MARKOV CHAIN: A STABILIZER TO MARKET EQUILIBRIUM MIX.

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Abstract.

In this reviewed paper, we studied the most satisfactory markov chain of the first order and showed that $\sum_{j=1}^{m} p_{ij} = 1$ for a matrix to be a transition probability matrix. We further gave critical analysis of customers' switching model with application to market strategies. It turned out that the variables interact optimally. Realising the importance , markov chain market share model was applied to inter temporal data of customers gains and losses of the three brands of beer. By estimating the transition probability matrix (TPM), the scope for probabilities of brand switching among its type were calculated to suggest the probable marketing mix on equilibrium market share. From the results, it was suggested that in the long-run, equilibrium market share were as follows; Star (38.1%), Gulder (34.4%), and Guiness (27,5%). The average staying time will be; Star (5), Gulder (4.5), and Guiness (4.2), while the expected return time will be Star (1.25), Gulder (1.29), and Guiness (1.32).

The study concluded that for profitable operation of any business, its marketing strategy must be designed to ensure that the variables; price, product quality, packaging advertising, and product availability interact optimally.

Keywords: Markov Chain Market Share Model, Steady-State Equilibrium, Brand Switching, Transition Probability Matrix.

Introduction

The main aim of setting up a business outfit is for profit maximization and satisfaction of wants for the teaming population. In order to realise this objective, depends to a large extent the successful production of a product, which meets certain needs and wants of a given market. Production in sizes and brands leads to the existence of preference among product buyers, resulting in certain brand attracting lesser or greater fraction of the market share than others Nworuh and Anyiam, (2009).

However, the study will give us a clearer picture of how to determine equilibrium market share in beer industry. The beer industry under study are; Star, Gulder, and Guiness. These brands of beer were chosen based on their increased and improvement in branch networks (depots), advertisement, product quality, price, and most of all their preference by the consumers. These brands of beer have different companies producing, distributing and selling them. A market share needs to take into account the following; total market size, market growth rate, and market segmentation. Over the year, beer industries in Nigeria had gone through structural changes in terms of increase in branch networks (depots) and provision of wide range of services. The survival of any industry normally depends on their ability to improve their efficiency and effectiveness in their product offering, Thyagarajan and Bin Mohmmed (2005). Our study is anchored on market segmentation which identifies the key factors that determine the market; price of the product, quality, advertisement, dissatisfaction, packaging and distribution network, Sharma (2011). It is known by the different beer industries that customers switch from one brand to another due to price, quality, dissatisfaction, packaging, and advertising. The customers in this study refers to the distributors of the product. These industries maintain records of the number of their customers and the brand company from which they obtain each new customer. It is very important that the producers know what quality or variables that attract the customers most, as a way of finding out optimum marketing mix that will lead to profit maximization. The current study attempts to device an equilibrium market mix policy using Markov chain market share model. Kosubuh and Stokes, (1980) suggest that markov chain application in the business situation application is rich in terms of economics and policy implications. In this study, an attempt has been made to estimate the transition matrix using data on customers' switching among brands of beer. This provides the probability of customers' switching among the brands.

The main objective of the study is aimed at attaining optimal marketing mix policy and steady-state equilibrium market share**.**

Methodology

The study used markov chain theory of the first order to analyse market data collected to obtain information about beer brands on market share. Markov chains are classified by their order. The case in which probability of occurrence of each state depends only upon the immediate preceding state, is said to be first order markov chain. In second order markov chain, it is assumed that the probability of occurrence in the forthcoming period depends upon the state in the last two periods. Similarly, in the third order markov chain, it is assumed that the probability of a state in the forthcoming period depends upon the states in the last three period. The movement of a system from one state to another depending upon the immediately preceding state with constant probability, forms the basis of markov chain. Therefore, markov chain satisfies the following properties;

(i) There are finite number of possible states

(ii) States are both collectively exhaustive and mutually exclusive.

(iii) The transition probabilities depend only on the current state of the system, ie. If current state is known, the conditional probability of the next state is independent of the states prior the present state.

(iv) The long-run probability of being in a particular state will be constant over time.

(v) The transition probabilities of moving to alternative states in the next time period must sum to 1.0

Markov Probability Model

The probability of brand switching from brand type i to brand type j is a conditional probability and can be represented by the transition matrix $P = \lfloor P_{ij} \rfloor$, where i refers to the number of brand types. For instance, P_{12} represents the probability of a change in brand

type 1 to brand type 2 in the next time period. While P_{ii} represents the probability of no change in brand type for brand type i. The model is represented by

$$P\lfloor S_t = j / S_1, S_2, ..., S_{t-1} = i \rfloor = P\lfloor S_t = j / S_{t-1} = i \rfloor, \ \forall i, j \quad (1)$$

Furthermore, it is assumed that the underlying variable that are responsible for the generation of brand switching do not change overtime, such that the transition probability has a stationary property ie.

$$P\lfloor S_t = j / S_{t-1} = i \rfloor = P_{ij}(t+1) = P_{ij}, \ \forall t \quad (2)$$

This can also be represented in a transition probability matrix as

$$P = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ \cdot \\ \cdot \\ \cdot \\ S_m \end{array} \begin{array}{c} \begin{array}{ccccccc} S_1 & S_2 & S_3 & \cdot & \cdot & \cdot & S_m \end{array} \\ \begin{bmatrix} p_{11} & p_{12} & \cdot & \cdot & \cdot & \cdot & p_{1m} \\ p_{21} & p_{22} & \cdot & \cdot & \cdot & \cdot & p_{2m} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ p_{m1} & p_{m2} & \cdot & \cdot & \cdot & \cdot & p_{mm} \end{bmatrix} \end{array}$$

With $\sum_{j=1}^{m} P_{ij} = 1$ and $0 \leq P_{ij} \leq 1, \ \forall i$.

Estimation of transition probabilities matrix.

The estimation of transition probability matrix plays a vital and crucial role in the study of markov process, Thyagarajan et al (2005). If a process that follows a known probability distribution, the estimation can be made with less difficulty, otherwise, the estimation procedure is a problem oriented. For micro economic data that trace the movement from any given state to another states, then, the estimation procedure follows that of multinomial distribution, that is $p_{ij} = \frac{n_{ij}}{n_{i.}}$, where n_{ij} is the number of time the process moves from state i to state j and $n_{i.}$ is the number of time the process is in state i, Thyagarajan et al (2005).

In this study, the estimation of transition probabilities is in steps, using $p_{ij} = \frac{n_{ij}}{n_{i.}}$

(i) Determine the retention probabilities; To determine the retention probabilities, divide the number of customers retained for the period under review by the number of customers at the beginning of the period. (the number of customers retained equals the original number of customers minus the number of customers that were lost.)

(ii) Determine gains and losses probabilities; (i) For those customers who switch brands, show gains and losses among brands for completing the matrix of transition. (ii) To convert the customer switching brand so that all gains and losses take the form of transition probabilities, divide the number of customers that each entity gained (lost) by the number of customers at the beginning of the period.

(iii) Develop matrix of transition probabilities. In a matrix of transition probabilities, retentions are shown as values on the main diagonal. The rows in the matrix show the retention and loss of customers, while the columns show the retention and gain of customers.

Calculation of future probable market share.

One of the advantages of using Markov model in the analysis of market equilibrium mix, besides understanding its basic characteristics is its ability to make forecast on the proportion Thyagarajan et al (2005).

The market share for any period n is determined by the following equation;

[market share in period 2] = [market share in period 1] x [transition probability matrix] (3)

[market share in period 3] = [market share in period 2] x [transition probability matrix] (4)

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.[market share in period n] = [market share in period n-1] x [transition probability matrix] (5)

Steady-state(Equilibrium) or Egordic Distribution

A markov chain is in equilibrium or egordic state if the absolute probability $p_j(n)$ converges to a limiting distribution independently of the initial distribution p_0 Udom (2010). The absolute probability $p_j(n)$ is the probability of being found in state j after n steps. A markov chain with m states is egordic, if there exist $\Pi = (\pi_1, \pi_2, \pi_3, ..., \pi_m)$ such that

(i) $\pi_i \geq 0, \forall_i$ (6)

(ii) $\sum_{i=1}^{m} \pi_i = 1$ (normalizing equation) ie $\pi_1 + \pi_2 + \pi_3 + ... + \pi_m = 1$ (7)

(iii) $$\lim_{n \to \infty} p_n = \Pi \tag{8}$$

(iv) $\Pi = \Pi P$. Udom (2010). (9)

$\Pi = (\pi_1, \pi_2, \pi_3, ..., \pi_m)$ is the stationary (egordic) or long run distribution of the markov chain.

As the number of transitions approaches infinity, a markov chain approaches a steady or equilibrium state, in which the probability distribution of its states becomes stationary. Thus, in the steady-state, the probability p_i that a markov chain is in any particular state s_i is constant from trail to trail.

Computationally, the stationary probability distribution $(\pi_1, \pi_2, \pi_3, ..., \pi_m)$ of the states of the markov chain is obtained by solving the equation given in a matrix form by

$$(\pi_1, \pi_2, \pi_3, ..., \pi_m) = (\pi_1 \quad \pi_2 \quad \cdot \quad \cdot \quad \cdot \quad \pi_m) \begin{bmatrix} p_{11} & p_{12} & \cdot & \cdot & \cdot & \cdot & p_{1m} \\ p_{21} & p_{22} & \cdot & \cdot & \cdot & \cdot & p_{2m} \\ \cdot & \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & \cdot & & & & \cdot \\ p_{m1} & \cdot p_{m2} & \cdot & \cdot & \cdot & \cdot & p_{mm} \end{bmatrix} \tag{10}$$

Where π_i is the probability (static) for states $i = 1,2,3,....,m$. This involves the solution of m-independent linear equations.

$\sum_{i=1}^{m} \pi_i p_{ij} = \pi_j$ for $j = 1,2,3,....,m$, (m-1 of which are independent) and $\sum_{i=1}^{m} \pi_i = 1$.

In general, once a steady-state is reached, multiplication of a state condition by the transition probabilities does not change the state condition. That is $p^n = p^{n-1}$ *P, for any value of n after a steady –state is reached.

Expected staying time; In the steady-state condition, the expected number of periods it will stay in state s_i is the reciprocal of the probability of leaving state s_i, that is

$$u_i = \frac{1}{1 - p_{ii}}, \ \forall_i . \tag{11}$$

In marketing, the expected staying time represents the expected number of successive periods in which customers buy the particular brand of the beer.

Expected return time; In the steady-state condition, the expected number of time periods it will return to state, s_i after leaving state, s_i is the reciprocal of remaining in state s_i. That is

$$v_i = \frac{1}{p_{ii}}, \ \forall_i . \qquad (12)$$

In marketing, the expected return time represent the expected number of successive periods before customers return to the particular brand of beer, after they have switched to other brands.

Data

All brand industries maintain records of the number of their customers and the brand industry from which they obtain each new customer.

The records of the number of different brands selection made for the period and their switching pattern to other brand in the preceding will be presented as in table 1,

Table 1; Layout

Gain and Loss	Star	Gulder	Guiness
Star	y_{11}	y_{12}	y_{13}
Gulder	y_{21}	y_{22}	y_{23}
Guiness	y_{31}	y_{32}	y_{33}

It is assumed that the new customer is allowed to enter the market and no old customer left the market during the period involved. The three brands of beer were chosen based on their large market, branch networks (depot), and preference by the consumers. The switching probabilities remain constant over the period (stationary condition).

Table 2; Gain and Losses

Brand	No. of customers as at Jan.1,2011	Change during the year Gain	Loss	No. of customers as at Jan. 1 2012
Star	3200	800	640	3360
Gulder	3120	650	700	3070
Guiness	2950	600	710	2840

Source: market survey, that is from the companies. This research is done within Enugu - the state capital of Enugu state of Nigeria.

Table 3; Gains from and Losses from.

Brand	No of customers as at Jan.1, 2011	Loss from Star Gulder Guiness	Gain from Star Gulder Guiness	No of customers As at Jan. 1, 2012
Star	3200	0 640 0	0 100 700	3360
Gulder	3120	100 0 600	640 0 10	3070
Guiness	2950	700 10 0	0 600 0	2840

Retention probabilities.

Table 4; Retention probabilities

Brand	Customer as at Jan. 1, 2011	No of customers Lost	No of customers Retained.	Probability of Retention
Star	3200	640	2560	(3200-640)/3200 =0.800
Gulder	3120	700	2420	(3120-700)/3120 = 0.776
Guiness	2950	710	2240	(2950-710)/2950 = 0.760

Probabilities associated with customer gains and losses.

Table 5; probabilities of Gain.

No of customers Originally served	Brand	Probability of Gain. From Star	From Gulder	From Guiness
3200	Star	0/3200 = 0	100/3120 = 0.032	700/2950 =0.237
3120	Gulder	640/3200 =0.200	0/3120 = 0	10/2950 = .003
2950	Guiness	0/3200 = 0	600/3120 = 0.192	0/250 = 0

Table 6; probabilities of Loss.

No of customers Originally served	Brand	Probability of Loss From Star	From Gulder	From Guiness
3200	Star	0/3200 = 0	640/3200 = 0.200	0/3200 = 0
3120	Gulder	100/3120 = 0.032	0/3120 = 0	600/3120 = 0.192
2950	Guiness	700/2950 = 0.237	10/2950 = .003	0/2950 = 0

Using either the table 5 or 6 above , the transition probability matrix is

Table 7; The transition probability matrix

$$\begin{array}{c} \\ Star \\ Gulder \\ Guiness \end{array} \begin{array}{c} \begin{array}{ccc} Star & Gulder & Guiness \end{array} \\ \begin{bmatrix} 0.800 & 0.200 & 0 \\ 0.032 & 0.776 & 0.192 \\ 0.237 & 0.003 & 0.760 \end{bmatrix} \end{array}$$

We note that for probability of loss, the rows represent the probabilities of retention and losses for each brand, while the columns represent the probabilities of retention and gains for each brand. For probability of gain, the rows represent the probabilities of retention and gains, while the columns represent the probabilities of retention and losses.

The market share as at Jan., 1 2012 is

Star = 0.363, Gulder = 0.331, Guiness = 0.306.

Future probable market share.

This is done by using (3) and (4)

For market share of Jan. 1, 2013, we have

$$\begin{pmatrix} 0.363 & 0.331 & 0.306 \end{pmatrix} \begin{bmatrix} 0.800 & 0.200 & 0 \\ 0.032 & 0.776 & 0.192 \\ 0.237 & 0.003 & 0.760 \end{bmatrix} = \begin{pmatrix} 0.374 & 0.330 & 0.296 \end{pmatrix} \quad (13)$$

That is Star = 0.374(37.4%), Gulder = 0.330(33%), and Guiness = 0.296(29.6%), of the market shares in Jan. 1, 2013. These estimates represent a net gain of 0.011 in the market share of Star, loss of .001 in the market share of Gulder, and loss of 0.01 in the market share of Guiness come Jan. 1, 2013.

For market share of Jan. 1, 2014, we have

$$\begin{pmatrix} 0.374 & 0.330 & 0.296 \end{pmatrix} \begin{bmatrix} 0.800 & 0.200 & 0 \\ 0.032 & 0.776 & 0.192 \\ 0.237 & 0.003 & 0.760 \end{bmatrix} = \begin{pmatrix} 0.380 & 0.332 & 0.288 \end{pmatrix} \quad (14)$$

That is Star = 0.380(38%), Gulder = 0.332(33.2%), and Guiness = 0.288(28.8%) of the market shares in Jan. 1, 2014. These estimates represent a net gain of 0.006 in the market share of Star, gain of 0.002 in the market share of Gulder, and loss of 0.008 in the market share of Guiness come Jan. 1, 2014.

For market share of Jan. 1, 2015, we have

$$\begin{pmatrix} 0.380 & 0.332 & 0.288 \end{pmatrix} \begin{bmatrix} 0.800 & 0.200 & 0 \\ 0.032 & 0.776 & 0.192 \\ 0.237 & 0.003 & 0.760 \end{bmatrix} = \begin{pmatrix} 0.383 & 0.334 & 0.283 \end{pmatrix} \quad (15)$$

That is Star = 0.383, Gulder = 0.334, and Guiness = 0.283 . In other words Star will have 38.3% , Gulder will have 33.4%, while Harp will have 28.3% of the market shares respectively in Jan. 1, 2015. The estimates also represent a net gain of 0.003 in the market share of Star, gain of 0.002 in the market share of Gulder, and loss of 0.005 in the market share of Guiness come Jan. 1, 2015.

Steady-state (equilibrium).

Using (10) subject to the condition that $\pi_1 + \pi_2 + \pi_3 = 1$.

$$(\pi_1, \pi_2, \pi_3) = (\pi_1 \quad \pi_2 \quad \pi_3)\begin{bmatrix} 0.800 & 0.200 & 0 \\ 0.032 & 0.776 & 0.192 \\ 0.237 & 0.003 & 0.760 \end{bmatrix}$$

That is ; $\pi_1 = 0.800\pi_1 + 0.032\pi_2 + 0.237\pi_3$

$\pi_2 = 0{,}200\pi_1 + 0.776\pi_2 + 0.003\pi_3$

$\pi_3 = 0.192\pi_2 + 0.760\pi_3$ and

$\pi_1 + \pi_2 + \pi_3 = 1$

and solving the resulting simultaneous equations, will yield $(0.381 \quad 0.344 \quad 0.275)$, that is $\pi_1 = 38.1\%$, $\pi_2 = 34.4\%$, and $\pi_3 = 27.5\%$ where (π_1, π_2, π_3) represent Star, Gulder and Guiness respectively.

Using equations (11) and (12), the expected staying time for Star, Gulder, and Guiness are 5, 4.5, and 4.2 respectively, while the expected return time for Star, Gulder, and Guiness are 1.25, 1.29, and 1.32 respectively.

Results and Discussion.

In table 7, the transition probability matrix obtained, the first row indicates that Star retains 80% of its customers, loses 20% of its customers to Gulder, and loses 0% of its customers to Guiness. The second row indicates that Gulder retains 77.6% of its customers, loses 3.2% of its customers to Star, and loses 19.2% of its customers to Guiness, while the third row indicates that Guiness retains 76% of its customers, loses 23.7% of its customers to Star, and loses 0.3% to f its customers to Gulder.

Similarly, the columns of the transition matrix yield the following information; Star retains 80% of its customers, gains 3.2% of gulder's customers, and gains 23.7% of Guiness' customers. Gulder retains 77.6% of its customers, gains 20% of Star's customers, and gains 0.3% of Guiness' customers, while Guiness retains 76% of its customers, gains 0% of Star's customers, and gains 19.2% of Gulder's customers.

The transition probability matrix indicates that the probability of brand switching from Star to Guiness cannot be made in one time period due to its zero probability. We also observed that the probabilities of retaining their customers among the brands of beer are almost the same; 0.800, 0776, and 0.76 respectively, indicating a very strong competition among the brands of beer for customers. The probability of brand switching from Star to Gulder is 0.200, and the probability of brand switching from Star to Guiness is 0. Other probability values should be interpreted accordingly.

With the aid of the transition probabilities, the periods brands market share predicted the future market share as follows;

	Star	Gulder	Guiness
Jan. 1, 2013	0.374	0.330	0.296
Jan. 1, 2014	0.380	0.332	0.288
Jan. 1, 2015	0.383	0.334	0.283

In the steady-state equilibrium, the values of the three unknowns imply that at steady-state equilibrium, the market share of Star, Gulder, and Guiness will be 38.1%, 34.4%, and 27.5% respectively. Hence, we conclude that market share of star will continue to grow from its current value of 36.3%, but stabilizes at 38.1%. The market share of Gulder will lose from its current value of 33.1% to 33% in the following period but continues to grow and stabilizes at 34.4%, while that of Stout continues to lose customers from its current share of 30.6%, but will fall to 27,5% in the long run.

Conclusion.

From the results obtained, we conclude that among the three brands of beer under study, Star is expected to constitute 38.1% of the market share. This is followed by Gulder 34.4%, and Guiness 27.5% of the market share. However, the optimum marketing mix can be reached and can be maintained only if no brands takes action that will alter the transition probability matrix.

In the expected staying time, that for star, customers will buy star for five successive periods (five years) before switching to other brands. Customers will buy Gulder for four and half successive periods (four and half years) before switching to other brands, and customers will buy Guiness for four successive periods (four years) before switching to other brands. Hence, the expected staying times are 5, 4.5, and 4.2 for Star, Gulder, and Guiness respectively. Similarly, the expected number of successive periods customers will start to buy Star brand again after switching to other brands is one period (one year). Same applied to Gulder and Guiness. Hence, the expected return times are 1.25, 1.29, and 1.32 for Star, Gulder and Guiness respectively.

From the results obtained, it was concluded that the markov chain is a good model for analysing market equilibrium share.

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