ELASTICO-VISCOUS EFFECTS ON AN OSCILLATORY HEAT AND MASS TRANSFER FLOW PAST A VERTICAL PLATE WITH THERMAL DIFFUSION AND PERIODIC SUCTION VELOCITY

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ABSTRACT

An analysis of three dimensional oscillatory flows with heat and mass transfer of a visco-elastic fluid past an infinite vertical porous plate embedded in porous medium is presented here. The plate is subjected to a transverse sinusoidal suction velocity fluctuating with time and thus generating a three-dimensional flow. The governing equations are solved by regular perturbation technique. The elastico-viscous fluid flow is characterized by Walters liquid (Model $B^{/}$). The expressions for the velocity field, temperature field, concentration field, shearing stress, rate of heat transfer, rate of mass transfer and current density of the fluid flow are derived in non-dimensional form. The effects of physical parameters are illustrated graphically and the results are interpreted physically.

Keywords: Thermal diffusion, oscillatory, skin friction, mass diffusivity, electrically conducting, visco-elastic.

1. INTRODUCTION

In recent years, the requirements of modern technology have stimulated interest in fluid flow, which involve the interaction of several phenomena. The process of heat and mass transfer phenomena is encountered in aeronautics, fluid fuel nuclear reactor, chemical process industries any many engineering applications in which fluid is the working medium. The problems of laminar flow control through a porous media have accomplished considerable significant in aeronautical engineering in view of its application to reduce drag and thus the vehicle power requirement by a substantial amount. Theoretical and experimental research have cited that the transition from laminar to the turbulent flow which causes the drag coefficient to increase may be prevented by suction of the fluid and the heat transfer from boundary layer to the wall. Lachmann (1961) has initiated this subject in the direction of its development.

Again, the method of cooling of the wall in controlling the laminar flow together with the application of suction is very useful for mankind as the energy consumption of the suction pump is not economical to get any desired reduction in the drag by increasing suction alone. Raptis and Kafousias (1982) have investigated magneto hydrodynamic free convective flow and mass transfer through a porous medium bounded by and infinite vertical porous plate with constant heat flux. Heat and mass transfer in a porous medium has been explained by Bejan and Khair (1985). Three dimensional convective flow and heat transfer in a porous medium has been studied by Singh *et al* (1985). Sattar and Alam (1994) have discussed thermal diffusion as well as transpiration effects on MHD free convection and mass transfer flow past an accelerated vertical porous plate. Choudhury and Chand (2002) has studied three dimensional flow and heat transfer through porous medium. Singh *et al* (1988), Jain and Gupta (2006) have investigated the effect of transverse sinusoidal injection velocity distribution on three dimensional free dimensional core flow of a viscous incompressible fluid in slip flow regime under the influence of heat sink. Considering the thermal diffusion effect (or Soret effect) into account, many researchers like Sattar and Alam and Goswami (1994), Singh *et al.*

(2007), Raju *et al.* (2008), Ahmed and Goswami (2011) have investigated some flow problems of practical interest.

All the above investigators, however, restrict their analysis to the flow of Newtonian fluid. Most fluids such as molten plastics, artificial fibres, crude oil blood and polymer solutions are considered non-Newtonian fluids. In this field of study, the analysis of visco-elastic fluid lies on the fact that the fluid possesses a certain degree of elasticity in addition to viscosity. Recently, authors like Choudhury and Dey (2010), Choudhury and Mahanta (2012), Choudhury and Das (2012), Choudhury and Debnath (2012) have studied some problems of physical interest in this field.

In this paper, we have analyzed the visco-elastic free convective flow and heat transfer past an oscillating porous plate in the slip flow regime. The visco-elastic fluid is characterized by Walters liquid (Model B´).

The constitutive equation for Walters liquid (Model B´) is

$$\sigma_{ik} = -pg_{ik} + \sigma'_{ik}, \qquad \sigma'^{ik} = 2\eta_0 e^{ik} - 2k_0 e'^{ik} \tag{1.1}$$

where σ^{ik} is the stress tensor, p is isotropic pressure, g_{ik} is the metric tensor of a fixed co-ordinate system x^i, v_i is the velocity vector, the contravarient form of e'^{ik} is given by

$$e'^{ik} = \frac{\partial e^{ik}}{\partial t} + v^m e^{ik}_{,m} - v^k_{,m} e^{im} - v^i_{,m} e^{mk} \tag{1.2}$$

It is the convected derivative of the deformation rate tensor e^{ik} defined by

$$2e^{ik} = v_{i,k} + v_{k,i} \tag{1.3}$$

Here η_0 is the limiting viscosity at the small rate of shear which is given by

$$\eta_0 = \int_0^\infty N(\tau)d\tau \text{ and } k_0 = \int_0^\infty \tau N(\tau)d\tau \tag{1.4}$$

N(τ) being the relaxation spectrum . This idealized model is a valid approximation of Walters liquid (Model B') taking very short memories into account so that terms involving

$$\int_0^\infty t^n N(\tau)d\tau, n \geq 2 \tag{1.5}$$

have been neglected.

2.FORMULATION OF THE PROBLEM

We now consider the unsteady, free and forced convection flow of an elastico-viscous electrically conducting fluid in presence of a heat sink taking into account the species concentration and thermal diffusion past a vertical porous plate with transverse sinusoidal suction velocity by making the following assumptions:

(i) All the fluid properties except the density in the buoyancy force term are constant.

(ii) The viscous dissipation, dissipations energy is negligible.

(iii) $\bar{T}_w > \bar{T}_\infty$ and $\bar{C}_w > \bar{C}_\infty$.

We introduce a co-ordinate system $(\bar{x}, \bar{y}, \bar{z})$ with $\bar{x}$-axis vertically upwards along the plate, $\bar{y}$-axis perpendicular to it directed into the fluid region and $\bar{z}$-axis along the width of the plate. Let u, v, w be the velocity components in the direction of $\bar{x}, \bar{y}, \bar{z}$ respectively.

The suction velocity distribution is taken as follows:

$$\bar{v}_w(\bar{z}) = -V_0\left(1 + \varepsilon cos\frac{\pi\bar{z}}{L}e^{i\bar{w}\bar{t}}\right) \tag{2.1}$$

which consists of a basic steady distribution $-V_0$ with a superimposed weak distribution $-\varepsilon V_0 cos\frac{\pi\bar{z}}{L}e^{i\bar{w}\bar{t}}$, where L is the wave length of the periodic suction and ε is a small reference parameter. Since the plate is infinite in length in $\bar{x}$-direction, therefore all the quantities except possibly the pressure are assumed to be independent of $\bar{x}$.

With these assumptions and under usual boundary layer approximations, the equations governing the flow become

Equation of continuity:

$$\frac{\partial\bar{v}}{\partial\bar{y}} + \frac{\partial\bar{w}}{\partial\bar{z}} = 0 \tag{2.2}$$

Momentum equations:

$$\begin{aligned}\frac{\partial\bar{u}}{\partial\bar{t}} + \bar{v}\frac{\partial\bar{u}}{\partial\bar{y}} + \bar{w}\frac{\partial\bar{u}}{\partial\bar{z}} &= \nu\left(\frac{\partial^2\bar{u}}{\partial\bar{y}^2} + \frac{\partial^2\bar{u}}{\partial\bar{z}^2}\right)\\ &- \frac{k_0}{\rho}\left[\frac{\partial^3\bar{u}}{\partial\bar{t}\partial\bar{y}^2} + \frac{\partial^3\bar{u}}{\partial\bar{t}\partial\bar{z}^2} + \bar{v}\left(\frac{\partial^3\bar{u}}{\partial\bar{y}^3} + \frac{\partial^3\bar{u}}{\partial\bar{y}\partial\bar{z}^2}\right) + \bar{w}\left(\frac{\partial^3\bar{u}}{\partial\bar{y}^2\partial\bar{z}} + \frac{\partial^3\bar{u}}{\partial\bar{z}^3}\right) - 2\frac{\partial^2\bar{u}}{\partial\bar{y}^2}\frac{\partial\bar{v}}{\partial\bar{y}}\right.\\ &\left. - \frac{\partial\bar{u}}{\partial\bar{y}}\frac{\partial^2\bar{v}}{\partial\bar{y}^2} - 2\frac{\partial^2\bar{u}}{\partial\bar{y}\partial\bar{z}}\frac{\partial\bar{v}}{\partial\bar{z}} - \frac{\partial\bar{u}}{\partial\bar{z}}\frac{\partial^2\bar{w}}{\partial\bar{z}^2} - \frac{\partial^2\bar{w}}{\partial\bar{y}^2}\frac{\partial\bar{u}}{\partial\bar{z}} - 2\frac{\partial^2\bar{u}}{\partial\bar{z}^2}\frac{\partial\bar{w}}{\partial\bar{z}} - 2\frac{\partial^2\bar{u}}{\partial\bar{y}\partial\bar{z}}\frac{\partial\bar{w}}{\partial\bar{y}} - \frac{\partial\bar{u}}{\partial\bar{y}}\frac{\partial^2\bar{v}}{\partial\bar{z}^2}\right]\\ &+ g\beta(\bar{T} - \bar{T}_\infty) + g\bar{\beta}(\bar{C} - \bar{C}_\infty)\end{aligned} \tag{2.3}$$

$$\begin{aligned}\frac{\partial\bar{v}}{\partial\bar{t}} + \bar{v}\frac{\partial\bar{v}}{\partial\bar{y}} + \bar{w}\frac{\partial\bar{v}}{\partial\bar{z}} &= -\frac{1}{\rho}\frac{\partial\bar{p}}{\partial\bar{y}} + \nu\left(\frac{\partial^2\bar{v}}{\partial\bar{y}^2} + \frac{\partial^2\bar{v}}{\partial\bar{z}^2}\right)\\ &- \frac{k_0}{\rho}\left[\frac{\partial^3\bar{v}}{\partial\bar{t}\partial\bar{y}^2} + \frac{\partial^3\bar{v}}{\partial\bar{t}\partial\bar{z}^2} + v\left(\frac{\partial^3\bar{v}}{\partial\bar{y}^3} - \frac{\partial^3\bar{w}}{\partial\bar{z}^3}\right) + \bar{w}\left(\frac{\partial^3\bar{v}}{\partial\bar{z}^3} - \frac{\partial^3\bar{v}}{\partial\bar{y}^2\partial\bar{z}} - 2\frac{\partial^3\bar{w}}{\partial\bar{y}\partial\bar{z}^2}\right)\right.\\ &\left. - 3\frac{\partial\bar{v}}{\partial\bar{y}}\frac{\partial^2\bar{v}}{\partial\bar{y}^2} + \frac{\partial\bar{v}}{\partial\bar{z}}\frac{\partial^2\bar{w}}{\partial\bar{z}^2} - \frac{\partial\bar{v}}{\partial\bar{z}}\frac{\partial^2\bar{w}}{\partial\bar{y}^2} - 3\frac{\partial^2\bar{v}}{\partial\bar{z}^2}\frac{\partial\bar{w}}{\partial\bar{z}} + 2\frac{\partial\bar{w}}{\partial\bar{y}}\frac{\partial^2\bar{w}}{\partial\bar{z}^2}\right]\end{aligned} \tag{2.4}$$

$$\frac{\partial \bar{w}}{\partial \bar{t}} + \bar{v}\frac{\partial \bar{w}}{\partial \bar{y}} + \bar{w}\frac{\partial \bar{w}}{\partial \bar{z}} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{z}} + \nu\left(\frac{\partial^2 \bar{w}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{w}}{\partial \bar{z}^2}\right) - \frac{k_0}{\rho}\left[\frac{\partial^3 \bar{w}}{\partial \bar{t}\partial \bar{y}^2} + \frac{\partial^3 \bar{w}}{\partial \bar{t}\partial \bar{z}^2} + \bar{v}\left(\frac{\partial^3 \bar{w}}{\partial \bar{y}^3} - \frac{\partial^3 \bar{w}}{\partial \bar{y}\partial \bar{z}^2} - 2\frac{\partial^3 \bar{v}}{\partial \bar{y}\partial \bar{z}^2}\right) + \bar{w}\left(\frac{\partial^3 \bar{w}}{\partial \bar{z}^3} - \frac{\partial^3 \bar{v}}{\partial \bar{y}^3}\right) + \frac{\partial \bar{w}}{\partial \bar{z}}\frac{\partial^2 \bar{w}}{\partial \bar{y}^2} - \frac{\partial^2 \bar{v}}{\partial \bar{z}^2}\frac{\partial \bar{w}}{\partial \bar{y}} - 3\frac{\partial \bar{w}}{\partial \bar{z}}\frac{\partial^2 \bar{w}}{\partial \bar{z}^2} + 2\frac{\partial^2 \bar{v}}{\partial \bar{y}^2}\frac{\partial \bar{v}}{\partial \bar{z}} + \frac{\partial \bar{w}}{\partial \bar{y}}\frac{\partial^2 \bar{v}}{\partial \bar{y}^2}\right] \quad (2.5)$$

Energy equation**:**

$$\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{v}\frac{\partial \bar{T}}{\partial \bar{y}} + \bar{w}\frac{\partial \bar{T}}{\partial \bar{z}} = \frac{k}{\rho C_p}\left(\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{T}}{\partial \bar{z}^2}\right) + \frac{\bar{Q}}{\rho C_p}(\bar{T}_\infty - \bar{T}) \quad (2.6)$$

Species concentration equation**:**

$$\frac{\partial \bar{C}}{\partial \bar{t}} + \bar{v}\frac{\partial \bar{C}}{\partial \bar{y}} + \bar{w}\frac{\partial \bar{C}}{\partial \bar{z}} = D_M\left(\frac{\partial^2 \bar{C}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{C}}{\partial \bar{z}^2}\right) + D_r\left(\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{T}}{\partial \bar{z}^2}\right) \quad (2.7)$$

where β is the co-efficient volume expansion for heat transfer, $\bar{\beta}$ is the co-efficient of volume expansion for mass transfer, k is the thermal conductivity, g is the acceleration due to gravity, ν is the kinematic viscosity, ρ is the density of the fluid, $\bar{p}$ is the pressure, $\bar{Q}$ is the first-order heat sink, $\bar{T}$ is the temperature in the boundary layer, $\bar{U}$ is the free stream velocity, $\bar{C}$ is the species concentration, $\bar{C}_\infty$ is the species concentration in the free stream, C_p is the specific heat at constant pressure, D_M is the co-efficient of chemical molecular diffusivity, D_r is the co-efficient of chemical thermal diffusivity and the other symbols have their usual meanings.

The relevant boundary conditions are:

$$\text{at } \bar{y} = 0 : \bar{u} = 0\,, \bar{v} = \bar{v}_w\,, \bar{w} = 0\,, \bar{T} = \bar{T}_w\,, \bar{C} = \bar{C}_w \quad (2.8)$$

$$\text{at } \bar{y} \to \infty : \bar{u} = \bar{U}\,, \bar{v} = -V_0\,, \bar{w} = 0\,, \bar{T} = \bar{T}_\infty\,, \bar{C} = \bar{C}_\infty\,, \bar{p} = \bar{p}_\infty \quad (2.9)$$

We introduce the following non-dimensional quantities:

$$y = \frac{\bar{y}}{L}\,, z = \frac{\bar{z}}{L}\,, u = \frac{\bar{u}}{V_0}\,, v = \frac{\bar{v}}{V_0}\,, w = \frac{\bar{w}}{V_0}\,, U = \frac{\bar{U}}{V_0}\,, \theta = \frac{\bar{T} - \bar{T}_\infty}{\bar{T}_w - \bar{T}_\infty}\,, \phi = \frac{\bar{C} - \bar{C}_\infty}{\bar{C}_w - \bar{C}_\infty}\,,$$

$$Pr = \frac{\mu C_p}{k}\,, Sc = \frac{\nu}{D_M}\,, Sr = \frac{D_r(\bar{T}_w - \bar{T}_\infty)}{\nu(\bar{C}_w - \bar{C}_\infty)}\,, Q = \frac{\bar{Q}L\nu}{V_0 k}\,, Gr = \frac{Lg\beta(\bar{T}_w - \bar{T}_\infty)}{V_0^2}\,,$$

$$Gm = \frac{Lg\bar{\beta}(\bar{C}_w - \bar{C}_\infty)}{V_0^2}\,, Re = \frac{V_0 L}{\nu}\,, p = \frac{\bar{p}}{\rho\left(\frac{\nu}{L}\right)^2}\,, p_\infty = \frac{\bar{p}_\infty}{\rho\left(\frac{\nu}{L}\right)^2}\,, t = \frac{V_0 \bar{t}}{L}\,, \omega = \frac{\bar{\omega} L}{V_0}$$

where (u, v, w) are the non dimensional components of the fluid velocity, U is the non dimensional free stream velocity, θ is the non dimensional temperature, ϕ is the non dimensional concentration, Pr is the Prandtl number, Sc is the Schmidt number, Sr is the Soret number, Q is the non dimensional first-order heat sink, Gr is the Grashof number for mass transfer, Re is the Reynolds number, p is the non dimensional pressure, p_∞ is the non dimensional pressure in the free stream, ω is the frequency parameter and the other symbols have their usual meanings.

The non dimensional forms of the equations (2.2) to (2.7) are

$$\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{2.10}$$

$$\begin{aligned}\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}
&=\frac{1}{Re}\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)\\
&-K\left[\frac{\partial^3 u}{\partial t\partial y^2}+\frac{\partial^3 u}{\partial t\partial z^2}+v\left(\frac{\partial^3 u}{\partial y^3}+\frac{\partial^3 u}{\partial y\partial z^2}\right)+w\left(\frac{\partial^3 u}{\partial y^2\partial z}+\frac{\partial^3 u}{\partial z^3}\right)-2\frac{\partial^2 u}{\partial y^2}\frac{\partial v}{\partial y}\right.\\
&\left.-\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2}-2\frac{\partial^2 u}{\partial y\partial z}\frac{\partial v}{\partial z}-\frac{\partial u}{\partial z}\frac{\partial^2 w}{\partial z^2}-\frac{\partial^2 w}{\partial y^2}\frac{\partial u}{\partial z}-2\frac{\partial^2 u}{\partial z^2}\frac{\partial w}{\partial z}-2\frac{\partial^2 u}{\partial y\partial z}\frac{\partial w}{\partial y}-\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial z^2}\right]\\
&+Gr\theta+Gm\emptyset\end{aligned} \tag{2.11}$$

$$\begin{aligned}\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}
&=-\frac{1}{Re^2}\frac{\partial p}{\partial y}+\frac{1}{Re}\left(\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)\\
&-K\left[\frac{\partial^3 v}{\partial t\partial y^2}+\frac{\partial^3 v}{\partial t\partial z^2}+v\left(\frac{\partial^3 v}{\partial y^3}-\frac{\partial^3 w}{\partial z^3}\right)+w\left(\frac{\partial^3 v}{\partial z^3}+\frac{\partial^3 v}{\partial y^2\partial z}-2\frac{\partial^3 w}{\partial y\partial z^2}\right)\right.\\
&\left.-3\frac{\partial v}{\partial y}\frac{\partial^2 v}{\partial y^2}+\frac{\partial v}{\partial z}\frac{\partial^2 w}{\partial z^2}-\frac{\partial v}{\partial z}\frac{\partial^2 w}{\partial y^2}-3\frac{\partial^2 v}{\partial z^2}\frac{\partial w}{\partial z}+2\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial z^2}\right]\end{aligned} \tag{2.12}$$

$$\begin{aligned}\frac{\partial w}{\partial t}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}
&=-\frac{1}{Re^2}\frac{\partial p}{\partial z}+\frac{1}{Re}\left(\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)\\
&-K\left[\frac{\partial^3 w}{\partial t\partial y^2}+\frac{\partial^3 w}{\partial t\partial z^2}+v\left(\frac{\partial^3 w}{\partial y^3}-\frac{\partial^3 w}{\partial y\partial z^2}-2\frac{\partial^3 v}{\partial y\partial z^2}\right)+w\left(\frac{\partial^3 w}{\partial z^3}-\frac{\partial^3 v}{\partial y^3}\right)+\frac{\partial w}{\partial z}\frac{\partial^2 w}{\partial y^2}\right.\\
&\left.-\frac{\partial^2 v}{\partial z^2}\frac{\partial w}{\partial y}-3\frac{\partial w}{\partial z}\frac{\partial^2 w}{\partial z^2}+2\frac{\partial^2 v}{\partial y^2}\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\frac{\partial^2 v}{\partial y^2}\right]\end{aligned} \tag{2.13}$$

$$\frac{\partial\theta}{\partial t}+v\frac{\partial\theta}{\partial y}+w\frac{\partial\theta}{\partial z}=\frac{1}{PrRe}\left(\frac{\partial^2\theta}{\partial y^2}+\frac{\partial^2\theta}{\partial z^2}\right)-\frac{Q}{Pr}\theta \tag{2.14}$$

$$\frac{\partial\emptyset}{\partial t}+v\frac{\partial\emptyset}{\partial y}+w\frac{\partial\emptyset}{\partial z}=\frac{1}{ScRe}\left(\frac{\partial^2\emptyset}{\partial y^2}+\frac{\partial^2\emptyset}{\partial z^2}\right)+\frac{Sr}{Re}\left(\frac{\partial^2\theta}{\partial y^2}+\frac{\partial^2\theta}{\partial z^2}\right) \tag{2.15}$$

subject to the boundary conditions

$$y=0: u=0\,,\ v=-\left(1+\varepsilon cos\pi z e^{i\omega t}\right),w=0\,,\theta=1\,,\emptyset=1 \tag{2.16}$$

$$y\to\infty: u=U\,,\ v=-1,w=0\,,\theta=0\,,\emptyset=0\,,p=p_\infty \tag{2.17}$$

3. METHOD OF SOLUTION

We assume the solutions of the equations (2.10) to (2.15) to be of the following forms

$$u = u_0(y) + \varepsilon u_1(y,z)e^{i\omega t} + o(\varepsilon^2) \quad (3.1)$$

$$v = v_0(y) + \varepsilon v_1(y,z)e^{i\omega t} + o(\varepsilon^2) \quad (3.2)$$

$$w = w_0(y) + \varepsilon w_1(y,z)e^{i\omega t} + o(\varepsilon^2) \quad (3.3)$$

$$p = p_0(y) + \varepsilon p_1(y,z)e^{i\omega t} + o(\varepsilon^2) \quad (3.4)$$

$$\theta = \theta_0(y) + \varepsilon \theta_1(y,z)e^{i\omega t} + o(\varepsilon^2) \quad (3.5)$$

$$\emptyset = \emptyset_0(y) + \varepsilon \emptyset_1(y,z)e^{i\omega t} + o(\varepsilon^2) \quad (3.6)$$

with $p_0 = p_\infty$, $w_0 = 0$

substituting these in equation (2.10) to (2.15), equating the coefficients of the similar terms and neglecting ε^2 , the following differential equations are obtained:

Zeroth-order equations**:**

$$\frac{dv_0}{dy} = 0 \quad (3.7)$$

$$v_0\frac{du_0}{dy} = \frac{1}{Re}\frac{d^2u_0}{dy^2} - Kv_0\frac{d^3u_0}{dy^3} + Gr\theta_0 + Gm\emptyset_0 \quad (3.8)$$

$$v_0\frac{d\theta_0}{dy} = \frac{1}{PrRe}\frac{d^2\theta_0}{dy^2} - \frac{Q}{Pr}\theta_0 \quad (3.9)$$

$$v_0\frac{d\emptyset_0}{dy} = \frac{1}{ScPr}\frac{d^2\emptyset_0}{dy^2} + \frac{Sr}{Re}\frac{d^2\theta_0}{dy^2} \quad (3.10)$$

First-order equations**:**

$$\frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0 \quad (3.11)$$

$$-\frac{\partial u_1}{\partial y} + v_1\frac{du_0}{dy} = Gr\theta_1 + Gm\emptyset_1 + \frac{1}{Re}\left(\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2}\right) - i\omega u_1 - K\left[i\omega\frac{\partial^2 u_1}{\partial y^2} + i\omega\frac{\partial^2 u_1}{\partial z^2} - \frac{\partial^3 u_1}{\partial y^3} + v_1\frac{\partial^3 u_0}{\partial y^3} + \frac{\partial^3 u_1}{\partial y\partial z^2} - 2\frac{d^2u_0}{dy^2}\frac{\partial v_1}{\partial y} - \frac{du_0}{dy}\frac{\partial^2 v_1}{\partial y^2} - \frac{du_0}{dy}\frac{\partial^2 v_1}{\partial z^2}\right] \quad (3.12)$$

$$-\frac{\partial v_1}{\partial y} = -\frac{1}{Re^2}\frac{\partial p_1}{\partial y} + \frac{1}{Re}\left(\frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_1}{\partial z^2}\right) - i\omega v_1 - K\left[i\omega\frac{\partial^2 v_1}{\partial y^2} + i\omega\frac{\partial^2 v_1}{\partial z^2} - \frac{\partial^3 v_1}{\partial y^3} + \frac{\partial^3 w_1}{\partial z^3}\right] \quad (3.13)$$

$$-\frac{\partial w_1}{\partial y} = -\frac{1}{Re^2}\frac{\partial p_1}{\partial z} + \frac{1}{Re}\left(\frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2}\right) - i\omega w_1 - K\left[i\omega\frac{\partial^2 w_1}{\partial y^2} + i\omega\frac{\partial^2 w_1}{\partial z^2} - \frac{\partial^3 w_1}{\partial y^3} + \frac{\partial^3 w_1}{\partial y \partial z^2} + 2\frac{\partial^3 v_1}{\partial y^2 \partial z}\right] \quad (3.14)$$

$$-\frac{\partial \theta_1}{\partial y} + v_1\frac{d\theta_0}{dy} = \frac{1}{PrRe}\left(\frac{\partial^2 \theta_1}{\partial y^2} + \frac{\partial^2 \theta_1}{\partial z^2}\right) - \frac{Q\theta_1}{Pr} - i\omega\theta_1 \quad (3.15)$$

$$-\frac{\partial \phi_1}{\partial y} + v_1\frac{d\varphi_0}{dy} = \frac{1}{ScRe}\left(\frac{\partial^2 \phi_1}{\partial y^2} + \frac{\partial^2 \phi_1}{\partial z^2}\right) + \frac{Sr}{Re}\left(\frac{\partial^2 \theta_1}{\partial y^2} + \frac{\partial^2 \theta_1}{\partial z^2}\right) - i\omega\phi_1 \quad (3.16)$$

The modified boundary conditions are:

$$y = 0: u_0 = 0, u_1 = 0, v_0 = -1, v_1 = -cos\pi z, w_0 = 0, w_1 = 0, \theta_0 = 1,$$
$$\theta_1 = 0, \phi_0 = 1, \phi_1 = 0 \quad (3.17)$$

$$y \to \infty: u_0 = U, u_1 = 0, v_0 = -1, v_1 = 0, w_0 = 0, w_1 = 0, \theta_0 = 0,$$
$$\theta_1 = 0, \phi_0 = 0, \phi_1 = 0, p_1 = 0 \quad (3.18)$$

To solve equation (3.8) we take K to be very small as $K \ll 1$ and assume

$$u_0 = u_{00} + Ku_{01} \quad (3.19)$$

$$\theta_0 = \theta_{00} + K\theta_{01} \quad (3.20)$$

$$\phi_0 = \phi_{00} + K\phi_{01} \quad (3.21)$$

Now using (31.9) to (3.21) in equations (3.8), and equating the coefficient of like powers of K and neglecting the higher powers of K, we get the following set of differential equations:

Zeroth order equations:

$$v_{00}\frac{du_{00}}{dy} = \frac{1}{Re}\frac{d^2u_{00}}{dy^2} + Gr\theta_{00} + Gm\phi_{00} \quad (3.22)$$

with the modified boundary conditions:

$$y = 0: u_{00} = 0, v_{00} = -1, w_{00} = 0, \theta_{00} = 1, \phi_{00} = 1$$
$$y \to \infty: u_{00} = U, v_{00} = -1, w_{00} = 0, \theta_{00} = 0, \phi_{00} = 0 \quad (3.23)$$

First-order equations:

$$v_{01}\frac{du_{00}}{dy} + v_{00}\frac{du_{01}}{dy} = \frac{1}{Re}\frac{d^2u_{01}}{dy^2} - v_{00}\frac{d^3u_{01}}{dy^3} + Gr\theta_{01} + Gm\phi_{01} \quad (3.24)$$

subject to the boundary conditions:

$$y = 0: u_{00} = 0, u_{01} = 0, v_{01} = 0, w_{01} = 0, \theta_{01} = 0, \phi_{01} = 0$$

$$y \to \infty: u_{00} = U, u_{01} = 0 \ , v_{01} = 0 \, , w_{01} = 0 \, , \theta_{01} = 0 \, , \emptyset_{01} = 0 \tag{3.25}$$

Solutions of the equations (3.22) and (3.24) subject to

$$u_{00} = U + A_1 e^{-\alpha y} + A_2 e^{-ScRey} + A_3 e^{-Rey} \tag{3.26}$$

$$u_{01} = A_4 e^{-\alpha y} + A_5 e^{-ScRey} + A_6 e^{-Rey} \tag{3.27}$$

The solutions of the equations (3.7), (3.9), (3.10) under the boundary conditions (3.17), (3.18) and using equations (3.26) and (3.27) we get

$$v_0 = -1$$

$$\theta_0 = e^{-\alpha y}$$

$$\emptyset_0 = (1 - A)e^{-ScRey} + Ae^{-\alpha y}$$

$$u_0 = u_{00} + Ku_{01}$$

$$= U + A_1 e^{-\alpha y} + A_2 e^{-ScRey} + A_3 e^{-Rey} + K(A_4 e^{-\alpha y} + A_5 e^{-ScRey} + A_6 e^{-Rey})$$

4. SOLUTION OF CROSS FLOW

We shall first consider the equations (3.12), (3.14) and (3.15) for $v_1(y, z), w_1(y, z)$ and $p_1(y, z)$ which are independent of the main flow component u_1, temperature field θ_1 and concentration field $\emptyset_1$.

The suction velocity distribution $v_w = -\left(1 + \varepsilon cos\pi z e^{i\omega t}\right)$ consists of a basic uniform distribution -1 with superimposed weak sinusoidal distribution $\varepsilon cos\pi z e^{i\omega t}$. Hence the velocity components v, w and p are also separated into mean and small sinusoidal components v_1, w_1 and p_1.

$$v_1 = -\pi v_{11}(y) cos\pi z \tag{4.1}$$

$$w_1 = v_{11}{}'(y) sin\pi z \tag{4.2}$$

$$p_1 = Re^2 p_{11}(y) cos\pi z \tag{4.3}$$

On substitution of the above, the equation (3.12) is satisfied and the equations (3.14) and (3.15) reduce to the following ordinary differential eqations:

$$v_{11}'' + Rev_{11}' - (\pi^2 + Rei\omega)v_{11} + KRe\left(v_{11}''' - i\omega v_{11}'' - \pi^2 v_{11}' + \pi^2 i\omega v_{11}\right) = -\frac{Rep_{11}'}{\pi} \tag{4.4}$$

$$v_{11}''' + Rev_{11}'' - (\pi^2 + Rei\omega)v_{11}{}' + KRe\left(v_{11}'''' - i\omega v_{11}''' - \pi^2 v_{11}'' + \pi^2 i\omega v_{11}'\right) = -\pi ReP_{11} \tag{4.5}$$

and the relevant boundary conditions are

$$y = 0: \ v_{11} = \frac{1}{\pi} \, , v_{11}' = 0 \tag{4.6}$$

$$y \to \infty: \ v_{11} = 0 \, , v_{11}' = 0 \tag{4.7}$$

Using multi-parameter perturbation technique and assuming K to be very small, we take

$$v_{11} = v_{110} + Kv_{111} \tag{4.8}$$

$$p_{11} = p_{110} + Kp_{111} \tag{4.9}$$

Using the above equations in (4.4) and (4.5) we get

Zeroth order equations:

$$v_{110}'' + Rev_{110}' - (\pi^2 + Rei\omega)v_{110} = -\frac{Re}{\pi}P_{110}' \tag{4.10}$$

$$v_{110}''' + Rev_{110}'' - (\pi^2 + Rei\omega)v_{110}' = -\pi ReP_{110} \tag{4.11}$$

First-order equations:

$$v_{111}'' + Rev_{111}' - (\pi^2 + Rei\omega)v_{111} + Rev_{110}''' - i\omega Rev_{110}'' - \pi^2 Rev_{110}' + \pi^2 i\omega Rev_{110} = -\frac{Re}{\pi}P_{111}' \tag{4.12}$$

$$v_{111}''' + Rev_{111}'' - (\pi^2 + Rei\omega)v_{111}' + Rev_{110}'''' - i\omega Rev_{110}''' - \pi^2 Rev_{110}'' + \pi^2 i\omega Rev_{110}' = -\pi ReP_{111} \tag{4.13}$$

The solutions of these equations are

$$v_{110} = \frac{1}{\pi - \xi}\left(-\frac{\xi}{\pi}e^{-\pi y} + e^{-\xi y}\right) \tag{4.14}$$

$$v_{111} = \xi_4 e^{-\pi y} + \xi_5 e^{-\xi y} \tag{4.15}$$

$$P_{110} = -\frac{\xi}{\pi Re(\pi - \xi)}\left(\xi_1 e^{-\pi y} + \xi_2 e^{-\xi y}\right) \tag{4.16}$$

$$P_{111} = \xi_6 e^{-\pi y} + \xi_7 e^{-\xi y} \tag{4.17}$$

Hence the solutions for the velocity components v_1, w_1 and pressure p_1 are as follows

$$v_1 = -\pi\left[\frac{1}{\pi - \xi}\left(-\frac{\xi}{\pi}e^{-\pi y} + e^{-\xi y}\right) + K\left(\xi_4 e^{-\pi y} + \xi_5 e^{-\xi y}\right)\right]cos\pi z$$

$$w_1 = \left[\frac{\xi}{\pi - \xi}\left(e^{-\pi y} + e^{-\xi y}\right) + K\left(-\pi\xi_4 e^{-\pi y} - \xi\xi_5 e^{-\xi y}\right)\right]sin\pi z$$

$$p_1 = Re^2\left[-\frac{\xi}{\pi Re(\pi - \xi)}\left(\xi_1 e^{-\pi y} + \xi_2 e^{-\xi y}\right) + K\left(\xi_6 e^{-\pi y} + \xi_7 e^{-\xi y}\right)\right]cos\pi z$$

5. SOLUTION FOR FIRST ORDER FLOW, CONCENTRATION AND TEMPERATURE FIELD

We now consider the equations (3.12), (3.15) and (3.16). The solutions for velocity components u, temperature field θ and concentration field $\emptyset$ are also separated into mean and sinusoidal components u_1, θ_1 and $\emptyset_1$. To reduce the partial differential equations (3.12), (3.15) and (3.16) into ordinary differential equations, we consider the following assumptions for u_1, θ_1 and $\emptyset_1$.

$$u_1 = u_{11}(y)cos\pi z \tag{5.1}$$

$$\theta_1 = \theta_{11}(y)cos\pi z \tag{5.2}$$

$$\phi_1 = \phi_{11}(y)cos\pi z \tag{5.3}$$

Substituting the above expressions in equations (3.12), (3.15) and (3.16), the following ordinary differential equations are derived.

$$Ku_{11}''' + \left(\frac{1}{Re} - Ki\omega\right)u_{11}'' + (K\pi^2 + 1)u_{11}' + \left(-\frac{\pi^2}{Re} - i\omega + K\pi^2 i\omega\right)u_{11}$$
$$= -Gr\theta_{11} - Gm\phi_{11} - \pi v_{11}u_0'$$
$$+ K\left(\pi v_{11}u_0''' + 2\pi u_0''v_{11}' + \pi u_0' v_{11}''\right.$$
$$\left. + \pi^2 u_0' v_{11}\right) \tag{5.4}$$

$$\theta_{11}'' + PrRe\theta_{11}' - (\pi^2 + QRe + i\omega PrRe)\theta_{11} = \pi PrRe\alpha e^{-\alpha y}v_{11} \tag{5.5}$$

$$\phi_{11}'' + ScRe\phi_{11}'' - (\pi^2 + i\omega ScRe)\phi_{11} = -\pi ScRe v_{11}\phi_0' - SrSc\left(\theta_{11}'' - \pi^2\theta_{11}\right) \tag{5.6}$$

with boundary conditions:

$$y = 0: \ u_{11} = 0, \theta_{11} = 0, \phi_{11} = 0 \tag{5.7}$$

$$y \to \infty: u_{11} = 0, \theta_{11} = 0, \phi_{11} = 0 \tag{5.8}$$

The solutions of the equations (5.5) and (5.6) subject to the boundary conditions are as follows:

$$\theta_{11} = B_1 e^{-(\alpha+\pi)y} + B_2 e^{-(\alpha+\xi)y} + B_3 e^{-hy}$$

$$\phi_{11} = C_1 e^{-(\pi+ScRe)y} + C_2 e^{-(\pi+\alpha)y} + C_3 e^{-(\xi+ScRe)y} + C_4 e^{-(\xi+\alpha)y} + C_5 e^{-hy} + C_6 e^{-m_2 y}$$

To solve equation (5.4), we assume K to be very small and the velocity and temperature in the neighbourhood of the plate as

$$u_{11} = u_{110} + Ku_{111} \tag{5.9}$$

Zeroth order equation:

$$\frac{1}{Re}u_{110}'' + u_{110}' - \left(\frac{\pi^2}{Re} + i\omega\right)u_{110} = -Gr\theta_{11} - Gm\phi_{11} - \pi v_{11}u_{00}' \tag{5.10}$$

First order equation:

$$u_{110}''' + \frac{1}{Re}u_{111}'' - i\omega u_{110}'' + \pi^2 u_{110}' + u_{111}' - \left(\frac{\pi^2}{Re} + i\omega\right)u_{111} + \pi^2 i\omega u_{110}$$
$$= -\pi v_{110}u_{01}' - \pi v_{111}u_{00}' + \pi v_{110}u_{00}''' + 2\pi u_{00}''v_{00}' + \pi u_{00}'v_{110}'' + \pi^2 u_{00}'v_{110} \tag{5.11}$$

subject to the boundary conditions are:

$$y = 0: \ u_{110} = 0, \quad u_{111} = 0 \tag{5.12}$$

$$y \to \infty: u_{110} = 0, \quad u_{111} = 0 \tag{5.13}$$

Now, solving equation (5.10) and (5.11) subject to the boundary conditions (5.12), (5.13), we get

$$u_{110} = H_1 e^{-hy} + H_2 e^{-(\xi+\alpha)y} + H_3 e^{-(\pi+\alpha)y} + H_4 e^{-(\pi+ScRe)y} + H_5 e^{-(\xi+ScRe)y} + H_6 e^{-m_2 y} + H_7 e^{-(\pi+Re)y} + H_8 e^{-(\xi+Re)y} + H_9 e^{-\xi y} \quad (5.14)$$

$$u_{111} = G_1 e^{-hy} + G_2 e^{-(\xi+\alpha)y} + G_3 e^{-(\pi+\alpha)y} + G_4 e^{-(\pi+ScRe)y} + G_5 e^{-(\xi+ScRe)y} + G_6 e^{-m_2 y} + G_7 e^{-(\pi+Re)y} + G_8 e^{-(\xi+Re)y} + G_9 e^{-\xi y} \quad (5.15)$$

With the use of above solutions, the velocity component is expressed by

$$u_1 = \left(M_0 e^{-\xi y} + M_1 e^{-hy} + M_2 e^{-(\xi+\alpha)y} + M_3 e^{-(\pi+\alpha)y} + M_4 e^{-(\pi+ScRe)y} + M_5 e^{-(\xi+ScRe)y} + M_6 e^{-m_2 y} + M_7 e^{-(\pi+Re)y} + M_8 e^{-(\xi+Re)y}\right) cos\pi z \quad (5.16)$$

6. SKIN FRICTION AND NUSSELT NUMBER AT THE PLATE

The non-dimensional skin-friction at the plate in the direction of the free stream is given by

$$\tau = \left[\frac{\partial u}{\partial y} - ReK\left(\frac{\partial^2 u}{\partial t \partial y} + v\frac{\partial^2 u}{\partial y^2} + w\frac{\partial^2 u}{\partial y \partial z} - 3\frac{\partial u}{\partial y}\frac{\partial v}{\partial y} - 2\frac{\partial u}{\partial z}\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y}\frac{\partial u}{\partial z}\right)\right]_{y=0}$$

$$= N_0 + N_1 \varepsilon e^{i\omega t} + N_2 \varepsilon i\omega e^{i\omega t} cos\pi z + N_3 \varepsilon e^{i\omega t} cos\pi z$$

The non dimensional heat flux at the plate $\bar{y} = 0$ in terms of Nusselt number Nu is given by

$$N_u = -\frac{1}{PrRe}\left[\theta_0'(0) + \varepsilon\theta_{11}'(0) cos\pi z e^{i\omega t}\right]$$

$$= \frac{1}{PrRe}\left[\alpha + \varepsilon\{hB_3 - B_2(\alpha+\xi) - B_1(\alpha+\pi)\} cos\pi z e^{i\omega t}\right]$$

8. THE CO-EFFICIENT OF RATE OF MASS TRANSFER

The mass transfer at the wall $\bar{y} = 0$ in terms of Sherwood number Sh is given by

$$Sh = \frac{-D_M}{v_0(\bar{C}_w - \bar{C}_\infty)}\left(\frac{\partial \bar{C}}{\partial \bar{y}}\right)_{\bar{y}=0} = \frac{-1}{ScRe}\left(\frac{\partial \phi}{\partial y}\right)_{y=0}$$

$$= \frac{1}{ScRe}\left[ScRe(1-a) + \alpha a + \varepsilon cos\pi z e^{i\omega t}\{C_1(\pi + ScRe) + C_2(\pi+\alpha) + C_3(\xi + ScRe) + C_4(\xi+\alpha) - C_5 h - C_6 m\}\right]$$

9. CURRENT DENSITY

The current density $\vec{J}$ is given by

$$\vec{J} = \sigma(\vec{q} \times \vec{B}) = \sigma B_0(\hat{\imath}\bar{w} + \hat{k}\bar{u})$$

The magnitude of $\vec{J}$ is given by

$$|\bar{J}| = \sigma B_0 \sqrt{\bar{u}^2 + \bar{w}^2} = \sigma B_0 v_0 \sqrt{u^2 + w^2}$$

The current density (in magnitude) in non-dimensional form is given by

$$J_c = \frac{|\bar{J}|}{\sigma B_0 v_0} = \sqrt{u^2 + w^2} = u\sqrt{1 + \left(\frac{w}{u}\right)^2} = u \quad \left(\because \frac{w}{u} < 1\right)$$

That is the magnitude of the non-dimensional current density is proportional to the boundary layer velocity.

10. RESULT AND DISCUSSION

The purpose of this present study is to bring out the effects of elastico-viscous parameter on the oscillatory three dimensional flow of a visco-elastic fluid past an infinite vertical porous surface by mass transfer and heat sink. The elastico-viscous effect is exhibited through the non-dimensional parameter K. The non zero values of the parameter K characterize the visco-elastic fluid and K=0 represents the Newtonian fluid flow phenomenon.

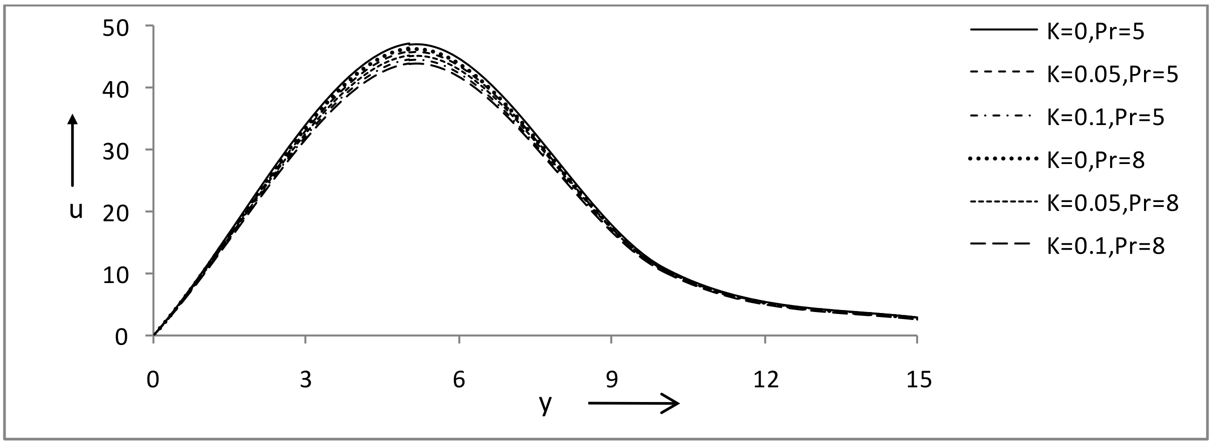

Figure 1: Variation of transient velocity u against y for Re=0.6, Sr=2, Sc=0.6, Gr=10, Q=1, ω=1.

The numerical calculations are to be carried out for free stream velocity U=1, Grashof number for mass transfer Gm=15, the frequency of oscillation ε =0.001, ω and t are chosen in such a way that $\omega t = \pi/2$ throughout the discussion. Figure 1-5 depict the velocity profile u against y for various values of the flow parameters involved in the solution. All the figures reveal that the velocity profile first enhances to a considerable extent and then diminishes for K≥0.

In fluid flow problems, the importance of Prandtl number cannot be ignored, as it studies the simultaneous behaviour of both momentum and thermal diffusions. Figure 1 represents the effect of Prandtl number on velocity profile. The rising value of Prandtl number raises the thickness of the fluid and hence the fluid experiences a decelerating trend. This physical phenomenon is observed in both Newtonian and elastico-viscous fluid. Again, the decelerating trend is observed in the velocity profile with the growth of elastico-viscous parameter as well as Prandtl number.

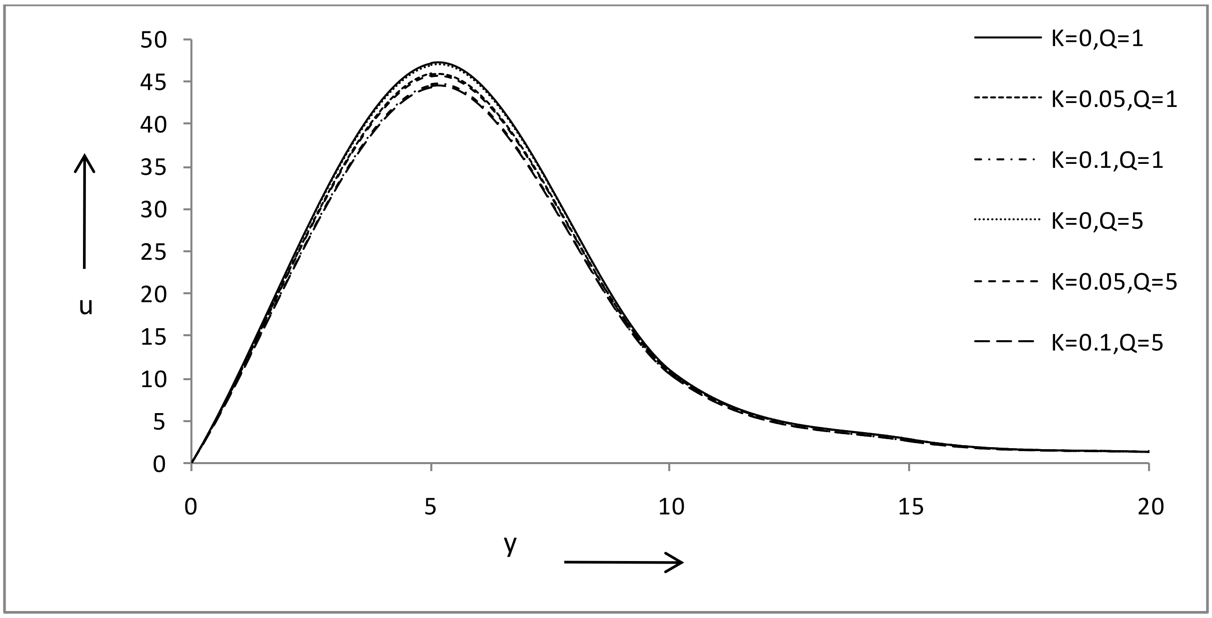

Figure 2: Variation of transient velocity u against y for Re=0.6, Pr=5, Sr=2, Sc=0.6, Gr=10, ω=1.

It is inferred from figure 2 that the velocity quickly increases up to some extent adjacent to the plate and after this the fluid velocity diminishing asymptotically towards 1 as $y \to \infty$. Also, the variations of heat source parameter Q and elastico-viscous parameter K reveal the deceleration pattern of the velocity profile.

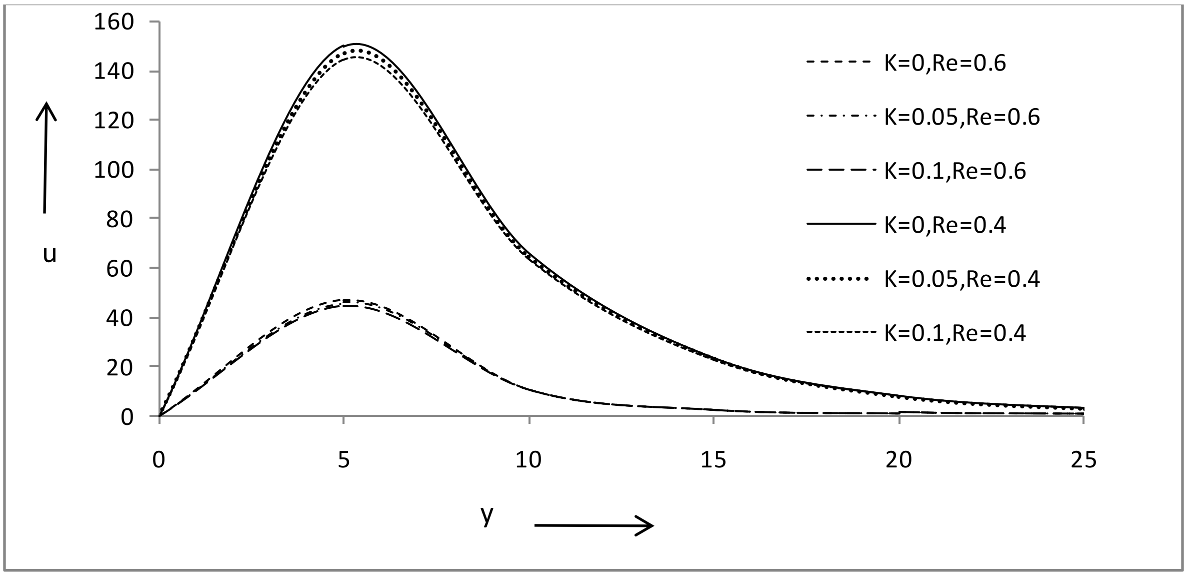

Figure 3: Variation of transient velocity u against y for Pr=5, Sr=2, Sc=0.6, Gr=10, Q=1, ω=1.

Figure 3 illustrates the behaviour of fluid flow for different values of Reynolds number with the variation of K. The impact of Reynolds number is very significant in case of visco-elastic fluid. The Reynolds number gives a measure of the ratio of inertial forces to viscous forces and consequently, it quantifies the relative importance of these two types of forces for given flow conditions. In the present study we see a large difference for small change in Reynolds number and the same diminishing trend is observed in Newtonian as well as visco-elastic fluid. The growth of visco-elastic parameter diminish the velocity profile with the variation of Reynolds number Re.

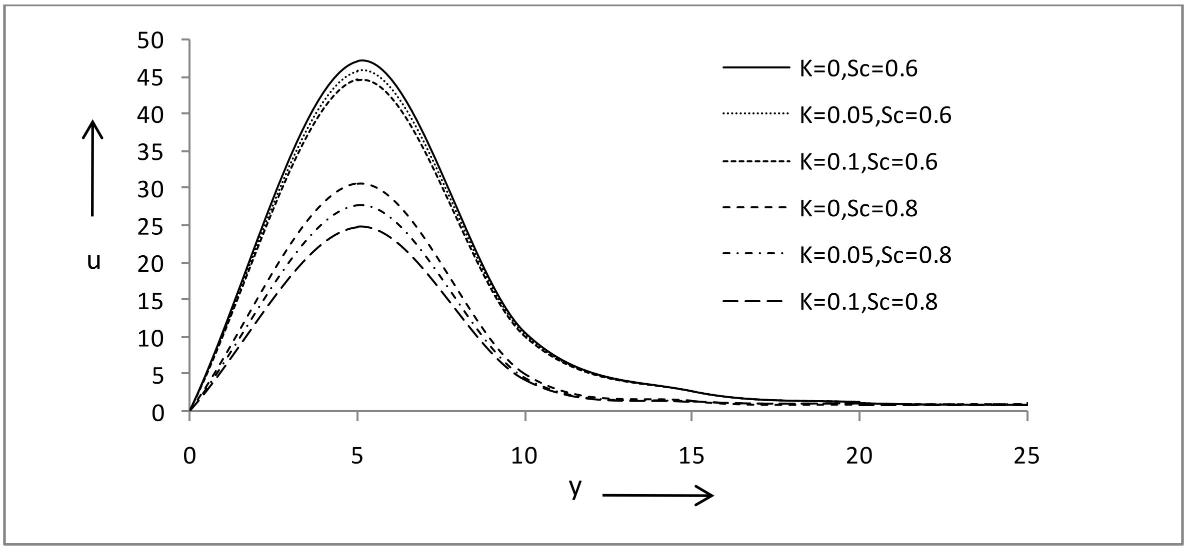

Figure 4: Variation of transient velocity u against y for Re=0.6, Pr=5, Sr=2, Gr=10, Q=1, ω=1.

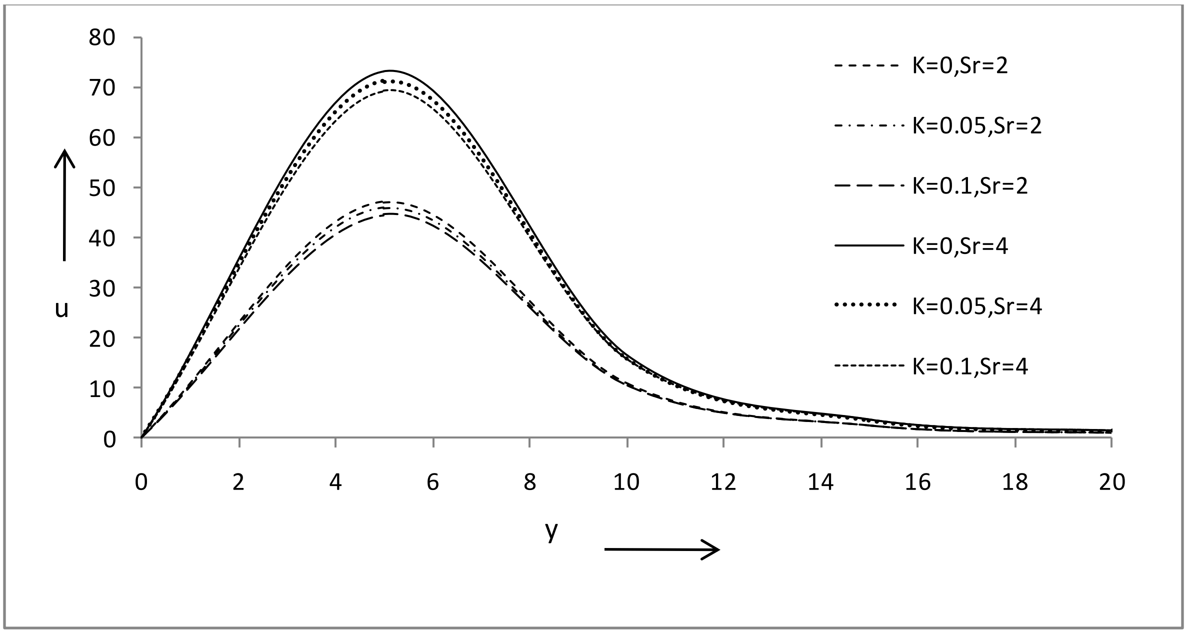

Figure 5: Variation of transient velocity u against y for Re=0.6, Pr=5, Sc=0.6, Gr=10, Q=1, ω=1.

The effects of Schimdt number and Soret number on the oscillatory elastico-viscous fluid and Newtonian fluid flow are exhibited in figures 4 and 5. The Schmidt number Sc is a dimensionless number defined as the ratio of momentum diffusivity (viscosity) and mass diffusivity. It is used to characterize fluid flows in which there are simultaneous momentum and mass diffusion convection processes. Figure 4 depicts that rising trend of Schmidt number diminish the speed of the fluid velocity whereas reverse behaviour is noticed with the growth of Soret number (figure 5).In both the figures , the increasing values of elastico-viscous parameter decelerate the fluid velocity.

Figures 6 to 11, have analyzed the nature of viscous drag formed by the fluids at the plate. The elasticity factor present in the visco-elastic fluid subdues the shearing stress at the plate in comparison with Newtonian fluid.

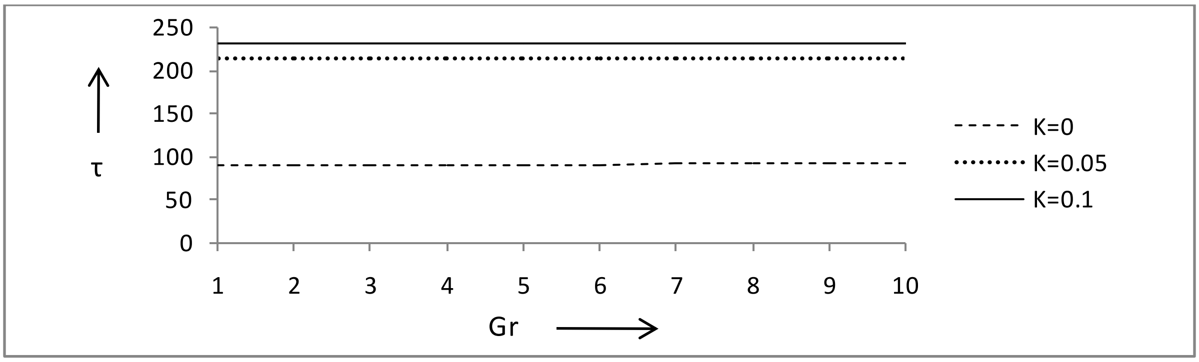

Figure 6: Skin friction versus Gr for Re=0.6, Pr=5, Sr=2, Sc=0.6, Q=1, ω=1.

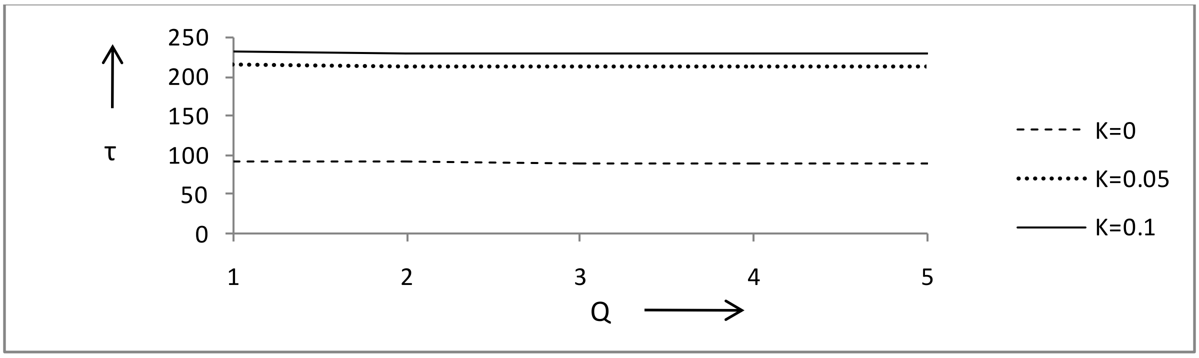

Figure 7: Skin friction versus Q for Re=0.6, Pr=5, Sr=2, Sc=0.6, Gr=10, ω=1.

Figures 6 and 7 show the respective behaviour of skin friction against Grashof number Gr (for externally cooled plate) and heat sink for Newtonian as well as non-Newtonian fluid flows. Grashof number studies the behaviour of free convection and it is defined as the ratio of buoyancy force to viscous force. It plays an important role in both heat and mass transfer mechanisms. In both the cases it is experienced that a steady growth of the skin friction at the plate due to the increase of the flow parameters (Grashof number and heat sink parameter). With the rise of elastico-viscous parameter the skin friction shows the nature of enhancement in both the figures.

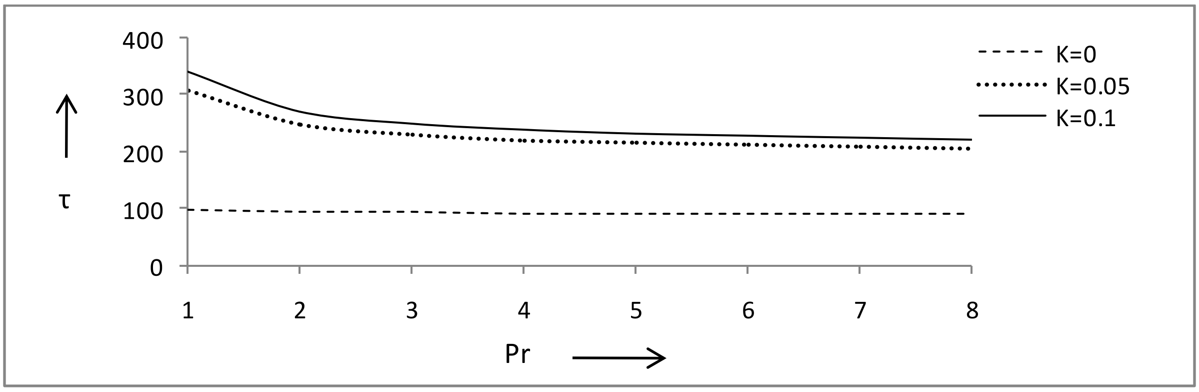

Figure 8: Skin friction versus Pr for Re=0.6, Sr=2, Sc=0.6, Gr=10, Gm=15, Q=1, ω=1.

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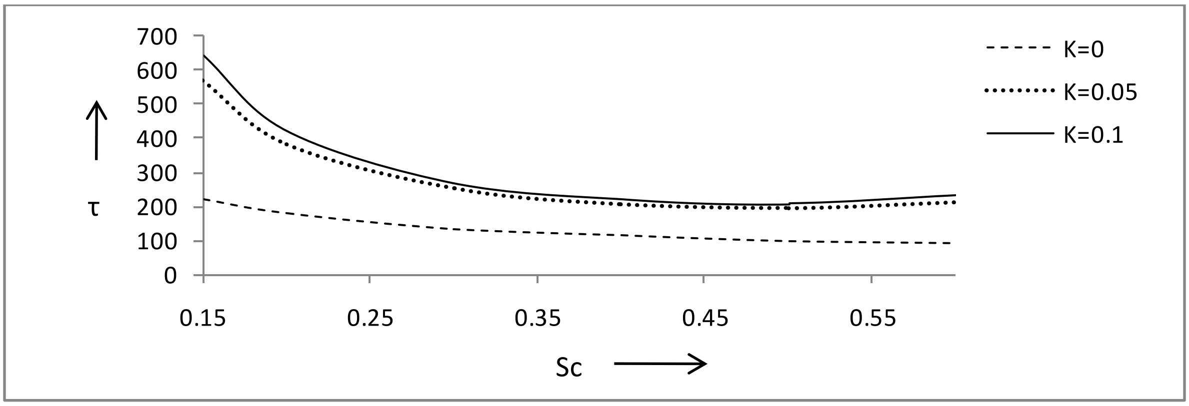

Figure 9: Skin friction versus Sc for Re=0.6, Pr=5, Sr=2,Gr=10, Q=1, ω=1.

Figures 8 and 9 exhibit the consequence of Prandtl number and Schimdt number respectively on the fluid flow. In both the cases skin friction rises with the growth of elastico-viscous parameter.

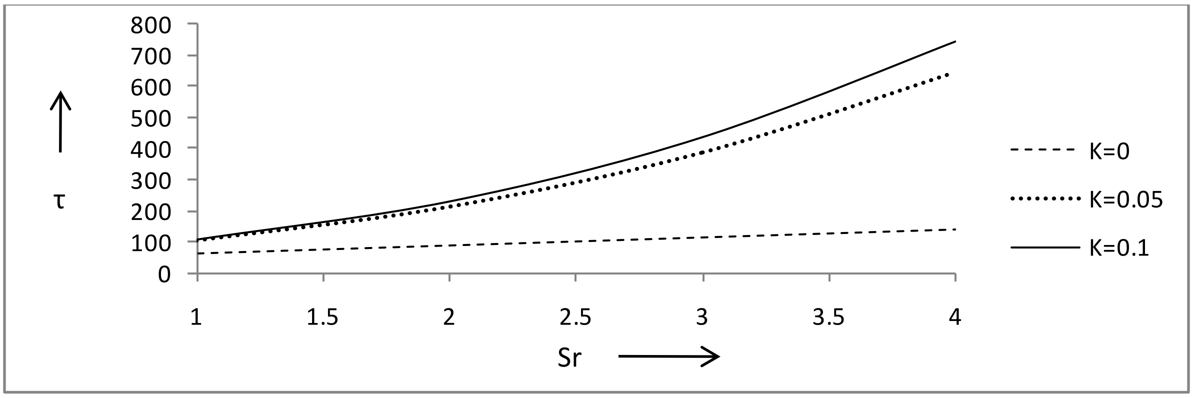

Figure 10: Skin friction versus Sr for Re=0.6, Pr=5, Sc=0.6, Gr=10, Q=1, ω=1.

Figure 10 characterizes the variation of shearing stress against Sr. The figure exhibits the increase of viscous drag with the increase of Sr for non-Newtonian flow. Simple fluid flow also shows slight swelling of shearing stress.

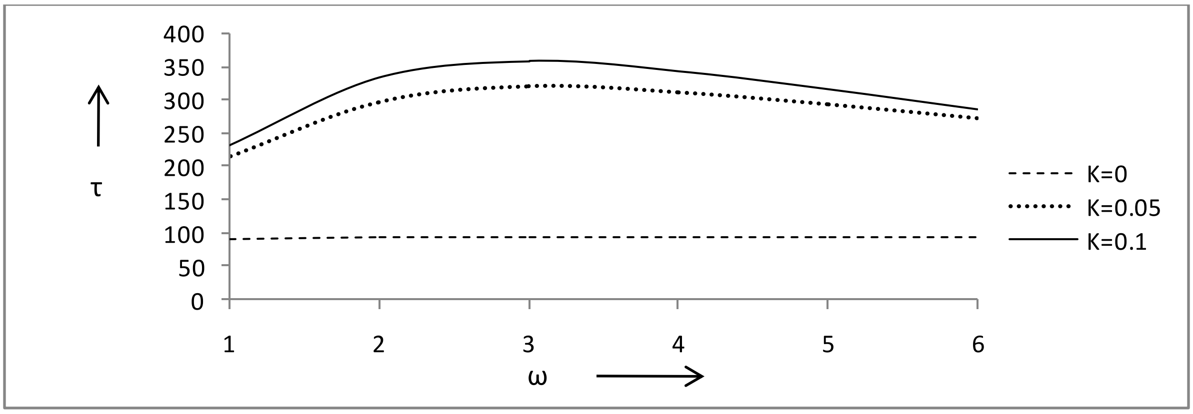

Figure 11: Skin friction versus ω for Re=0.6, Pr=5, Sr=2, Sc=0.6, Gr=10, Q=1.

Figure 11 shows the effect of frequency parameter on the governing flow. It is observed that magnitude of viscous drag first enhances and then retarded with the increase of frequency parameter for K>0 but a steady flow for Newtonian fluid flow mechanism.

The temperature profile and rate of heat transfer are not affected significantly during the changes made in visco-elasticity of the fluid flow. Constants are obtained but not given due to brevity.

11. CONCLUSIONS

The unsteady three dimensional oscillatory Walters liquid (Model $B^{/}$) past an infinite vertical plate in presence of heat and mass transfer has been investigated. Some of the important points are enlisted as below:

- The velocity profile first enhances and then diminishes in both Newtonian and non-Newtonian cases.
- The visco-elasticity factor decelerates the speed of fluid flow in comparison with the Newtonian fluid.
- The shearing stress formed at the plate is subdued with the growing trend of visco-elastic parameter.
- A declined trend of skin friction is observed in case of rising values of Schimdt number Sc and Prandtl number Pr with that of visco elastic parameter.
- The viscous drag slightly enhances with the increasing values of Soret number (Sr) in both fluid flow mechanism.
- The skin friction first increases and then decreases with the increasing values of frequency parameter. With that of increasing value of K when K>0 but for K=0 a steady formation is observed.

REFERENCES

[1] Ahmed N and Goswami J K. Effect of thermal diffusion on an oscillatory three dimensional flow with mass transfer past an infinite vertical porous plate in presence of a heat sink. Int. J. of Appl. Math and Mech. 2011, 7, 4, 29-52.

[2] Bejan A and Khair KR. Heat and mass transfer in a porous medium. Int. J. Heat Transfer. 1985, 28,902-918,

[3] Choudhury RC and Chand T. Three dimensional flow and heat transfer through porous medium. Int. J. Appl. Mech. Engineering.2002, 7, 4, 1141-1156.

[4] Choudhury R and Das A. Three dimensional free convective flow and heat transfer in a visco-elastic fluid through a porous medium. Int. J. App. Math. Analysis and Appl. 2006, 1, 1, 87-100.

[5] Choudhury R and Dey D. Free convective visco-elastic flow with geat and mass transfer through a porous medium with periodic permeability. Int. J. Heat and mass transfer (Elsevier). 2010, 53, 1666-1672.

[6] Choudhury R and Mahanta M. MHD mixed convective oscillatory flow of visco-elastic fluid in a porous channel. JP. J. Heat and Mass transfer. 6(2), 177-190, 2012.

[7] Choudhury R and Das UJ. Heat transfer to MHD oscillatory visco-elastic flow ub a channel filled with porous media. Phy. Res. Int. Ar ID 879537, 5 pages, 2012.

[8] Choudhury R and Debnath K, Unsteady oscillatory flow of a visco-elastic fluid past a porous vertical plate with a periodic suction. Int. J. Math. Archive, 3(5), 1764-1771, 2012.

[9] Jain NC and Gupta P. Three dimensional free convection Couette flow with transpiration cooling. J. Zhejiang University.2006, A, 7, 3, 340-346.

[10] Lachmann GV (1961). Boundary layers and flow control. Its properties and applications, Vol. I and II, Pergamon Press.

[11] Raju MC, Varma SVK, Reddy PV and Saha S. Soret effects due to natural convection between heated inclined plates with magnetic field. J. Mech. Eng. 2008, 39, 2, 65-70.

[12] Raptis A and Kafousias N. Magnetohydrodynamic free convective flow and mass transfer through a porous medium bounded by and infinite vertical porous plate with constant heat flux. Can. J. Phy. 1982, 60, 1725-1729 .

[13] Sattar MA and Alam MM. Thermal diffusion as well as transpiration effects on MHD free convection and mass transfer flow past an accelerated vertical porous plate. Indian J. Pure. Appl. Math. 1994, 25, 6, 679-688

[14] Singh NP, Singh Atul K and Singh Ajay K.MHD free convection MHD mass transfer flow past of flat plate. The Arabian J. Sci. Eng. 2007, 32, 1A, 93-112.

[15] Singh P, Misra JK and Narayan KA. Three dimensional convective flow and heat transfer in a porous medium. Indian J. Pure and Appl. Math. 1988, 19, 1130-1135.