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Finite element analysis of convective heat and mass transfer flow of a viscous fluid in a rectangular cavity with chemical reaction

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Abstract:

In this chapter we investigate effect of Magnetic field on convective Heat and Mass Transfer flow of a viscous electrically conducting fluid through a Porous Medium in a rectangular cavity with radiation and dissipative effects. The equations governing the flow, heat and mass transfer are solved by employing Galerkine finite element analysis with 3 noded triangular elements. The temperature and Concentration distributions are analyzed for different values of governing parameters. The rate of Heat and Mass transfer evaluated numerically for a different parametric values.
Keywords: porous medium, Dissipative viscous fluid, Rectangular duct.
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1. INTRODUCTION:

The combination of temperature and concentration gradients in the fluid will lead to buoyancy-driven flows. This has an importance influence on the solidification process in a binary system. When heat and mass transfer occurs simultaneously, it leads to a complex fluid motion called double-diffusive convection. Ostrach [7] and Viskanta et. al., [13] reported complete reviews on the subject. Bejan [4] reported a fundamental study of scale analysis relative to heat and mass transfer within cavities submitted to horizontal combined and pure temperature and concentration gradients. Kamotani et. al., [5] considered an experimental study of natural convection in shallow enclosures with horizontal temperature and concentration gradients.Acharya and Goldstein [1] studied numerically two-dimensional natural convection of air in an externally heated vertical or inclined square box containing uniformly distributed internal energy sources. Verschoor et. al., [12] have studied the effect of viscous dissipation and radiation on unsteady magneto hydrodynamic free convection flow fast vertical plate in porous medium. Badruddin et. al., [3] have investigated the radiation and viscous dissipation on convective heat transfer in porous cavity. Recently Padmavathi [8] Nagaradhika [6] and Sreenivas [11] have analyzed the connective heat transfer through a porous medium in a rectangular cavity with heat sources and dissipation under varied conditions. Ranga Reddy [9] has discussed the natural convective Heat and Mass transfer in Porous Rectangular Cavity with a differentially heated side walls using Brinkman model. Reddaih et. al., [10] have analyzed the effect of viscous dissipation on convective heat and mass transfer flow of a viscous fluid in a duct of rectangular cross section by employing Galerkin finite element analysis.

2. FORMULATION OF THE PROBLEM

We consider the mixed convective heat and mass transfer flow of a viscous incompressible fluid in a saturated porous medium confined in the rectangular duct (Fig. 1) whose base length is a and height b. The heat flux on the base and top walls is maintained constant. The Cartesian coordinate system O (x,y) is chosen with origin on the central axis of the duct and its base parallel to x-axis.

We assume that

i) The convective fluid and the porous medium are everywhere in local thermodynamic equilibrium.
ii) There is no phase change of the fluid in the medium.
iii) The properties of the fluid and of the porous medium are homogeneous and isotrophic.
iv) The porous medium is assumed to be closely packed so that Darcy's momentum law is adequate in the porous medium.
v) The Boussinesq approximation is applicable.

Under these assumption the governing equations are given by

$$\frac{\partial u'}{\partial x'}+\frac{\partial v'}{\partial y'}=0 \tag{2.1}$$

$$u'=-\frac{k}{\mu}\left(\frac{\partial p'}{\partial x'}\right) \tag{2.2}$$

$$v'=-\frac{k}{\mu}\left(\frac{\partial p'}{\partial y'}+\rho' g\right)-\left(\frac{\sigma\mu_e^2 H_o^2}{(\mu/\rho)}\right)v \tag{2.3}$$

$$\rho_\sigma c_p\left(u'\frac{\partial T'}{\partial x'}+v'\frac{\partial T'}{\partial y'}\right)=K_1\left(\frac{\partial^2 T'}{\partial x'^2}+\frac{\partial^2 T'}{\partial y'^2}\right)+Q(T_0-T)+\left(\frac{\mu}{K}\right)(u^2+v^2)-\frac{\partial(q_r)}{\partial x} \tag{2.4}$$

$$\rho_\sigma c_p\left(u'\frac{\partial C}{\partial x'}+v'\frac{\partial C}{\partial y'}\right)=D_1\left(\frac{\partial^2 C}{\partial x'^2}+\frac{\partial^2 C}{\partial y'^2}\right)+k_{11}\left(\frac{\partial^2 T}{\partial x'^2}+\frac{\partial^2 T}{\partial y'^2}\right) \tag{2.5}$$

$$\rho'=\rho_0\left\{1-\beta(T'-T_0)-\beta_1(T-T_0)^2-\beta^\bullet(C'-C_0)\right\} \tag{2.6}$$

$$T_0=\frac{T_h+T_c}{2},\quad C_0=\frac{C_h+C_c}{2}$$

where u′ and v′ are Darcy velocities along θ(x, y) direction. T′, C, p′ and g′ are the temperature, Concentration, pressure and acceleration due to gravity, T_c ,Cc and T_h ,C_h are the temperature and Concentration on the cold and warm side walls respectively. ρ′, μ, ν, and β are the density, coefficients of viscosity, kinematic viscosity and thermal expansion of he fluid, k is the permeability of the porous medium, K_1 is the thermal conductivity, C_p is the specific heat at constant pressure , Q is the strength of the heat source,k_{11} is the cross diffusivity , β* is the volume coefficient of expansion with mass fraction concentration and q_r is the radiative heat flux. σ is the electrically conductivity, μ_e is the magnetic permeability of the medium and H_0 is the strength of the magnetic field.

The boundary conditions are

$$
\begin{array}{ll}
u' = v' = 0 & \text{on the boundary of the duct} \\
T' = T_c, C = C_c & \text{on the side wall to the left} \\
T' = T_h, C = C_h & \text{on the side wall to the right} \\
\dfrac{\partial T'}{\partial y} = 0, \ \dfrac{\partial C}{\partial y} = 0 & \text{on the top } (y = 0) \text{ and bottom} \\
u = v = 0 & \text{walls } (y = 0) \text{which are insulated.}
\end{array} \quad (2.7)
$$

Invoking Rosseland approximation for radiation

$$q_r = \frac{4\sigma^*}{3\beta_R}\frac{\partial T'^4}{\partial y}$$

Expanding T^4 in Taylor's series about T_e and neglecting higher order terms

$$T'^4 \cong 4T_e^3 T - 3T_e^4$$

We now introduce the following non-dimensional variables

$$
\begin{array}{lll}
x' = ax; & y' = by; & c = b/a \\
u' = (\nu/a)u; & v' = (\nu/a)v; & p' = (\nu^2\rho/a^2)p \\
T' = T_0 + \theta(T_h - T_c) & C' = C_0 + \phi(T_h - T_c) &
\end{array} \quad (2.8)
$$

The governing equations in the non-dimensional form are

$$u = -\left(\frac{K}{a^2}\right)\frac{\partial p}{\partial x} \quad (2.9)$$

$$v = -\frac{k}{a^2}\frac{\partial p}{\partial y} - \frac{kag}{v^2} + \frac{k(ag\beta(T_h - T_c)\theta + \beta_1 g\theta^2)}{v^2} + \frac{kag\beta^{\bullet}(C_h - C_c)\phi}{v^2} \quad (2.10)$$

$$P\left(u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y}\right) = \left(1 + \frac{4N}{3}\right)\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right) - \alpha\theta + E_C\left(u^2 + v^2\right) \quad (2.11)$$

$$Sc\left(u\frac{\partial\phi}{\partial x} + v\frac{\partial\phi}{\partial y}\right) = \left(\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2}\right) - K\phi \quad (2.12)$$

In view of the equation of continuity we introduce the stream function ψ as

$$u = \frac{\partial\psi}{\partial y} \ ; \quad v = -\frac{\partial\psi}{\partial x} \quad (2.13)$$

Eliminating p from the equation (2.9) and (2.10) and making use of (2.11) the equations in terms of ψ and θ are

$$((1 + M^2)\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}) = -Ra(\frac{\partial\theta}{\partial x} + 2\theta\gamma\frac{\partial\theta}{\partial x} + N\frac{\partial\phi}{\partial x}) \quad (2.14)$$

$$P\left(\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}\right) = \left(1 + \frac{4}{3N_1}\right)\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right) - \alpha\theta + E_C\left(\left(\frac{\partial\psi}{\partial y}\right)^2 + \left(\frac{\partial\psi}{\partial x}\right)^2\right) \quad (2.15)$$

$$Sc\left(\frac{\partial\psi}{\partial y}\frac{\partial\phi}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\phi}{\partial y}\right) = \left(\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2}\right) - K\phi \quad (2.16)$$

where

$G = \frac{g\beta(T_h - T_c)a^3}{\nu^2}$ (Grashof number) $P = \mu c_p / K_1$ (Prandtl number)

$\alpha = Qa^z/K_1$ (Heat source parameter) $Ra = \frac{\beta g(T_g - T_c)Ka}{\nu^2}$ (Rayleigh Number)

$N_1 = \frac{3\beta_R K_1}{4\sigma^{\bullet} : T_e^3}$ (Radiation parameter) $Sc = \frac{\nu}{D}$ (Schmidt Number)

$N = \frac{\beta^*(C_h - C_c)}{\beta(T_h - T_c)}$ (Buoyancy ratio) $Ec = \left(\frac{a^4}{\mu K K_1 \Delta T}\right)$ (Eckert number)

$\gamma = \frac{\beta_1 \Delta T}{\beta_0}$ (Density ratio) $K = \frac{K' L^2}{D_1}$ (Chemical reaction parameter)

The boundary conditions are

$$\frac{\partial\psi}{\partial x} = 0, \frac{\partial\psi}{\partial y} = 0 \ on \quad x = 0 \ \& 1 \tag{2.17}$$

$$\theta = 1 \quad \phi = 1 \quad on \quad x = 0 \tag{2.18}$$
$$\theta = 0 \quad \phi = 0 \quad on \quad x = 1$$

3. FINITE ELEMENT ANALYSIS AND SOLUTION OF THE PROBLEM:

The region is divided into a finite number of three node triangular elements, in each of which the element equation is derived using Galerkin weighted residual method. In each element f_i the approximate solution for an unknown f in the variational formulation is expressed as a linear combination of shape function. $(N_k^i) k = 1,2,3,$ which are linear polynomials in x and y. This approximate solution of the unknown f coincides with actual values at each node of the element. The variational formulation results in a 3 x 3 matrix equation (stiffness matrix) for the unknown local nodal values of the given element. These stiffness matrices are assembled in terms of global nodal values using inter element continuity and boundary conditions resulting in global matrix equation.

In each case there are r distinct global nodes in the finite element domain and f_p (p = 1,2,……r) is the global nodal values of any unknown f defined over the domain then

$$f = \sum_{i=1}^{8} \sum_{p=1}^{r} f_p \ \Phi_p^i ,$$

where the first summation denotes summation over s elements and the second one represents summation over the independent global nodes and

$\Phi_p^i = N_N^i$, if p is one of the local nodes say k of the element e_i

= 0, otherwise.

f_p' s are determined from the global matrix equation. Based on these lines we now make a finite element analysis of the given problem governed by (2.14)- (2.16) subjected to the conditions (2.17) – (2.18).

Let ψ^i , θ^i and ϕ^i be the approximate values of ψ ,θ and ϕ in an element θ_i.

$$\psi^i = N_1^i \ \psi_1^i + N_2^i \ \psi_2^i + \ N_3^i \ \psi_3^i \tag{3.1a}$$

$$\theta^i = N_1^i \ \theta_1^i + N_2^i \ \theta_2^i + \ N_3^i \ \theta_3^i \tag{3.1b}$$

$$\phi = N_1^i \ \phi_1^i + N_2^i \ \phi_2^i + \ N_3^i \ \phi_3^i \tag{3.1c}$$

Substituting the approximate value ψ^i , θ^i and ϕ^i for ψ ,θ and ϕ respectively in (2.13), the error

$$E_1^i = \left(1 + \frac{4}{3N_1}\right)\frac{\partial^2\theta^i}{\partial x^2} + \frac{\partial^2\theta^i}{\partial y^2} - P\left(\frac{\partial\psi^i}{\partial y}\frac{\partial\theta^i}{\partial x} - \frac{\partial\psi^i}{\partial x}\frac{\partial\theta^i}{\partial y}\right) - \alpha\theta + E_C\left[\left(\frac{\partial\psi}{\partial y}\right)^2 + \left(\frac{\partial\psi}{\partial x}\right)^2\right] \tag{3.2}$$

$$E_2^i = \frac{\partial^2\phi^i}{\partial x^2} + \frac{\partial^2\phi^i}{\partial y^2} - Sc\left(\frac{\partial\psi^i}{\partial y}\frac{\partial\phi^i}{\partial x} - \frac{\partial\psi^i}{\partial x}\frac{\partial\phi^i}{\partial y}\right) - K\phi$$

(3.3)

Under Galerkin method this error is made orthogonal over the domain of e_i to the respective shape functions (weight functions) where

$$\int_{ei} E_1^i \ N_k^i \ d\Omega = 0 \qquad ,$$

$$\int_{ei} E_2^i N_k^i d\Omega = 0$$

$$\int_{ei=} N_k^i \left(\left(1 + \frac{4}{3N_1}\right)\left(\frac{\partial^z\theta^i}{\partial x^2} + \frac{\partial^z\theta^i}{\partial y^2}\right) - P\left(\frac{\partial\psi^i}{\partial y}\frac{\partial\theta^i}{\partial x} - \frac{\partial\psi^i}{\partial x}\frac{\partial\theta^i}{\partial y}\right) - \alpha\theta + \left[E_C\left(\frac{\partial\psi}{\partial y}\right)^2 + \left(\frac{\partial\psi}{\partial x}\right)^2\right] d\Omega = 0 \tag{3.4}$$

$$\int_{ei=} N_k^i \left(\left(\frac{\partial^z\phi^i}{\partial x^2} + \frac{\partial^z\phi^i}{\partial y^2}\right) - Sc\left(\frac{\partial\psi^i}{\partial y}\frac{\partial\phi^i}{\partial x} - \frac{\partial\psi^i}{\partial x}\frac{\partial\phi^i}{\partial y}\right) - \mathrm{K}\phi\right) d\Omega = 0 \tag{3.5}$$

Using Green's theorem we reduce the surface integral (3.4) & (3.5) without affecting ψ terms and obtain

$$\int_{ei} N_k^i \left\{ \left(1 + \frac{4}{3N_1}\right)\frac{\partial N_k^i}{\partial x}\frac{\partial\theta^i}{\partial x} + \frac{\partial N_k^i}{\partial y}\frac{\partial\theta^i}{\partial y} - p^i N_k\left(\frac{\partial\psi^i}{\partial y}\frac{\partial\theta^i}{\partial x} - \frac{\partial\psi^i}{\partial x}\frac{\partial\theta^i}{\partial y}\right) - \alpha\theta + E_C\left(\frac{\partial\psi}{\partial y}\right)^2 + \left(\frac{\partial\psi}{\partial x}\right)^2 \right\} d\Omega$$

$$= \int_{\Gamma i} N_k^i \left(\frac{\partial\theta^i}{\partial x} n_x + \frac{\partial\theta^i}{\partial y} n_y \right) d\Gamma_i$$

$$\int_{eiei} N_k^i \left\{ \frac{\partial N_k^i}{\partial x}\frac{\partial\phi^i}{\partial x} + \frac{\partial N_k^i}{\partial y}\frac{\partial\phi^i}{\partial y} - Sc^i N_k\left(\frac{\partial\psi^i}{\partial y}\frac{\partial\phi^i}{\partial x} - \frac{\partial\psi^i}{\partial x}\frac{\partial\phi^i}{\partial y}\right) - K\phi) \right\} d\Omega \tag{3.6}$$

$$= \int_{\Gamma i} N_k^i \left(\frac{\partial\theta^i}{\partial x}{}_x + (\frac{\partial\theta^i}{\partial y}{}_y \right) d\Gamma_i \tag{3.7}$$

where Γ_I is the boundary of e_i.

Substituting L.H.S. of (3.1a) - (3.1c) for ψ^i , θ^i and ϕ^i in (3.6) & (3.7) we get

$$\sum_1 \int_{ei} \left(1 + \frac{4N}{3}\right)\frac{\partial N_k^i}{\partial x}\frac{\partial N_L^i}{\partial x} + \frac{\partial N_L^i}{\partial y}\frac{\partial N_k^i}{\partial y} - P\sum_1 \psi_m^i \int_{ei}\left(\frac{\partial N_m^i}{\partial y}\frac{\partial N_L^i}{\partial x} - \frac{\partial N_m^i}{\partial x}\frac{\partial N_L^i}{\partial y}\right) d\Omega \quad -\alpha\sum\theta^i \int_{ei} N_k N_l d\Omega_i + E_C \int_{ei}\left(\left(\frac{\partial\psi}{\partial y}\right)^2 + \left(\frac{\partial\psi}{\partial x}\right)^2\right) d\Omega$$

$$= \int_{\Gamma_i} N_k^i \left(\frac{\partial \theta^i}{\partial x} n_x + \frac{\partial \theta^i}{\partial y} n_y \right) d\Gamma_i = Q_k^i \qquad \text{(l, m, k = 1,2,3)} \tag{3.8}$$

$$\sum_1 \int_{ei} \phi^i \left(\frac{\partial N_k^i}{\partial x}\frac{\partial N_L^i}{\partial x} + \frac{\partial N_L^i}{\partial y}\frac{\partial N_k^i}{\partial y} \right) - Sc \sum_1 \psi_m^i \int_{ei} \left(\frac{\partial N_m^i}{\partial y}\frac{\partial N_L^i}{\partial x} - \frac{\partial N_m^i}{\partial x}\frac{\partial N_L^i}{\partial y} \right) d\Omega - K\phi \int N_{ij}^i d\Omega^i$$

$$= \int_{\Gamma i} N_k^i \left(\frac{\partial \theta^i}{\partial x} + \frac{ScSo}{N}\frac{\partial \phi^i}{\partial x})n_x + (\frac{\partial \theta^i}{\partial y})_y \right) d\Gamma_i = Q_i^C \qquad \text{(l, m, k = 1,2,3)} \tag{3.9}$$

where

$Q_k^i = Q_{k1}^i + Q_{k2}^i + Q_{k3}^i, Q_k^i$'s being the values of Q_k^i on the sides s = (1,2,3) of the element e_i. The sign of Q_k^i 's depends on the direction of the outward normal w.r.t the element.

Choosing different N_k^i 's as weight functions and following the same procedure we obtain matrix equations for three unknowns (Q_p^i) viz.,

$$(a_p^i)(\theta_p^i) = (Q_k^i) \tag{3.10}$$

where (a_{pk}^i) is a 3 x 3 matrix, $(\theta_p^i),(Q_k^i)$ are column matrices.

Repeating the above process with each of s elements, we obtain sets of such matrix equations. Introducing the global coordinates and global values for θ_p^i and making use of inter element continuity and boundary conditions relevant to the problem the above stiffness matrices are assembled to obtain a global matrix equation. This global matrix is r x r square matrix if there are r distinct global nodes in the domain of flow considered.

Similarly substituting ψ^i, θ^i and ϕ^i in (2.12) and defining the error

$$E_3^i = (1+M^2)\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} - \mathrm{Ra}\,(\frac{\partial \theta}{\partial x} + N\frac{\partial \phi}{\partial x}) \tag{3.11}$$

and following the Galerkin method we obtain

$$\int_\Omega E_3^i \psi_j^i d\Omega = 0 \tag{3.12}$$

Using Green's theorem (3.8) reduces to

$$\int_\Omega \left((1+M^2)\frac{\partial N_k^i}{\partial x}\frac{\partial \psi^i}{\partial x} + \frac{\partial N_k^i}{\partial y}\frac{\partial \psi^i}{\partial y} + \mathrm{Ra}\,(\theta^i \frac{\partial N_k^i}{\partial x} + \phi^i \frac{\partial N_k^i}{\partial x} \right) d\Omega$$

$$= \int_\Gamma N_k^i \left(\frac{\partial \psi^i}{\partial x} n_x + \frac{\partial \psi^i}{\partial y} n_y \right) d\Gamma_i + \int_\Gamma N_k^i n_x \theta^i d\Gamma_i \tag{3.13}$$

In obtaining (3.13) the Green's theorem is applied w.r.t derivatives of ψ without affecting θ terms.

Using (3.1) and (3.2) in (3.13) we have

$$\sum_m \psi_m^i \left\{ \begin{array}{l} \int_\Omega \left((1+M^2)\frac{\partial N_k^i}{\partial x}\frac{\partial N_m^i}{\partial x} + \frac{\partial N_m^i}{\partial y}\frac{\partial N_k^i}{\partial y} \right) d\Omega_i + \\ \mathrm{Ra}\sum_L (\theta_L^i \int_{\Omega^i} (\mathrm{N_k^i} + 2\gamma)\frac{\partial N_L^i}{\partial x} d\Omega + \phi_L^i N \int_{\Omega^i} \mathrm{N_k^i}\frac{\partial N_L^i}{\partial x} d\Omega \end{array} \right\}$$

$$= \int_{\Gamma} N_k^i \left(\frac{\partial \psi^i}{\partial x} n_x + \frac{\partial \psi^i}{\partial y} n_y \right) d\Gamma_i + \int_{\Gamma} N_k^i \theta^i d\Omega_i = \Gamma_k^i \tag{3.14}$$

In the problem under consideration, for computational purpose, we choose uniform mesh of 10 triangular element (Fig. ii). The domain has vertices whose global coordinates are (0,0), (1,0) and (1,c) in the non-dimensional form. Let e_1, e_2…..e_{10} be the ten elements and let θ_1, θ_2, …..θ_{10} be the global values of θ and ψ_1, ψ_2,……ψ_{10} be the global values of ψ at the ten global nodes of the domain (Fig. ii).

4. SHAPE FUNCTIONS AND STIFFNESS MATRICES

Range functions in $\underset{i,j}{n}$; i = element, j = node.

$\underset{1,1}{n} = 1 - 3x$	$\underset{1,2}{n} = 3x - \frac{3y}{C}$
$\underset{2,1}{n} = 1 - \frac{3y}{C}$	$\underset{2,2}{n} = -1 + \frac{3y}{C}$
$\underset{2,3}{n} = 1 - 3x + \frac{3y}{C}$	$\underset{3,1}{n} = 2 - 3x$
$\underset{3,2}{n} = -1 + 3x - \frac{3y}{C}$	$\underset{3,3}{n} = \frac{3y}{C}$
$\underset{4,1}{n} = 1 - \frac{3y}{C}$	$\underset{4,2}{n} = -2 + 3x$
$\underset{4,3}{n} = 2 - 3x + \frac{3y}{C}$	$\underset{5,1}{n} = 2 - 3x$
$\underset{5,2}{n} = -1 + 3x - \frac{3y}{C}$	$\underset{5,3}{n} = \frac{3y}{C}$
$\underset{6,1}{n} = 2 - 3x$	$\underset{6,2}{n} = 3x - \frac{3y}{C}$
$\underset{6,3}{n} = 1 + \frac{3y}{C}$	$\underset{7,1}{n} = 2 - \frac{3y}{C}$
$\underset{7,2}{n} = -2 + 3x$	$\underset{7,3}{n} = 1 - 3x + \frac{3y}{C}$
$\underset{8,1}{n} = 3 - 3x$	$\underset{8,2}{n} = -1 + 3x - \frac{3y}{C}$
$\underset{9,2}{n} = 3x - \frac{3y}{C}$	$\underset{9,3}{n} = -1 + \frac{3y}{C}$

Substituting the above shape functions in (3.8), (3.9) & (3.14) w.r.t each element and integrating over the respective triangular domain we obtain the element in the form (3.8). The 3x3 matrix equations are assembled using connectivity conditions to obtain a 8x8 matrix equations for the global nodes ψ_p,θ_p and ϕ_p.

The global matrix equation for θ is

$$A_3 X_3 = B_3 \tag{4.1}$$

The global matrix equation for ϕ is

$$A_4 X_4 = B_4 \tag{4.2}$$

The global matrix equation for ψ is

$$A_5 X_5 = B_5 \tag{4.3}$$

Where A_3 ,A_4, A_5 are not given due to space constraints.

5. NUMERICAL RESULTS AND DISCUSSIONS:

Figs 1-4 represent θ with radiation parameter N_1. It is found that the actual temperature experiences an enhancement with increase in the radiation parameter N_1 at all the levels. It is found that in the degenerating chemical reaction case the actual temperature reduces at $y = \frac{2h}{3}$ and $x = \frac{2}{3}$ levels while in the generating case it reduces with increase in |k|. The effect of chemical reaction on θ is shown in figs. (5-8). It is found that in the degenerating chemical reaction case the actual temperature reduces at $y = \frac{2h}{3}$ and $x = \frac{2}{3}$ levels while in the generating case it reduces with increase in |k| (figs. 6&8). At $y = \frac{h}{3}$ & $x = \frac{1}{3}$ levels the actual temperature reduces with |k| while it reduces with k≤1.5 and for k≥2.5, it reduces at $x = \frac{1}{3}$ and at $y = \frac{h}{3}$ it enhances in the region (0.333≤ x≤0.663) and reduces within the region (0.729≤x≤0.921) (fig. 5&7). From figs(9-12) we find that higher the radiative heat flux larger the actual concentration at all horizontal and vertical levels.

The effect of chemical reaction on C is exhibited in figs.(13-16). It is found that in the degenerating reaction case the actual concentration at $y = \frac{h}{4}$ level enhances with k≤1.5 and reduces with k≥2.5 while at $y = \frac{2h}{3}$, it enhances with all values of k (figs. 13&14). At the vertical levels $x = \frac{1}{3}$ and $\frac{2}{3}$ the actual concentration reduces with k≤1.5 and enhances with higher k≥2.5. In the generating case the actual concentration enhances with |k|≤1.5 and reduces with higher |k|≥2.5 at both horizontal levels. The actual concentration reduces with |k| at $x = \frac{1}{3}$ level and enhances it at $x = \frac{2}{3}$ level (figs.15&16).

6. TABLES:

From table.1 we find that higher the radiative heats flux lesser the Nusselt number at all three quadrants. Higher the radiative heat flux lesser the Sherwood number at the first and middle quadrants and enhances at the upper quadrant (table. 2).

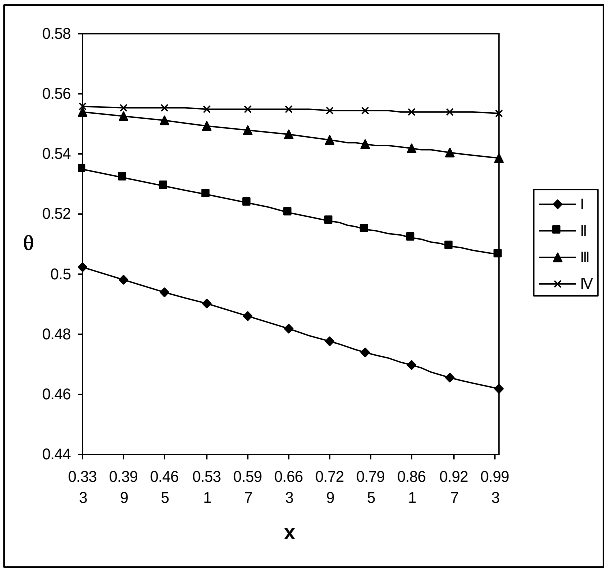

Fig. 1: Variation of θ with N_1 at $y = \frac{h}{3}$ level

	I	II	III	IV
N_1	0.01	0.03	0.05	0.07

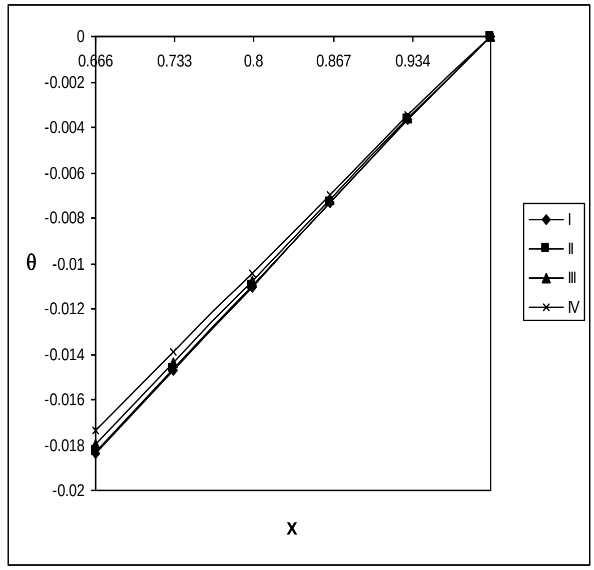

Fig. 2: Variation of θ with N_1 at $y = \frac{2h}{3}$ level

	I	II	III	IV
N_1	0.01	0.03	0.05	0.07

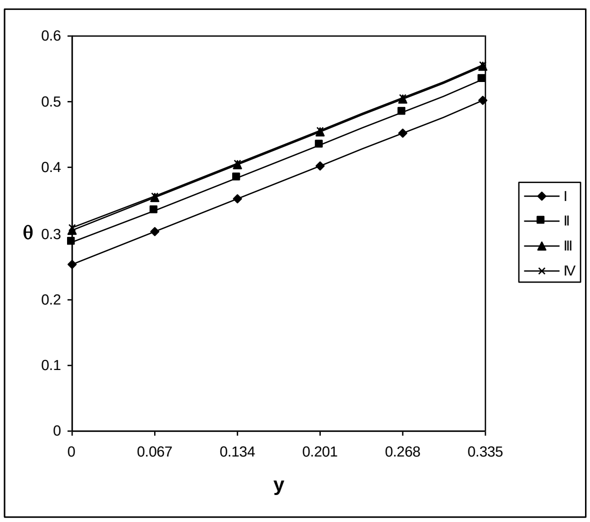

Fig. 3: Variation of θ with N_1 at $x = \frac{1}{3}$ level

	I	II	III	IV
N_1	0.01	0.03	0.05	0.07

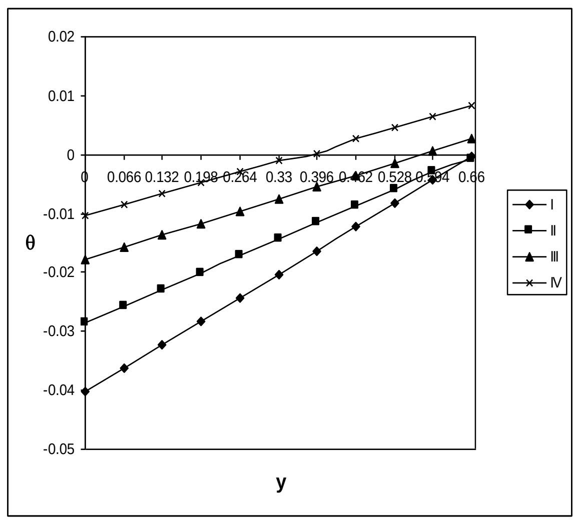

Fig.4 : Variation of θ with N_1 at $x = \frac{2}{3}$ level

	I	II	III	IV
N_1	0.01	0.03	0.05	0.07

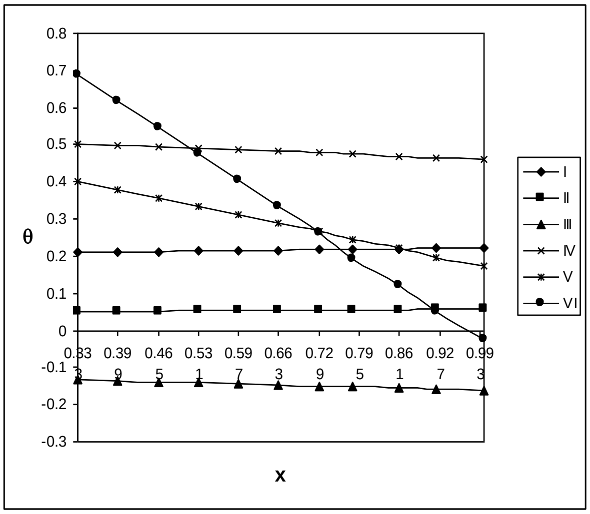

Fig. 5: Variation of θ with k at $y = \frac{h}{3}$ level

	I	II	III	IV	V	VI
k	-0.5	-1.5	-2.5	0.5	1.5	2.5

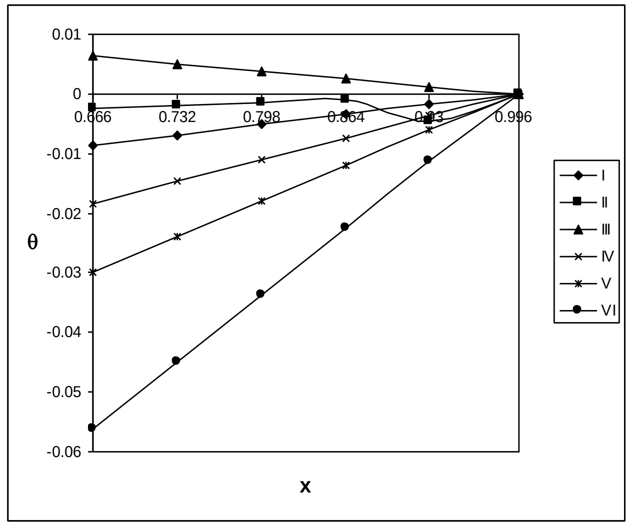

Fig. 6: Variation of θ with k at $y = \frac{2h}{3}$ level

	I	II	III	IV	V	VI
k	-0.5	-1.5	-2.5	0.5	1.5	2.5

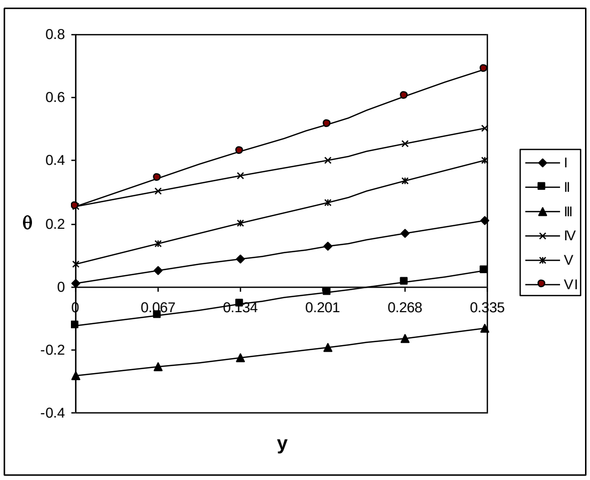

Fig. 7: Variation of θ with k at $x = \frac{1}{3}$ level

	I	II	III	IV	V	VI
K	-0.5	-1.5	-2.5	0.5	1.5	2.5

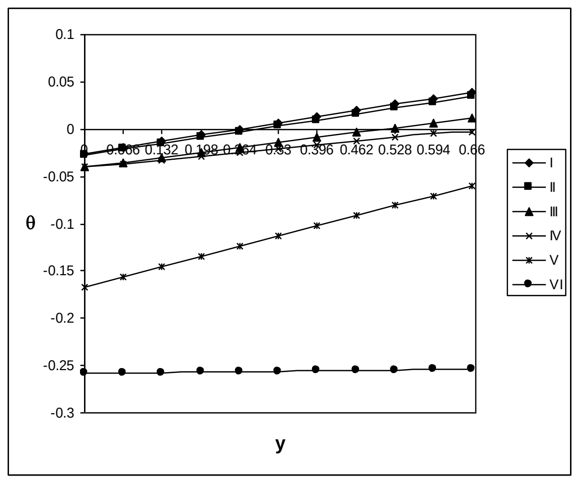

Fig. 8 : Variation of θ with k at $x = \frac{2}{3}$ level

	I	II	III	IV	V	VI
k	-0.5	-1.5	-2.5	0.5	1.5	2.5

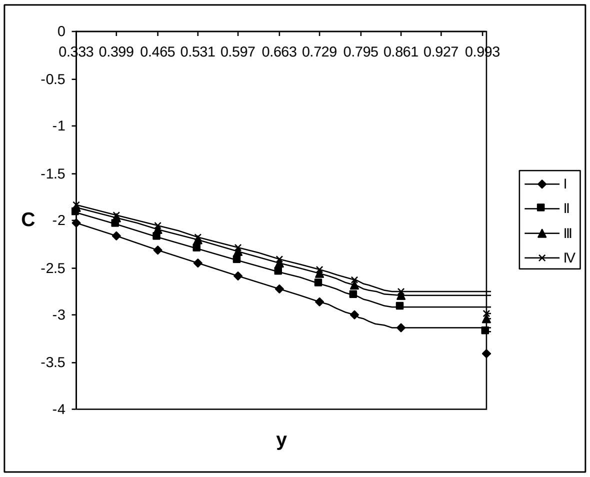

Fig. 9: Variation of C with N_1 at $y = \frac{h}{3}$ level

	I	II	III	IV
N_1	0.01	0.03	0.05	0.07

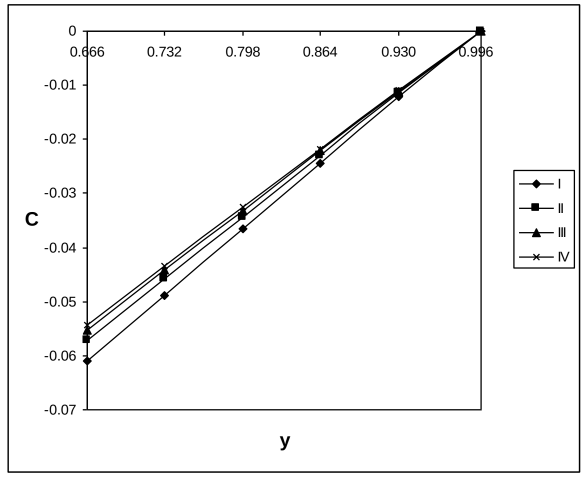

Fig. 10: Variation of C with N_1 at $y = \frac{2h}{3}$ level

	I	II	III	IV
N_1	0.01	0.03	0.05	0.07

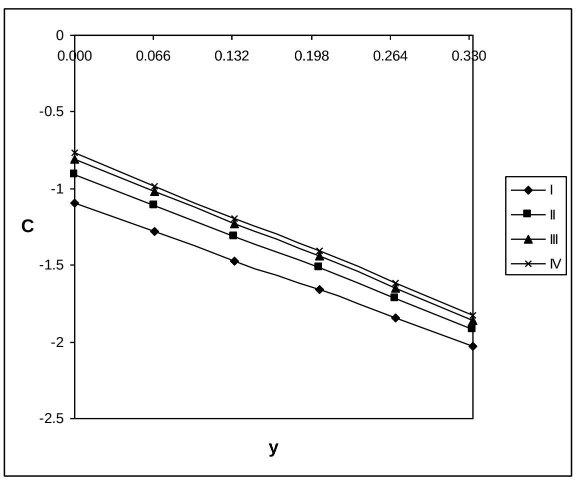

Fig.11: Variation of C with N_1 at $x = \frac{1}{3}$ level

	I	II	III	IV
N_1	0.01	0.03	0.05	0.07

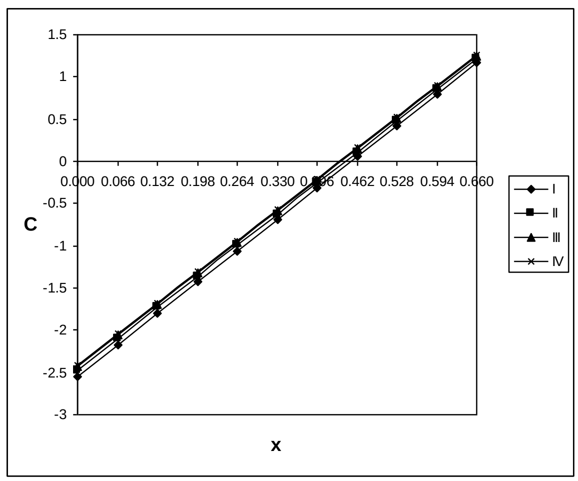

Fig. 12 : Variation of C with N_1 at $x = \frac{2}{3}$ level

	I	II	III	IV
N_1	0.01	0.03	0.05	0.07

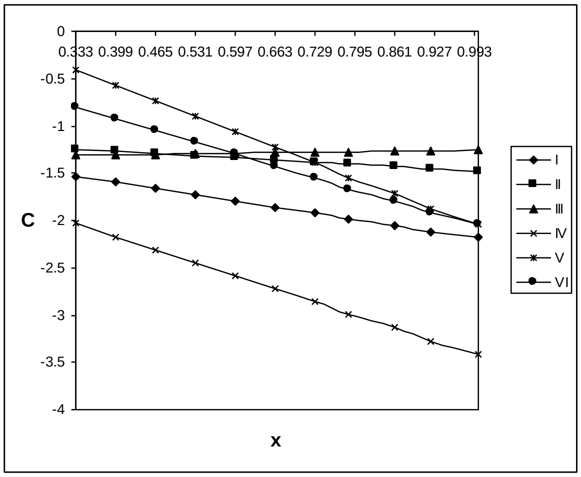

Fig. 13: Variation of C with k at $y = \frac{h}{3}$ level

	I	II	III	IV	V	VI
k	-0.5	-1.5	-2.5	0.5	1.5	2.5

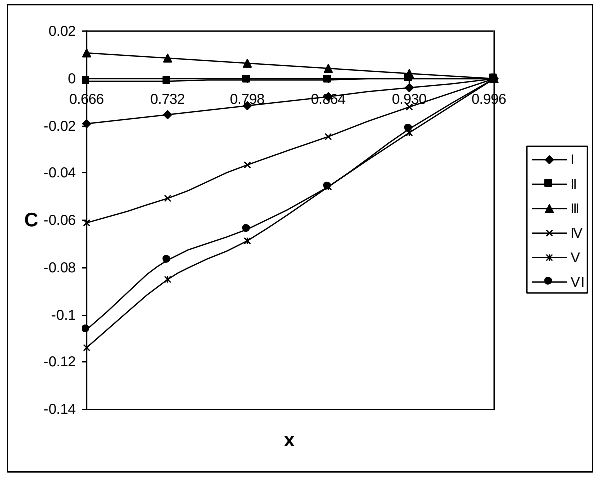

Fig. 14: Variation of C with k at $y = \frac{2h}{3}$ level

	I	II	III	IV	V	VI
k	-0.5	-1.5	-2.5	0.5	1.5	2.5

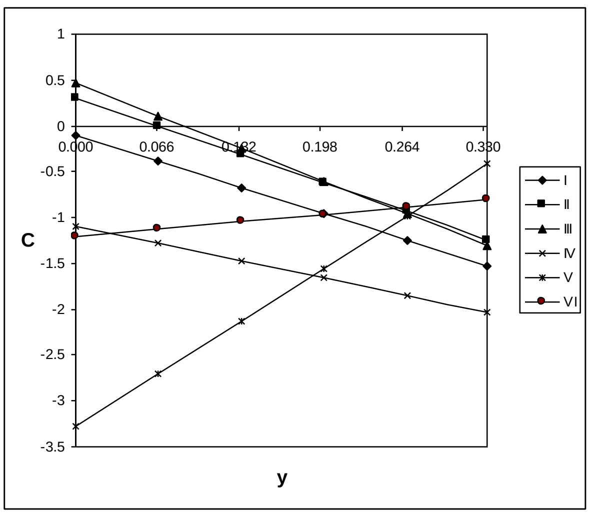

Fig. 15 : Variation of C with k at $x = \frac{1}{3}$ level

	I	II	III	IV	V	VI
k	-0.5	-1.5	-2.5	0.5	1.5	2.5

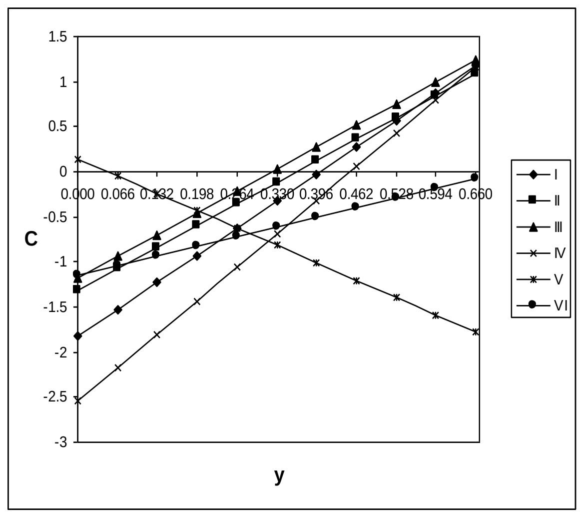

Fig. 16 : Variation of C with k at $x = \frac{2}{3}$ level

	I	II	III	IV	V	VI
k	-0.5	-1.5	-2.5	0.5	1.5	2.5

Table – 1
Nusselt Number (Nu) at x = 1 at different levels

	I	II	III	IV
Nu_1	2.1612316	2.11454	2.0711624	2.041476
Nu_2	2.0812544	2.0574208	2.0303218	2.00400504
Nu_3	2.001277332	2.0003017	1.9894812	1.996653424
N_1	-0.5	-0.8	1	2

Table – 2
Sherwood number (Sh) at x = 1 at different levels

	I	II	III	IV
Sh_1	12.18088	11.87376	11.71532	11.64928
Sh_2	4.754888	4.50434	4.359728	4.300824
Sh_3	-2.67112	-2.86508	-2.99588	-3.04764
N_1	-0.5	-0.8	1	2

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