated items. They are sumed up after evaluating the above cost individually

The ordering cost OC = $C_3$

The holding cost $HC=h\int_0^{T_0}q(t)dt+h\int_{T_0}^{T_1}q(t)dt$

$$=h\left[\int_0^{T_0}\left\{\frac{-a}{\alpha+\theta}e^{\alpha t}+\left(S+\frac{a}{\alpha+\theta}e^{-\theta t}\right)\right\}dt+\int_{T_0}^{T_1}\left\{\frac{-D}{\theta}+\left(S_0+\frac{D}{\theta}\right)e^{-\theta(t-T_0)}\right\}dt\right]$$

$$HC=h\left(\frac{-a}{\alpha+\theta}\left(\frac{e^{\alpha t}}{\alpha}\right)_0^{T_0}+\left(S+\frac{a}{\alpha+\theta}\right)\left(\frac{e^{-\theta t}}{-\theta}\right)_0^{T_0}\right)-\frac{D}{\theta^2}\log\frac{D+\theta S_0}{D}+\frac{S_0}{\theta}$$

$$HC=h\left[\frac{-a}{\alpha+\theta}\left(\frac{e^{\alpha T_0}}{\alpha}-\frac{1}{\alpha}\right)+\left(S+\frac{a}{\alpha+\theta}\right)\left(\frac{e^{-\theta T_0}}{-\theta}+\frac{1}{\theta}\right)-\frac{D}{\theta^2}\log\left(\frac{D+\theta S_0}{D}\right)+\frac{S_0}{\theta}\right]\rightarrow 23$$

The deteriorted items = $q(0)-\left(\int_0^{T_0}(ae^{\alpha t})dt+\int_{T_0}^{T_1}Ddt\right)=S-\left(a\int_0^{T_0}e^{\alpha t}dt+D(t)_{T_0}^{T_1}\right)$

$$=S_0-a\left(\frac{e^{\alpha T_0}}{\alpha}-\frac{1}{\alpha}\right)-\frac{D}{\theta}\log\frac{D+\theta S_0}{D}\rightarrow(24)$$

The Selling price of deteriorated items

$$DP=U\left(S_0+a\left(\frac{e^{\alpha T_0}}{\alpha}-\frac{1}{\alpha}\right)-\frac{D}{\theta}\log\frac{D+\theta S_0}{D}\right)\rightarrow(25)$$

The shortage cost SHC for a cycle is

SHEC = $-c_2\int_{T_1}^{T}q(t)dt$

$$=-C_2\int_{T_1}^{T}-(Q-S)dt$$

$$=C_2\int_{T_1}^{T}(Q-S)dt$$

SHE = $C_2\frac{(Q-S)^2}{2D}\rightarrow(26)$

The purchase cost PC fer a quantity of amount $S(f_{i-1}<S<f_i)$ is

$$PC=P_iS\rightarrow(27)$$

The total cost function

TVC = OC + HC + SHC + PC – DP

$$=C_3+h\left(\frac{-a}{\alpha+\theta}\left(\frac{e^{\alpha T_0}}{\alpha}-\frac{1}{\alpha}\right)+\left(S+\frac{a}{\alpha+\theta}\right)\left(\frac{e^{-\theta T_0}}{-\theta}+\frac{1}{\theta}\right)-\frac{D}{\theta^2}\log\left(\frac{D+\theta S_0}{D}\right)+\frac{S_0}{\theta}\right)$$

$$+C_2\frac{(Q-S)^2}{2D}+P_iS-U\left(S_0+a\left(\frac{e^{\alpha T_0}}{\alpha}-\frac{1}{\alpha}\right)-\frac{D}{\theta}\log\frac{D+\theta S_0}{D}\right)\rightarrow(28)$$

The selling price SR for a quantity of non - deteriorated amount is

$$SR=V_i\left(S-\left\{S_0-a\left(\frac{e^{\alpha T_0}}{\alpha}-\frac{1}{\alpha}\right)-\frac{D}{\theta}\log\left(\frac{D+\theta S_0}{D}\right)\right\}\right)\rightarrow(29)$$

The total profit functics TP is defined as $TP(S,Q)=SR-TVC$

$$=V_i\left(S-\left\{S_0+a\left(\frac{e^{\alpha T_0}}{\alpha}-\frac{1}{\alpha}\right)-\frac{D}{\theta}\log\left(\frac{D+\theta S_0}{\theta}\right)\right\}\right)$$

$$-\left(C_3+h\left\{\frac{-a}{\alpha+\theta}\left(\frac{e^{\alpha T_0}}{\alpha}-\frac{1}{\alpha}\right)+\left(S+\frac{a}{\alpha+\theta}\right)\left(\frac{e^{-\theta T_0}}{-\theta}+\frac{1}{\theta}\right)\right\}-\frac{D}{\theta^2}\log\left(\frac{D+\theta S_0}{D}\right)+\frac{S_0}{\theta}\right)$$

$$+C_2\frac{(Q-S)^2}{2D}+P_iS-U\left(S+a\left(\frac{e^{\alpha T_0}}{\alpha}-\frac{1}{\alpha}\right)-\frac{D}{\theta}\log\frac{D+\theta S_0}{D}\right)$$

$$=(V_i-P_i)S-C_3+\frac{ha}{\alpha+\theta}\left(\frac{e^{\alpha T_0}}{\alpha}-\frac{1}{\alpha}\right)-h\left(S+\frac{a}{\alpha+\theta}\right)\left(\frac{e^{-\theta T_0}}{-\theta}+\frac{1}{\theta}\right)$$

$$+\frac{hD}{\theta^2}\log\left(\frac{D+\theta S_0}{D}\right)-\frac{hS_0}{\theta}+C_2\frac{(Q-S)^2}{2D}+U\left(S+a\left(\frac{e^{\alpha T_0}}{\alpha}-\frac{1}{\alpha}\right)-\frac{D}{\theta}\log\frac{D+\theta S_0}{D}\right)$$

$$-V_iS_0+a\left(\frac{e^{\alpha T_0}}{\alpha}-\frac{1}{\alpha}\right)-\frac{D}{\theta}\log\left(\frac{D+\theta S_0}{D}\right)$$

TP= $(V_i-P_i)S-C_3+\frac{ha}{\alpha+\theta}\left(\frac{e^{\alpha T_0}}{\alpha}-\frac{1}{\alpha}\right)-\left(S+\frac{a}{\alpha+\theta}\right)\left(\frac{e^{-\theta T_0}}{-\theta}+\frac{1}{\theta}\right)+\frac{hD}{\theta^2}\log\left(\frac{D+\theta S_0}{D}\right)$

$$-\frac{hS_0}{\theta}+C_2\frac{(Q-S)^2}{2D}+(U-V_i)\left(S_0+a\left(\frac{e^{\alpha T_0}}{\alpha}-\frac{1}{\alpha}\right)-\frac{D}{\theta}\log\left(\frac{D+\theta S_0}{D}\right)\right)\rightarrow 30$$

The total Profit per unit time $TPU(S,Q)=\frac{TP(S,Q)}{T}-S$

$$=\frac{1}{T}(V_i-P_i)S-C_3+\frac{ha}{\alpha+\theta}\left(\frac{e^{\alpha T_0}}{\alpha}-\frac{1}{\alpha}\right)-\left(S+\frac{a}{\alpha+\theta}\right)\left(\frac{e^{-\theta T_0}}{-\theta}+\frac{1}{\theta}\right)$$

$$+\frac{hD}{\theta^2}\log\left(\frac{D+\theta S_0}{D}\right)-\frac{hS_0}{\theta}+C_2\frac{(Q-S)^2}{2D}+(U-V_1)\left\{S_0+\alpha\left(\frac{e^{\alpha T_0}}{\alpha}-\frac{1}{\alpha}\right)-\frac{D}{\theta}\log\left(\frac{D+\theta S_0}{D}\right)\right\}\rightarrow 31$$

Here the profit function (31) is to be maximized

## 1.5NUMERICAL EXAMPLE AND SENSITIVITY ANALLYSIS

A Numerical Example is Considered to illustrate the effect of the developed model.

The Set-up cost (C3) - 8250/- Order

The demand (D) - 300/Month

The Shortage Cost (C2) $ 0.3/Unit

The holiday cost per Unit (h) $ 10

The Selling Price of deteriorated Items (u) = $ 15
and the price breaks arc

| Quantity | Purchase Cost | Selling Price |
|---|---|---|
| O<S< 500 | 30 | 40 |
| 500<S≤1000 | 28 | 37 |
| 1000<S≤2000 | 25 | 33 |
| 2000<S | 18 | 26 |

The Problem is solved by Genetic Algorithm and is given in appendix. The Solutions for different parametric Values of θ,a,b,c and so are given in Tables 1 to 4 .

**TABLE No1 : (θ < S≤500)**

| Parameters | Changing Paramieters | Change in Parameters | S | Q | Total Profit Per Unit time (TPU) |
|---|---|---|---|---|---|
| a=4 | | 0.1 | 497.72 | 543.09 | 4180.72 |
| $S_0$ = 90 | θ | 0.4 | 476.32 | 532.08 | 3999.83 |
| | | 0.6 | 465.24 | 525.14 | 3895.72 |
| θ =0.1 | | 4 | 497.72 | 546.09 | 4180.72 |
| $S_0$ = 90 | a | 12 | 496.57 | 552.08 | 4328.53 |
| | | 16 | 485.23 | 562.62 | 4520.14 |
| θ =0.1 | $S_0$ | 90 | 497.72 | 546.09 | 4180.72 |
| a=4 | | 110 | 498.54 | 548.04 | 3725.06 |
| | | 125 | 512.13 | 552.24 | 3624.16 |

**TABLE No2 : (500< S≤1000)**

| Parameters | Changing Paramieters | Change in Parameters | S | Q | Total Profit Per Unit time (TPU) |
|---|---|---|---|---|---|
| a=4 | | 0.1 | 992.16 | 1086.57 | 5990.76 |
| $S_0$ = 100 | **θ** | 0.4 | 980.2 | 1094.12 | 6102.27 |
| | | 0.6 | 965.24 | 1098.24 | 6120.25 |
| θ =0.1 | | 4 | 992.16 | 1086.57 | 5990.76 |
| $S_0$ = 100 | a | 12 | 999.75 | 1099.36 | 6125.26 |
| | | 16 | 1000.26 | 1100.76 | 6224.28 |
| θ =0.1 | | 100 | 992.16 | 1086.57 | 5990.76 |
| a=4 | $S_0$ | 130 | 995.26 | 1096.56 | 5362.96 |
| | | 140 | 1000.02 | 1099.23 | 5120.14 |

| TABLE No3 : (1000< S≤2000) | | | | | |
|---|---|---|---|---|---|
| **Parameters** | **Changing Paramieters** | **Change in Parameters** | **S** | **Q** | **Total Profit Per Unit time (TPU)** |
| a=4 | | 0.1 | 1983.76 | 2096.29 | 8956.79 |
| $S_0$ = 100 | **θ** | 0.4 | 1968.26 | 2089.63 | 8691.03 |
| | | 0.6 | 1852.06 | 2076.23 | 8532.04 |
| θ =0.1 | | 4 | 1983.76 | 2096.29 | 8956.79 |
| $S_0$ = 100 | a | 12 | 1996.06 | 2102.24 | 9266.16 |
| | | 16 | 1982.13 | 2108.13 | 9462.26 |
| θ =0.1 | | 100 | 1983.76 | 2096.29 | 8956.79 |
| a=4 | $S_0$ | 130 | 1987.48 | 2098.36 | 8238.02 |
| | | 140 | 1978.16 | 2086.24 | 8432.15 |

| TABLE No4 : (2000< S) | | | | | |
|---|---|---|---|---|---|
| **Parameters** | **Changing Paramieters** | **Change in Parameters** | **S** | **Q** | **Total Profit Per Unit time (TPU)** |
| a=4 | | 0.1 | 2492.86 | 2649.71 | 5699.31 |
| $S_0$ = 100 | **θ** | 0.4 | 2471.5 | 2631.02 | 5542.8 |
| | | 0.6 | 2462.16 | 2612.14 | 5438.24 |
| θ =0.1 | | 4 | 2492.86 | 2649.71 | 5699.31 |
| $S_0$ = 100 | a | 12 | 2499.56 | 2657.72 | 5798.26 |
| | | 16 | 2504.21 | 2672.18 | 5892.14 |
| θ =0.1 | | 100 | 2492.86 | 2649.71 | 5699.41 |
| a=4 | $S_0$ | 130 | 2496.76 | 2651.32 | 5425.35 |
| | | 140 | 2502.36 | 2702.24 | 5380.06 |

## 1.6 Sensitivity analysis of the above model reveals the following observations

- When the Parameter ‘θ’ increases, the corresponding order level and optimal profit will decrease.
- In case of change in Parameter ‘a’ i.e., the demand change increases the optimal order quantity as well as optimal profit increases.

- ❖ When initial value of order level increases the corresponding EOQ and Profit will increase.

- ❖ From the above tables one can be observe that the total profit per minute time is maximum in range $1000 < S \leq 2000$ i.e., Rs.9462.26.

## 1.7 CONCLUSION.

In this paper an attempt is made with two component demand i.e., initially the demand is increasing with exponential rate and after certain period the demand would be constant. In real word, it is a fact that when a product is introduced it will have high sells and gradually its demand will come down this type of situation can be seen in a electric products like I.C. Chips, L.D. bulbs, etc. It will be more interesting when their product of a commodities demand initially constant and later it will pick up demand exponentially, this type of situation can be found in case of detergent products.

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# Appendix

## (D) Genetic Algorithm

Genetic Algorithm is a class of adaptive search technique based on the principle of population genetics. The algorithm is an example of a search procedure that uses random choice as a tool to guide a highly exploitative search through a coding of parameter space. Genetic Algorithm work according to the principles of natural genetics on a population of string structure representing the problem variable. All these features make genetic algorithm search robust allowing them to be applied to a wide variety of problems.

### Implementing GA

The following are adopted in the proposed GA to solve the problem:

(1) Parameters
(2) Chromosome representation
(3) Initial population production
(4) Evaluation
(5) Selection
(6) Crossover
(7) Mutation
(8) Termination

### Parameters

Firstly, we set the different parameters on which this GA depends. All these are the number of generation (MAXGEN), population size(POPSIZE), probability of crossover(PCROS), probability of mutation (PMUTE)

### Chromosome Representation

An important issue in applying a GA is to design an appropriate chromosome representation of solutions of the problem together with genetic operators. Traditional binary vectors used to represent the chrosones are not effective in many non-linear problems. Since the proposed problem is highly non-linear, hence to overcome the difficulty, a real-number representation is used. In this representation, each chromosome Vi is a string of n numbers of genes G (j = 1,2,….n) where these n numbers of genes respectively denote n number of decision variables (Xi, i=1,2,….n).

**Initial Population Production**

The population generation technique proposed in the present GA is illustrated by the following procedure. For each chromosome Vi, every gene $G_{ij}$ is randomly generated between its boundary ($LB_j$, $UB_j$ 0 where $LB_j$, and $UB_j$ are the lower and upper bounds of the variables $X_1$ i=1,2,….n, POPSIZE.

**Evaluation**

Evaluation function plays the some role in GA as that which the environment plays in natural evaluation. Now, evaluation function (EVAL) for the chromosome $V_1$ is equivalent to the objective function PF(X). These are following steps of evaluation.

**Step 1:** find EVAL ($V_1$) by EVAL ($V_1$) = f($X_1$, $X_2$…..$X_n$)

Where the genes G represent the decision variable $X_1$, j= 1,2….n, POPSIZE and f is the objective function.

**Step 2 :** find total fitness of the population: F= $\sum_{i=1}^{popsize} EVAL(V_1)$

**Step 3 :** calculate the probability pi of selection for each chromosome V1 as

$Y_i = \sum_{i=1}^{1} p_1$

**Selection**

The selection scheme in GA determines which solutions in the current population are to be selected for recombination. Many selection schemes, such as Stochastic random sampling, Roulette wheel selection have been proposed for various problems. In this paper we adopt roulette wheel selection process.

This roulette selection process is based on spinning the roulette wheel POPSIZE times, each time we select a single chromosome for the new population in the following way:

(a) Generate a random (float) number r between 0 to 1.
(b) If r< Y, then the first chromosome is Vi otherwise select the $i^{th}$ chromosome $V_i$ $(2 \le i \le POPSIZE)$ such that $T_{i-1} \le r \le Y_1$

**Crossover**

Crossover operator is mainly responsible for the search of new string. The exploration of the solution space is made possible by exchanging genetic information of the current chromosomes. Crossover operates on two parent solutions at a time and generates offspring solutions by recombining both parent solution features. After selection chromosomes for new population, the crossover operator is applied. Here, the whole arithmetic crossover operation is used. It is defined as a linear combination of two consecutive selected chromosomes $V_m$ and $V_n$ and the resulting offspring $V^1_m$ and are $V^1_n$ calculated as:

$$V_m^1 = c.V_m + (l-c).V_n$$
$$V_n^1 = c.V_n + (l-c).V_m$$

Where c is a random number between 0 and 1.

**Mutation**

Mutation operator is used to prevent the search process from converging to local optima rapidly. It is applied to a single chromosome Vi the selection of a chromosome for mutation is performed in the following way:

Step 1: Set i ←1

Step 2: Generate a random number u from the range [0,1]

Step 3: If u<PMUTE, then go to step 2.

Step 4: Set i ← i+1

Step 5: If i $\leq$ POPSIZE, then go to Step 2.

Then the particular gene $G_{ij}$ of the chromosome $V_1$ selected by the above mentioned steps is randomly selected in this problem, the mutation is defined as

$G_y^{mut}$ random number from the range (0,1)

**Termination**

If the number of iteration is less than or equal to MAXGEN then the process is going on, otherwise it terminates.
The GA's procedure is given below:

```
Begin

do {
        t ← 0
        while (all constraints are not satisfied)
        {
        Initialize Population (t)
        }
        Evaluate Population (t)
```

```
while (not terminate)
{
t ← t + 1
select Population(t) from Population (t-1)
crossover and mutate Population(t)
evaluate Population(t)
}
Print Optimum Result
}
End.
```