

## Finding Exact and Approximate Solution of Fractional Burger's Equation

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### Abstract

In this paper we study fractional Burger's equation of the form  $u_t^{(\alpha)} + uu_x - u_{xx} = 0$ ,  $x \in \mathbb{R}$ ,  $t > 0$  to find the exact solution. Sumudu transform is used to find the approximate solution and comparing this solution with the exact solution.

**Keywords** Fractional calculus, Burger's equation, Sumudu Decomposition method.

### 1. Introduction

Fractional differential equations (FDEs) appear more and more frequently in different research areas of physics and engineering such as chemistry, electromagnetics, acoustics, viscoelasticity, electrochemistry, and material science, ..., etc[2]. In general, there exists no method that yields an exact solution for fractional partial differential equations. Only approximate and numerical solutions can be derived.

Burger's equation is a fundamental partial differential equation in fluid mechanics. It is also a very important model encountered in several areas of applied mathematics such as heat conduction, acoustic waves, gas dynamics and traffic flow. It was actually first introduced by Bateman (1915) [6] when he mentioned it as worthy of study and gave its steady solutions. It was later proposed by Burger(1948) [7] as one of a class of equation describing mathematical models of turbulence. In the context of gas dynamics it was discussed by Hopf, and Cole(1950)[8]. In (1972) Benton and Platzman [16] Surveyed exact solution of one dimensional Burger's equation. In (2005) Gorguis [9] gives comparison between Cole - Hopf transformation and Decomposition method for solving Burger's equation. In (2006) Kadalbajoo and Awasti [10] developed stable numerical method based on Crank Nicolson to solve Burger's equation. In (2008) Djoko [11] examine the stability of a finite difference Approximation for Burger's equation by approximating the nonlinear term by a linear expression using techniques based on the boundaries of the solution sequence with respect to  $\Delta t$  for  $t \in (0, \infty)$  and with the help of discrete Aronwall lemma stability is achieved. In (2009)Wani and Thakar [12] analysed stability of Mixed Euler Method for one Dimensional nonlinear Burger's equation. In (2009) Pandey and Verma [13] wrote on difference scheme for Burger's equation. In (2011) Kanti Pandey and Lajja Verma [15] gave a note on Crank Nicolson scheme for Burger's equation without Hopf -Cole transformation solutions are obtained by ignoring nonlinear term. The Sumudu transformation method is one of the most important transform methods introduced in the early 1993s by Watugala [14] it is a powerful tool for solving many kinds of PDEs in various field of science and engineering. In (2014) Saad N. and Muna S. [17] used Bernoulli equation to solve Burger's equation.

## 2. Preliminaries

In this section, we provide some basic definitions and properties of the fractional calculus theory and Sumudu transform which are used in this paper.

### Definition 2.1[1]

A real valued function  $f(x)$ ,  $x > 0$ , is said to be in the space  $C_\mu$ ,  $\mu \in \mathbb{R}$ , if there exists a real number  $p$ ,  $p > \mu$ , such that  $f(x) = x^p f_1(x)$ , where  $f_1(x) \in C[0, \infty)$ , and it is said to be in the space  $C_\mu^n$  if  $f^{(n)}(x) \in C_\mu$ ,  $n \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$ .

### Definition 2.2 [1]

The Riemann-Liouville fractional integral operator of order  $\alpha \geq 0$ , of a function  $f(x) \in C_\mu$ ,  $\mu \geq -1$  is defined as

$$J^\alpha f(x) = \frac{1}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} f(t) dt, \quad x > 0 \tag{1}$$

$$J^0 f(x) = f(x).$$

Properties of the operator  $J^\alpha$  can be found for  $f \in C_\mu$ ,  $\mu \geq -1$ ,  $\alpha, \beta \geq 0$  and  $\gamma > -1$ , we have

1.  $J^\alpha J^\beta f(x) = J^{\alpha+\beta} f(x) = J^\beta J^\alpha f(x)$
2.  $J^\alpha C = \frac{C}{\Gamma(\alpha+1)} x^\alpha$ , C is constant
3.  $J^\alpha x^\gamma = \frac{\Gamma(\gamma+1)}{\Gamma(\alpha+\gamma+1)} x^{\alpha+\gamma}$

### Definition 2.3[1]

The Caputo definition of fractional derivative operator is given by

$$D_*^\alpha f(x) = \frac{1}{\Gamma(n-\alpha)} \int_0^x (x-t)^{n-\alpha-1} f^{(n)}(t) dt, \quad \alpha > 0 \tag{2}$$

For  $n-1 < \alpha \leq n$ ,  $n \in \mathbb{N}$ ,  $x > 0$  and  $\Gamma(\cdot)$  is the gamma function.

and satisfies the following properties:

1.  $D_*^\alpha C = 0$ , C is constant
2.  $D_*^\alpha x^\gamma = \frac{\Gamma(\gamma+1)}{\Gamma(\gamma-\alpha+1)} x^{\gamma-\alpha}$ ,  $x > 0$ ,  $\gamma > -1$
3.  $D_*^\alpha (\sum_{i=0}^m c_i f_i(x, t)) = \sum_{i=0}^m c_i D_*^\alpha f_i(x, t)$ , where  $c_0, c_1, \dots, c_m$  are constant.

### Definition 2.4[2]

For  $m$  to be the smallest integer that exceeds  $\alpha$ , the Caputo time- fractional derivatives of order  $\alpha > 0$  is defined as

$$D_{*t}^\alpha u(x, t) = \frac{\partial^\alpha u(x, t)}{\partial t^\alpha} = \begin{cases} \frac{1}{\Gamma(m-\alpha)} \int_0^t (t-\tau)^{m-\alpha-1} \frac{\partial^m u(x, \tau)}{\partial \tau^m} d\tau, & m-1 < \alpha < m \\ \frac{\partial^m u(x, t)}{\partial t^m}, & \alpha = m \in \mathbb{N} \end{cases}$$

### Lemma 2.1[2]

If  $n-1 < \alpha \leq n$ ,  $f \in C_\mu^n$ ,  $n \in \mathbb{N}$  and  $\mu \geq -1$ , then

$$D_*^\alpha J^\alpha f(x) = f(x) \text{ and}$$

$$J^\alpha D_*^\alpha f(x) = f(x) - \sum_{k=0}^{n-1} f^{(k)}(0^+) \frac{x^k}{k!}, \text{ where } x > 0$$

### Definition 2.5[3]

The Sumudu transform over the following set of functions

$$A = \{f(t) \mid \exists M, \tau_1, \tau_2 > 0, |f(t)| < M e^{\frac{|t|}{\tau_1}}, \text{ if } t \in (-1)^j \times [0, \infty)\} \tag{3}$$

Is defined as, for  $u \in (\tau_1, \tau_2)$ , we have:

$$\mathbb{S} [f(t)] = G(u) = \int_0^\infty f(ut) e^{-t} dt = \int_0^\infty \frac{1}{u} f(t) e^{-\frac{t}{u}} dt \quad (4)$$

Some special properties of the Sumudu transform are as follows:

1.  $\mathbb{S} [1] = 1$
2.  $\mathbb{S} \left[ \frac{t^n}{\Gamma(n+1)} \right] = u^n, n > 0$
3.  $\mathbb{S} [e^{at}] = \frac{1}{1-au}$
4.  $\mathbb{S} [\alpha f(x) \pm \beta g(x)] = \alpha \mathbb{S} [f(x)] \pm \beta \mathbb{S} [g(x)].$

### Theorem 2.1[3]

Let  $G(u)$  be the Sumudu transform of  $f(t)$ , such that

1.  $G(1/s)/s$ , is a meromorphic function, with singularities having  $\text{Re}(s) < \gamma$ , and
2. There exists a circular region  $\Gamma$  with radius  $R$  and positive constants  $M$  and  $k$  with

$$\left| \frac{G(1/s)}{s} \right| < M R^{-k}$$

Then the function  $f(t)$  is given by

$$f(t) = \mathbb{S}^{-1}[G(t)] = \frac{1}{2\pi i} \int_{\gamma-i\infty}^{\gamma+i\infty} e^{st} G\left(\frac{1}{s}\right) \frac{ds}{s} = \sum \text{residuse} \left[ e^{st} \frac{G(1/s)}{s} \right].$$

To solve fractional differential equations, the following lemma of Sumudu transform will be needed.

### Lemma 2.2[3]

The Sumudu transform  $\mathbb{S} [f(t)]$  of the Caputo fractional derivative is defined as follows:

$$\mathbb{S} [D_t^\alpha f(t)] = \frac{G(u)}{u^\alpha} - \sum_{k=0}^{n-1} \frac{f^{(k)}(0)}{u^{\alpha-k}}, \text{ where } G(u) = \mathbb{S} [f(t)]$$

Then it can be easily understood that

$$\mathbb{S} [D_t^\alpha f(x, t)] = \frac{\mathbb{S}[f(x, t)]}{u^\alpha} - \sum_{k=0}^{n-1} \frac{f^{(k)}(x, 0)}{u^{\alpha-k}}, n-1 < \alpha \leq n \quad (5)$$

## 3. Exact Solution of the Fractional Burger's Equation

The procedure is divided into four steps. In the first step we reduce the second form of the fractional Burger's equation to second order nonlinear ordinary differential equation. In the second step this nonlinear ordinary differential equation is integrated to get first order ordinary differential equation. In the third step the first order is reduced to the Bernoulli equation. The fourth step we find the exact solution of the second form of the fractional Burger's equation. And shows some figures represents the exact solution.

The second form of fractional Burger's equation is

$$u_t^{(\alpha)} + uu_x - u_{xx} = 0, 0 < \alpha \leq 1, x \in \mathbb{R}, t \geq 0 \quad (6)$$

To solve equation (6) we take the transformation

$$\xi(x, t) = kx + \frac{ct^\alpha}{\Gamma(\alpha+1)} + \xi_0 \quad (7)$$

Where  $k, c, \xi_0$  are constants, by using this transformation we get

$$u_t^{(\alpha)} = u_t^{(\alpha)}(\xi) = u'(\xi) D_t^\alpha \xi = u'(\xi) \frac{c}{\Gamma(\alpha+1)} \frac{\Gamma(\alpha+1)}{\Gamma(\alpha-\alpha+1)} = cu'(\xi)$$

And

$$uu_x = u(\xi) (u(\xi))_x = ku(\xi)u'(\xi)$$

Also

$$u_{xx} = (u(\xi))_{xx} = (ku'(\xi))_x = k(ku''(\xi)) = k^2u''(\xi)$$

So by this transformation for  $\xi$ , equation (6) can be turned into the following nonlinear ordinary differential equation of second order with respect to the variable  $\xi$ ;

$$cu' + kuu' - k^2u'' = 0, \quad ' = \frac{d}{d\xi} \tag{8}$$

Integrating equation (8) from 0 to  $\xi$  we obtain

$$cu(\xi) - cu(0) + \frac{ku^2(\xi)}{2} - \frac{ku^2(0)}{2} - k^2u'(\xi) + k^2u'(0) = 0 \tag{9}$$

$$k^2u'(\xi) - cu(\xi) = \frac{ku^2(\xi)}{2} + L \tag{10}$$

$$\text{Where } L = k^2u'(0) - cu(0) - \frac{ku^2(0)}{2} \tag{11}$$

We shall reduce equation (10) to Bernoulli equation by letting

$$\bar{u}(\xi) = u(\xi) + m, \quad m \text{ is constant} \tag{12}$$

Substituting (12) into (10) we have

$$k^2\bar{u}'(\xi) + (km - c)\bar{u}(\xi) = \frac{k\bar{u}^2(\xi)}{2} + \frac{km^2}{2} - cm + L \tag{13}$$

Now we can choose  $m$  such that

$$km^2 - 2cm + 2L = 0 \tag{14}$$

That is

$$m_{1,2} = \frac{c \pm \sqrt{c^2 - 2kL}}{k}$$

Therefore equation (13) becomes first order ODE's

$$\bar{u}' + \frac{(km-c)}{k}\bar{u} = \frac{\bar{u}^2}{2k} \tag{15}$$

Notice that equation (15) represents Bernoulli equation.

To solve it, suppose

$$z = \bar{u}^{-1} \text{ Then } z_\xi = -\bar{u}^{-2}\bar{u}_\xi \tag{16}$$

Substituting equation (16) into (15) we obtain

$$\bar{u}^{-2}\bar{u}_\xi + \frac{(km-c)}{k^2}\bar{u} = \frac{1}{2k}$$

To get;

$$z_\xi + \frac{(c-km)}{k^2}z = -\frac{1}{2k} \tag{17}$$

This equation is first-order linear differential equation which has unique solution for a given initial condition according to the existence and uniqueness theorem [4].

The integrating factor of (17) is

$$I(\xi) = e^{\left(\frac{c-km}{k^2}\right)\xi}$$

And its solution is given by

$$e^{\left(\frac{c-km}{k^2}\right)\xi} z = -\frac{k}{2(c-km)} e^{\left(\frac{c-km}{k^2}\right)\xi} + c_1$$

Where  $c_1$  is an arbitrary constant.

From (16) we get the solution of Bernoulli equation (15)

$$\bar{u}(\xi) = \frac{e^{\left(\frac{c-km}{k^2}\right)\xi}}{\frac{k}{2(km-c)} e^{\left(\frac{c-km}{k^2}\right)\xi} + c_1} \tag{18}$$

Now from (12),  $u(\xi) = \bar{u}(\xi) + m$ , so that

$$u(\xi) = \frac{e^{\frac{(c-km)}{k^2}\xi}}{\frac{k}{2(km-c)}e^{\frac{(c-km)}{k^2}\xi} + c_1} - m \tag{19}$$

Substituting equation (7) into (19) to get

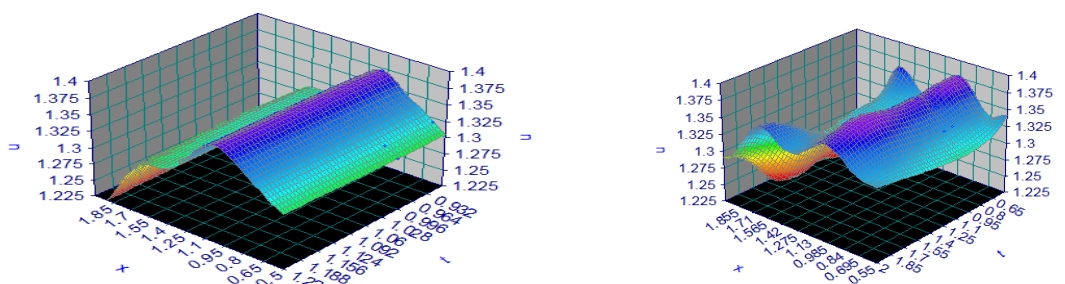
$$u(x, t) = \frac{e^{\frac{(c-km)}{k^2}(kx + \frac{ct^\alpha}{\Gamma(\alpha+1)} + \xi_0)}}{\frac{k}{2(km-c)}e^{\frac{(c-km)}{k^2}(kx + \frac{ct^\alpha}{\Gamma(\alpha+1)} + \xi_0)} + c_1} - m \tag{20}$$

Which is the solution of the second form of fractional Burger's equation (6).

As a special case let  $L = \xi_0 = 0$ ,  $c = -\frac{1}{8}$ ,  $k=1$  and from equation (14) to obtain either  $m = 0$  or  $m = -\frac{1}{4}$ , if we

take  $m = -\frac{1}{4}$  and  $c_1 = 6$  from (20) to get;

$$u(x, t) = \frac{e^{\frac{1}{8}(x - \frac{t^\alpha}{8\Gamma(\alpha+1)})}}{-4e^{\frac{1}{8}(x - \frac{t^\alpha}{8\Gamma(\alpha+1)})} + 6} + \frac{1}{4} \tag{21}$$



**Figure 1:** represents the solution equation (21) when  $\alpha = 0.75$ ,  $0.5 < x < 2$ ,  $0.9 < t < 1.22$  and  $1.225 < u < 1.4$ .

**Figure 2:** represents the solution equation (21) when  $\alpha = 0.9$ ,  $0.55 < x < 2$ ,  $0.5 < t < 2$  and  $1.225 < u < 1.4$ .

#### 4. Numerical Solution of the Fractional Burger's Equation Using Sumudu Decomposition Method[2]

In this section Sumudu transform is used to find the approximate solution for the fractional Burger's equation.

We can extended the value of  $\alpha$  to be any positive real number to solve equation (6) and we solve the case  $0 < \alpha \leq 1$ . Subject to the initial condition

$$u(x, 0) = \frac{e^{\frac{1}{8}x}}{-4e^{\frac{1}{8}x} + 6} + \frac{1}{4}$$

Taking the Sumudu transform of equation (6) and using the property of Sumudu transform together with the initial condition, we obtain

$$\mathbb{S} [u(x, t)] = \frac{e^{\frac{1}{8}x}}{-4e^{\frac{1}{8}x} + 6} + \frac{1}{4} + u^\alpha \mathbb{S} \left[ \frac{\partial^2}{\partial x^2} u(x, t) - u(x, t) \frac{\partial u(x, t)}{\partial x} \right].$$

The inverse of Sumudu transform implies that

$$\sum_{n=0}^{\infty} u_n(x, t) = \frac{e^{\frac{1}{8}x}}{-4e^{\frac{1}{8}x} + 6} + \frac{1}{4} + \mathbb{S}^{-1}[u^\alpha \mathbb{S} [\frac{\partial^2}{\partial x^2} \sum_{n=0}^{\infty} u_n(x, t) - \sum_{n=0}^{\infty} A_n(u)]] \quad (22)$$

Where  $A_n(u)$  are Adomian polynomials[5]. The first few components polynomials are given by

$$A_0 = u_0 u_{0x}, A_1 = u_0 u_{1x} + u_1 u_{0x}, A_2 = u_0 u_{2x} + u_1 u_{1x} + u_2 u_{0x} \text{ and so on.}$$

The recursive relation is given by

$$u_0(x, t) = \frac{e^{\frac{1}{8}x}}{-4e^{\frac{1}{8}x} + 6} + \frac{1}{4}$$

$$u_1(x, t) = \mathbb{S}^{-1}[u^\alpha \mathbb{S} [\frac{\partial^2}{\partial x^2} u_0(x, t) - A_0(u)]]$$

⋮

$$u_n(x, t) = \mathbb{S}^{-1}[u^\alpha \mathbb{S} [\frac{\partial^2}{\partial x^2} u_{n-1}(x, t) - A_{n-1}(u)]]$$

Therefore;

$$u_1(x, t) = \frac{t^\alpha}{\Gamma(\alpha+1)} \left( \frac{\frac{9}{16} e^{\frac{1}{8}x} - \frac{3}{8} e^{\frac{1}{4}x}}{(-4e^{\frac{1}{8}x} + 6)^3} - \frac{\frac{3}{16} e^{\frac{1}{8}x}}{(-4e^{\frac{1}{8}x} + 6)^2} \right)$$

$$u_2(x, t) = \frac{t^{2\alpha}}{\Gamma(2\alpha+1)} \left( \frac{\frac{81}{256} e^{\frac{1}{8}x} - \frac{27}{128} e^{\frac{1}{4}x} - \frac{9}{64} e^{\frac{3}{8}x} + \frac{3}{32} e^{\frac{1}{2}x}}{(-4e^{\frac{1}{8}x} + 6)^5} - \frac{\frac{27}{128} e^{\frac{1}{8}x} - \frac{9}{16} e^{\frac{1}{4}x} + \frac{3}{32} e^{\frac{3}{8}x}}{(-4e^{\frac{1}{8}x} + 6)^4} + \frac{\frac{9}{256} e^{\frac{1}{8}x} - \frac{9}{128} e^{\frac{1}{4}x}}{(-4e^{\frac{1}{8}x} + 6)^3} \right)$$

$$u_3(x, t) = \frac{t^{3\alpha}}{\Gamma(3\alpha+1)} \left( \frac{\frac{729}{4096} e^{\frac{1}{8}x} + \frac{243}{256} e^{\frac{1}{4}x} - \frac{189}{64} e^{\frac{3}{8}x} + \frac{207}{256} e^{\frac{1}{2}x} + \frac{31}{128} e^{\frac{5}{8}x} - \frac{3}{32} e^{\frac{3}{4}x}}{(-4e^{\frac{1}{8}x} + 6)^7} \right. \\ \left. - \frac{\frac{243}{1024} e^{\frac{1}{8}x} - \frac{2349}{2048} e^{\frac{1}{4}x} - \frac{375}{512} e^{\frac{3}{8}x} + \frac{17}{256} e^{\frac{1}{2}x} - \frac{3}{256} e^{\frac{5}{8}x}}{(-4e^{\frac{1}{8}x} + 6)^6} + \frac{\frac{243}{4096} e^{\frac{1}{8}x} - \frac{513}{2048} e^{\frac{1}{4}x} - \frac{117}{1024} e^{\frac{3}{8}x} + \frac{15}{512} e^{\frac{1}{2}x}}{(-4e^{\frac{1}{8}x} + 6)^5} \right. \\ \left. - \frac{\frac{27}{4096} e^{\frac{1}{8}x} - \frac{9}{512} e^{\frac{1}{4}x} - \frac{9}{1024} e^{\frac{3}{8}x}}{(-4e^{\frac{1}{8}x} + 6)^4} + \frac{\Gamma(2\alpha+1)t^{3\alpha}}{\Gamma(\alpha+1)^2\Gamma(3\alpha+1)} \left( -\frac{243}{1024} e^{\frac{1}{4}x} + \frac{81}{512} e^{\frac{3}{8}x} + \frac{27}{256} e^{\frac{1}{2}x} - \frac{9}{128} e^{\frac{5}{8}x} \right) \right. \\ \left. + \frac{\frac{81}{512} e^{\frac{1}{4}x} - \frac{27}{128} e^{\frac{3}{8}x} + \frac{9}{128} e^{\frac{1}{2}x}}{(-4e^{\frac{1}{8}x} + 6)^6} - \frac{\frac{27}{1024} e^{\frac{1}{4}x} - \frac{27}{512} e^{\frac{3}{8}x}}{(-4e^{\frac{1}{8}x} + 6)^5} \right)$$

And so on.

The solution by Sumudu transformation up to four terms is

$$u(x, t) = \sum_{i=0}^3 u_i(x, t) = u_0(x, t) + u_1(x, t) + u_2(x, t) + u_3(x, t) \\ = \frac{e^{\frac{1}{8}x}}{-4e^{\frac{1}{8}x} + 6} + \frac{1}{4} + \frac{t^\alpha}{\Gamma(\alpha+1)} \left( \frac{\frac{9}{16} e^{\frac{1}{8}x} - \frac{3}{8} e^{\frac{1}{4}x}}{(-4e^{\frac{1}{8}x} + 6)^3} - \frac{\frac{3}{16} e^{\frac{1}{8}x}}{(-4e^{\frac{1}{8}x} + 6)^2} \right) \\ + \frac{t^{2\alpha}}{\Gamma(2\alpha+1)} \left( \frac{\frac{81}{256} e^{\frac{1}{8}x} - \frac{27}{128} e^{\frac{1}{4}x} - \frac{9}{64} e^{\frac{3}{8}x} + \frac{3}{32} e^{\frac{1}{2}x}}{(-4e^{\frac{1}{8}x} + 6)^5} - \frac{\frac{27}{128} e^{\frac{1}{8}x} - \frac{9}{16} e^{\frac{1}{4}x} + \frac{3}{32} e^{\frac{3}{8}x}}{(-4e^{\frac{1}{8}x} + 6)^4} \right. \\ \left. + \frac{\frac{9}{256} e^{\frac{1}{8}x} - \frac{9}{128} e^{\frac{1}{4}x}}{(-4e^{\frac{1}{8}x} + 6)^3} \right) + \frac{t^{3\alpha}}{\Gamma(3\alpha+1)} \left( \frac{\frac{729}{4096} e^{\frac{1}{8}x} + \frac{243}{256} e^{\frac{1}{4}x} - \frac{189}{64} e^{\frac{3}{8}x} + \frac{207}{256} e^{\frac{1}{2}x} + \frac{31}{128} e^{\frac{5}{8}x} - \frac{3}{32} e^{\frac{3}{4}x}}{(-4e^{\frac{1}{8}x} + 6)^7} \right.$$

$$\begin{aligned}
 & - \frac{\frac{243}{1024}e^{\frac{1}{8}x} - \frac{2349}{2048}e^{\frac{1}{4}x} - \frac{375}{512}e^{\frac{3}{8}x} + \frac{17}{256}e^{\frac{1}{2}x} - \frac{3}{256}e^{\frac{5}{8}x}}{\left(-4e^{\frac{1}{8}x} + 6\right)^6} + \frac{\frac{243}{4096}e^{\frac{1}{8}x} - \frac{513}{2048}e^{\frac{1}{4}x} - \frac{117}{1024}e^{\frac{3}{8}x} + \frac{15}{512}e^{\frac{1}{2}x}}{\left(-4e^{\frac{1}{8}x} + 6\right)^5} \\
 & - \frac{\frac{27}{4096}e^{\frac{1}{8}x} - \frac{9}{512}e^{\frac{1}{4}x} - \frac{9}{1024}e^{\frac{3}{8}x}}{\left(-4e^{\frac{1}{8}x} + 6\right)^4} + \frac{\Gamma(2\alpha+1)t^{3\alpha}}{\Gamma(\alpha+1)^2\Gamma(3\alpha+1)} \left( \frac{-\frac{243}{1024}e^{\frac{1}{4}x} + \frac{81}{512}e^{\frac{3}{8}x} + \frac{27}{256}e^{\frac{1}{2}x} - \frac{9}{128}e^{\frac{5}{8}x}}{\left(-4e^{\frac{1}{8}x} + 6\right)^7} \right. \\
 & \left. + \frac{\frac{81}{512}e^{\frac{1}{4}x} - \frac{27}{128}e^{\frac{3}{8}x} + \frac{9}{128}e^{\frac{1}{2}x}}{\left(-4e^{\frac{1}{8}x} + 6\right)^6} - \frac{\frac{27}{1024}e^{\frac{1}{4}x} - \frac{27}{512}e^{\frac{3}{8}x}}{\left(-4e^{\frac{1}{8}x} + 6\right)^5} \right).
 \end{aligned}$$

**Table1.** Comparison between the exact solution of equation (21) and the approximate solution by Sumudu

where  $\alpha = 0.75$ .

t	x	u(x, t) Sumudu	u(x, t) Exact	Error
0.2	0.2	0.7827	0.7814	0.0013
	0.5	0.8526	0.8505	0.0021
	0.7	0.9082	0.9056	0.0026
0.5	0.2	0.7788	0.7732	0.0056
	0.5	0.8496	0.8404	0.0092
	0.7	0.9066	0.8939	0.0127
0.8	0.2	0.7790	0.7664	0.0126
	0.5	0.8529	0.8320	0.0209
	0.7	0.9133	0.8842	0.0291

**Table 2.** Comparison between the exact solution of equation (21) and the approximate solution by Sumudu

where  $\alpha = 0.9$

t	x	u(x, t) Sumudu	u(x, t) Exact	Error
0.2	0.2	0.7842	0.7835	0.0007
	0.5	0.8541	0.8531	0.0010
	0.7	0.9097	0.9086	0.0011
0.5	0.2	0.7788	0.7755	0.0033
	0.5	0.8486	0.8432	0.0054
	0.7	0.9045	0.8970	0.0078
0.8	0.2	0.7767	0.7835	-0.0068
	0.5	0.8482	0.8341	0.0141
	0.7	0.9059	0.8866	0.0193

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