Linear Reduced Order System based on Pade Approximation and Time Moment and Markov Parameter

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Abstract:-A new technique developed for order reduction of linear time invariant system of single input and single output system (SISO) using two methods, one method which is used to determine the denominator of reduced order model based on Pade approximation and other one is time moment and markov parameter which is used to determine the numerator of reduced order model. Proposed method for reduced order model guarantees the stability if original high order system is stable. It replaces the characteristics and quality of original high order system.

Key words:- Model reduction, Pade Approximation, Time moment and Markov Parameter, stability.

I. INTRODUCTION

The problem of reducing high order system into low order system is desired, because of the analysis and design of high order system is complicated. The simulation and design of high order system is great difficult. It is also not economic. It is highly possible and keen interest for engineers to approximate high order system. As the reduced order model helps the engineers to get analysis and design of the system very simple. So

the high order system is replaced by a low order system. There are several papers regarding the approximation of high order system has been proposed by several authors and its responses are given by step as well as frequency responses. A large number of model reduction have been proposed on time moment and markov parameters (1-3). Bosely and Lees,has also proposed a method for model reducton (4) but this method has advantages and disadvantages, because of reduced order model may be unstable even original system is stable. To overcome problems generated into reduced model, methods given by Hikin J and Sinha NK, continued fraction (chen and sheih). The pade approximation is one of the methods to obtain reduced order model (5-9). Some reduction methods have been developed by error minimization technique. (10-12)

In present method pade approximation technique and time moment and markov parameters are employed. Pade approximation technique(14) is used to determine the denominator of reduced order model and time moment and Markov parameters is used to determine the numerator of reduced order model.

II. ALGORITHM USED FOR REDUCED ORDER MODEL.

Let high n^{th} order transfer function of linear time invariant system (LTIS) of single input and single output) is given as.

$$G(s)=\frac{D_0+D_1s+D_2s^2+.........+D_{n-1}s^{n-1}}{d_0+d_1s+d_2s^2+........d_{n-1}s^{n-1}+d_ns^n}$$

$$G(s)=\frac{\sum_{j=0}^{n-1}D_js^j}{\sum_{i=0}^{n}d_is^i} \qquad (1)$$

Its reduced order model is expressed in form of

$$G_r(s)=\frac{E_0+E_1S+E_2S^2+.......+E_{P-1}S^{P-1}}{e_0+e_1s+e_2s^2+........+e_{p-1}s^{p-1}+e_ps^p} \quad G_r(s)=\frac{\sum_{j=0}^{p-1}E_js^j}{\sum_{i=0}^{p}e_is^i} \qquad (2)$$

Determination of denominator of reduced order system is given as.

Consider the denominator polynomial of high order system G(s).

$$p(s)=d_0+d_1s+d_2s^2+......+d_{n-1}s^{n-1}+d_ns^n \qquad (3)$$

P(s) is separated into even and odd parts

$$p_e(s)=d_0+d_2s^2+d_4s^4 \qquad (4)$$

$$p_o(s)=d_1s+d_3s^3+d_5s^5 \qquad (5)$$

Even part and odd part in s^2 are defined by

$$a(s^2)=p_e(s) \qquad (6)$$

$$b(s^2)=\frac{p_o(s)}{s} \qquad (7)$$

$$p(s)=a(s^2)+sb(s^2) \qquad (8)$$

The phase angle θ of polynomial at

frequency ω is given by

$$j\tan\theta=\frac{p_o(s)}{p_e(s)}\ |\ s=j\omega$$

and rational function

$$T = \frac{p_o(s)}{p_e(s)} = \frac{sb(s^2)}{a(s^2)}$$

Putting z for s^2 , by Maclaurin series expansion

of rational, we have

$$\frac{b(s^2)}{a(s^2)} = \frac{b(z)}{a(z)} = \sum_{j=0}^{\infty} (-1)^j \alpha_j z^j \qquad (9)$$

$$A(s) = a(s^{2)} + sb(s^2) \qquad (10)$$

of degree n=2 ρ is said to be [$\rho-1, \rho$]

Pade approximant of p(s) if, for $\rho < \mu$

$$\frac{b(z)}{a(z)} = \sum_{j=0}^{\infty} (-1)^j \alpha_j z^j \qquad (11)$$

Determination of numerator coefficients of reduced order model is given as,

Original high order transfer function G(s) of eqn. (1) can be expanded into power series about s=0, thus

$$G(s) = Q_0 + Q_1 s + Q_2 s^2 + \ldots\ldots \qquad (12)$$

Where $Q_0 = \frac{D_0}{d_0}$

$Q_i = \frac{1}{d_0}(d_i - \sum_{j=1}^{i} d_j Q_{i-j})$, i>0

d_i =0 for i>n-1

Time moments of G(s) are directly proportional to the Q_i's.

Transfer function is also expressed in power series about s=∞

$$G(S) = \frac{M_1}{s} + \frac{M_2}{s^2} + \frac{M_3}{s^3} + \ldots\ldots\ldots\ldots$$

$$G(s) = M_1 s^{-1} + M_2 s^{-2} + M_3 s^{-3} + \ldots\ldots \quad (13)$$

$$\sum_{i=1}^{\infty} m_i s^{-i} = M_1 s^{-1} + M_2 s^{-2} + m_3 s^{-3} + \ldots \ldots$$

$$M_i = \frac{D_{n-1}}{d_n}$$

$$M_i = \frac{1}{d_n}\left[D_{n-i} - \sum_{j=1}^{i-1} d_{n-j} M_{i-j} \right], \text{ for i >1}$$

$d_i = 0$ for i>n-1

M_i's are called the Markov parameters of the system. The reduced order $G_{r1}(s)$ is obtained by matching Initial time moments, given by

$$G_{r1}(s) = \frac{\sum_{j=0}^{p} e_j s^j}{\sum_{j=0}^{p} e_j s^j}\left(Q_0 + Q_1 s + Q_2 s^2 + \ldots\ldots..\right) \quad (14)$$

Collecting the term up to (p-1) of power of s in numerator, we have

$$G_{r1}(s) = \frac{\sum_{j=0}^{p-1} E_j s^j}{\sum_{j=0}^{p} e_j s^j}$$

Alternatively the reduced model $G_{r2}(s)$ is obtained by matching initial Markov parameters ,given by

$$G_{r2}(s) = \frac{\sum_{j=0}^{p} e_j s^j}{\sum_{j=0}^{p} e_j s^j}\left(M_1 s^{-1} + M_2 s^{-2} + \ldots.\right) \quad (15)$$

Neglecting the term up to (p-1) of negative power of s in numerator.

The steady state is matched if the time moments are matched however if the Markov parameters are matched, there will be steady state error between the output of original system and reduced model. To avoid the steady state error, condition is given by

$$\frac{D_0}{d_0} = k\frac{E_0}{e_0} \quad (16)$$

The final reduced model is obtained by multiplying gain correction factor 'k' with numerator of G_{r2}

III. Results:- Numerical Analysis.

Consider 4th order transfer function of original high order system be (12)

$$G(s) = \frac{s^3 + 7s^2 + 24s + 24}{s^4 + 10s^3 + 35s^2 + 50s + 24} \qquad (17)$$

From above system, 2nd order reduced model is to be determined.

Denominator polynomial of original system is given, as

$$p(s) = s^4 + 10s^3 + 35s^2 + 50s + 24$$

$$\frac{b(z)}{a(z)} = \frac{50 + 10z}{24 + 35z + z^2}$$

$$= 2.08 - 2.62z + ...$$

$$\alpha = 2.08 \ , \ \gamma = 1.26$$

$$\frac{b(z)}{a(z)} = \alpha\left[1 - \gamma z + (\gamma z)^2 - (\gamma z)^3 + ...\right]$$

$$= \frac{\alpha}{1 + \gamma z}$$

$A(s) = 1.26s^2 + 2.08s + 1$ is reduced order denominator. Normalizing it, we get

$$A(s) = s^2 + 1.6508s + 0.7936 \qquad (18)$$

Power series expansion of G(s) is given about s=o

$$G(s) = 1 - 1.0833s + ...$$

Power series expansion of G(s) is given about s=∞

$$G(s) = s^{-1} + 3s^{-2} + \ \ldots\ldots\ldots$$

Reduced order model is obtained by matching initial time moment as,

$G_{r1}(s)=\frac{s^2+1.6508s+0.7936}{s^2+1.6508s+0.7936}\left(1-0.0833s+...\right)$ Collecting the terms up to (p-1) in numerator then transfer function of reduced order model is given by

$$G_{r1}(s)=\frac{0.7911s+0.7936}{s2+1.6508s+0.7936} \qquad (19)$$

Reduced order model is obtained by matching initial markov parameters as

$$G_{r2}(s)=\frac{s^2+1.6508s+0.7936}{s^2+1.6508s+0.7936}\left(s^{-1}+3s^{-2}+.....\right)$$

Collecting the terms up to (p-1) in numerator the transfer function of reduced model is given by

$$G_{r2}(s)=\frac{s-1.3492}{s2+1.6508+0.7936} \qquad (20)$$

To avoid the steady state error, numerator of $G_{r2(}$s) of eqn. (20) is multiplied by gain factor

$k=\frac{1}{1.7001}$ then we get transfer function of reduced order model as

$$G_{r2}(s)=\frac{-0.5882s+0.7936}{s^2+1.6508s+0.7936} \qquad (21)$$

Unit step responses of original (sys1) and reduced order (sys2) systems are shown in Fig. 1. From this Fig steady state responses of original and reduced order model are exactly matching. Here considering transfer function of reduced order model obtained by matching initial time moments which gives exact approximation of the original system.

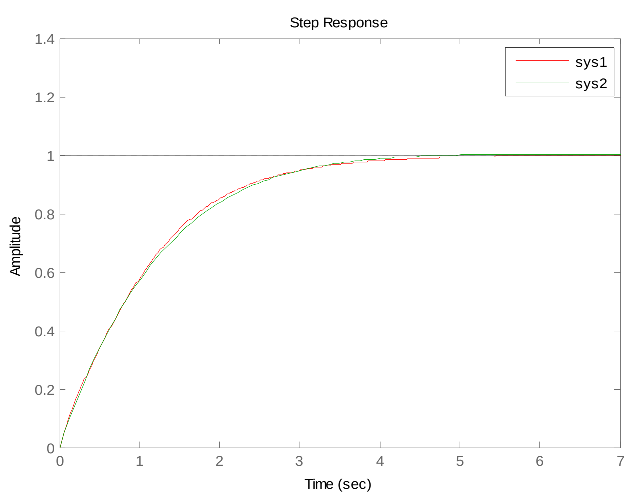

Fig 1 : Comparison of step responses of reduced model with original system

IV. Discussion:-

In this paper two methods have been proposed for reducing high order system into low order system. First method is pade approximation which is used to determine the denominator of reduced order model .second one is time moment and Markov parameters which is used to determine the numerator of reduced order model .Reduced order model replaces the performance and characteristics of original system. Proposed method provides the steady state stability in reduced order model if original system is stable.

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