

α^* -closed sets in bitopological spaces

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ABSTRACT

In this paper we introduce α^* -closed sets in bitopological spaces. Properties of these sets are investigated and we introduce seven new bitopological spaces namely, $(i,j)\text{-}T_\alpha^*$, $(i,j)\text{-}\alpha_g T_\alpha^*$, $(i,j)\text{-}g_s T_\alpha^*$, $(i,j)\text{-}g T_\alpha^*$, $(i,j)\text{-}g_{sp} T_\alpha^*$, $(i,j)\text{-}g_p T_\alpha^*$, $(i,j)\text{-}g_{pr} T_\alpha^*$.

Key words: $(i,j)\text{-}\alpha^*$ -closed sets, $(i,j)\text{-}T_\alpha^*$ spaces, $(i,j)\text{-}\alpha_g T_\alpha^*$ spaces, $(i,j)\text{-}g_s T_\alpha^*$ spaces, $(i,j)\text{-}g T_\alpha^*$ spaces, $(i,j)\text{-}g_{sp} T_\alpha^*$ spaces, $(i,j)\text{-}g_p T_\alpha^*$ spaces, $(i,j)\text{-}g_{pr} T_\alpha^*$ spaces.

1. INTRODUCTION

A triple (X, τ_1, τ_2) where X is a non-empty set and τ_1 and τ_2 are topologies in X is called a bitopological space and Kelly [7] initiated the study of such spaces. In 1985, Fukutake [3] introduced the concepts of g -closed sets in bitopological spaces. Levine [9] introduced the class of generalized closed sets, a super class of closed sets in 1970. M. K. R. S. Veerakumar [17] introduced and studied the concepts of g^* -closed sets and g^* -continuity in topological spaces. Sheik John. M and Sundaram. P [14] introduced and studied the concepts of g^* -closed sets in bitopological spaces in 2002. Pauline Mary Helen, Ponnuthai Selvarani and Veronica Vijayan [13] introduced g^{**} -closed sets in topological spaces in 2012. The purpose of this paper is to introduce the concepts of $(i,j)\text{-}\alpha^*$ -closed sets, $(i,j)\text{-}T_\alpha^*$ spaces, $(i,j)\text{-}\alpha_g T_\alpha^*$ spaces, $(i,j)\text{-}g_s T_\alpha^*$ spaces, $(i,j)\text{-}g T_\alpha^*$ spaces, $(i,j)\text{-}g_{sp} T_\alpha^*$ spaces, $(i,j)\text{-}g_p T_\alpha^*$ spaces, $(i,j)\text{-}g_{pr} T_\alpha^*$ spaces in bitopological spaces and investigate some of their properties.

2. PRELIMINARIES

Definition 2.1

A subset A of a topological space (X, τ) is said to be

1. a pre-open set [10] if $A \subseteq \text{int}(\text{cl}(A))$ and a pre-closed set if $\text{cl}(\text{int}(A)) \subseteq A$
2. a semi-open [8] if $A \subseteq \text{cl}(\text{int}(A))$ and a semi-closed set if $\text{int}(\text{cl}(A)) \subseteq A$
3. a regular open set [10] if $A = \text{int}(\text{cl}(A))$

4. a generalized closed set[9] (briefly g-closed set) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .
5. a α -open set [11] if $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$ and an α -closed if $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$
6. a semi-preopen set[1] if $A \subseteq \text{cl}(\text{int}(\text{cl}(A)))$ and a semi – preclosed set if $\text{int}(\text{cl}(\text{int}(A))) \subseteq A$.
7. a α^* -closed set [18] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is α -open in (X, τ) .

If A is a subset of X with topology τ , then the closure of A is denoted by $\tau\text{-cl}(A)$ or $\text{cl}(A)$, the interior of A is denoted by $\tau\text{-int}(A)$ or $\text{int}(A)$ and the complement of A in X is denoted by A^c .

For a subset A of (X, τ_i, τ_j) , $\tau_j\text{-cl}(A)$ (resp. $\tau_i\text{-int}(A)$) denotes the closure (resp. interior) of A with respect to the topology τ_j .

Definition 2.2

A subset A of a topological space (X, τ_i, τ_j) is called

1. (i, j) – g-closed[3] if $\tau_j\text{-cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in τ_i .
2. (i, j) – rg-closed [12] if $\tau_j\text{-cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in τ_i .
3. (i, j) – gpr-closed [5] if $\tau_j\text{-pcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in τ_i .
4. (i, j) – ω g-closed [4] if $\tau_j\text{-cl}(\tau_i\text{-int}(A)) \subseteq U$ whenever $A \subseteq U$ and U is open in τ_i .
5. (i, j) – ω -closed [6] if $\tau_j\text{-cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in τ_i .
6. (i, j) – gs-closed[16] if $\tau_j\text{-scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in τ_i .
7. (i, j) – gsp-closed[2] if $\tau_j\text{-spcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in τ_i .
8. (i, j) – α g-closed[16] if $\tau_j\text{-}\alpha\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in τ_i .

Definition 2.3

A bitopological space (X, τ_i, τ_j) is called

1. an (i, j) – $T_{1/2}$ space[3] if every (i, j) –g-closed set is τ_j –closed.
2. an (i, j) – T_b space [16] if every (i, j) –gs-closed set is τ_j –closed.
3. an (i, j) – T_d space [16] if every (i, j) –gs-closed set is (i, j) –g-closed.
4. an (i, j) – ${}_aT_d$ space [3] if every (i, j) – α g-closed set is (i, j) –g-closed.
5. an (i, j) – ${}_aT_b$ space [16] if every (i, j) – α g-closed set is τ_j –closed.

3. (i, j) – α^* -closed sets

We introduce the following definition.

Definition 3.1 A subset A of a topological space (X, τ_1, τ_2) is said to be an (i, j) - α^* -closed set if

$\tau_j\text{-cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is α -open in τ_i . We denote the family of all (i, j) - α^* -closed sets in (X, τ_1, τ_2) by $\alpha^*C(i, j)$.

Remark 3.2 By setting $\tau_1 = \tau_2$ in definition (3.1), a (i, j) - α^* -closed set is a α^* -closed set.

Proposition 3.3 Every τ_j -closed subset of (X, τ_1, τ_2) is (i, j) - α^* -closed.

The converse of the above proposition is not true as seen in the following example.

Example 3.4 Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, \{c\}, \{a, c\}, X\}$ and $\tau_2 = \{\emptyset, \{a\}, X\}$. Then the set $A = \{b\}$ is $(1, 2)$ - α^* -closed but not τ_2 -closed in (X, τ_1, τ_2) .

Proposition 3.5 If A is (i, j) - α^* -closed and τ_i - α -open, then A is τ_j - α -closed.

Proposition 3.6 If A is both (i, j) - α^* -closed and τ_i - α -open, then it is τ_j -closed.

Proposition 3.7 In a bitopological space (X, τ_1, τ_2) every (i, j) - α^* -closed set is

- (i) (i, j) -g-closed
- (ii) (i, j) -rg-closed (iii) (i, j) -gpr-closed (iv) (i, j) - ω g-closed.

The following examples show that the converse of the above proposition is not true.

Example 3.8 Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, \{a\}, X\}$ and $\tau_2 = \{\emptyset, \{a\}, \{a, b\}, X\}$. Then the set $A = \{b\}$ is $(1, 2)$ -g-closed but not $(1, 2)$ - α^* -closed.

Example 3.9 Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, \{a, b\}, X\}$ and $\tau_2 = \{\emptyset, \{a\}, X\}$. Then the set $A = \{b\}$ is $(1, 2)$ -rg-closed but not $(1, 2)$ - α^* -closed.

Example 3.10 Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, \{c\}, \{a, b\}, X\}$ and $\tau_2 = \{\emptyset, \{a\}, X\}$. Then the set $A = \{c\}$ is $(1, 2)$ -gpr-closed but not $(1, 2)$ - α^* -closed.

Example 3.11 Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, \{a, b\}, X\}$ and $\tau_2 = \{\emptyset, \{a\}, X\}$. Then the set $A = \{a\}$ is $(1, 2)$ - ω g-closed but not $(1, 2)$ - α^* -closed.

Theorem 3.12 Every (i, j) - α^* -closed set is a (i, j) -gs-closed set.

The converse of the above theorem need not be true.

Example 3.13 Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, \{a\}, X\}$ and $\tau_2 = \{\emptyset, \{a\}, \{a, b\}, X\}$. Then the set $A = \{b\}$ is $(1, 2)$ -gs-closed but not $(1, 2)$ - α^* -closed.

Theorem 3.14 Every (i, j) - α^* -closed set is a (i, j) -gp-closed set.

The converse of the above is not true as seen in the following example.

Example 3.15 Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, \{c\}, \{a, b\}, X\}$ and $\tau_2 = \{\emptyset, \{a\}, X\}$. Then the set $A = \{b\}$ is $(1, 2)$ -gp-closed but not $(1, 2)$ - α^* -closed.

Theorem 3.16 Every (i, j) - α^* -closed set is a (i, j) -g α -closed set.

The converse of the above theorem need not be true.

Example 3.17 Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, \{a, b\}, X\}$ and $\tau_2 = \{\emptyset, \{a\}, X\}$. Then the set $A = \{b\}$ is $(1, 2)$ -g α -closed but not $(1, 2)$ - α^* -closed.

Theorem 3.18 Every (i, j) - α^* -closed set is a (i, j) -g α -closed set.

The following example support that the converse of the above theorem is not true.

Example 3.19 Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, \{a, b\}, X\}$ and $\tau_2 = \{\emptyset, \{a\}, X\}$. Then the set $A = \{b\}$ is $(1, 2)$ -g α -closed but not $(1, 2)$ - α^* -closed.

Theorem 3.20 Every (i, j) - α^* -closed set is a (i, j) -gsp-closed set.

The converse of the above is not true as seen in the following example.

Example 3.21 Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, \{a, b\}, X\}$ and $\tau_2 = \{\emptyset, \{a\}, X\}$. Then the set $A = \{b\}$ is $(1, 2)$ -gsp-closed but not $(1, 2)$ - α^* -closed.

Theorem 3.22 Every (i, j) - ω -closed set is a (i, j) - α^* -closed set.

The converse of the above is not true as seen in the following example.

Example 3.23 Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, \{c\}, \{a, c\}, X\}$ and $\tau_2 = \{\emptyset, \{a\}, X\}$. Then the $A = \{b\}$ is $(1, 2)$ - α^* -closed but not $(1, 2)$ - ω -closed.

Proposition 3.24 If $A, B \in \alpha^*C(i, j)$ then $A \cup B \in \alpha^*C(i, j)$

Remark 3.25 The intersection of two (i,j) - α^* -closed set need not be (i,j) - α^* -closed as seen from the following example.

Example 3.26 Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, \{a\}, \{b, c\}, X\}$ and $\tau_2 = \{\emptyset, \{b\}, \{b, c\}, \{c\}, \{a, c\}, X\}$. Let $A = \{a, b\}$ and $B = \{b, c\}$. Then A and B are $(2,1)$ - α^* -closed sets but $A \cap B = \{b\}$ is not a $(2,1)$ - α^* -closed set.

Remark 3.27 $\alpha^*C(1,2)$ is generally not equal to $\alpha^*C(2,1)$

Example 3.28 In example (3.26), $A = \{b\} \notin \alpha^*C(2,1)$ but $A = \{b\} \in \alpha^*C(1,2)$.

Hence $\alpha^*C(2,1) \neq \alpha^*C(1,2)$

Proposition 3.29 If $\tau_1 \subseteq \tau_2$ in (X, τ_1, τ_2) , then $\alpha^*C(1,2) \subseteq \alpha^*C(2,1)$

Proposition 3.30 If A is a (i,j) - α^* -closed, then $\tau_j \text{-Cl}(A) \setminus A$ contains no non-empty τ_i - α -closed set.

Proof Let A be a (i,j) - α^* -closed set and let F be a τ_i - α -closed set such that $F \subseteq \tau_j \text{-Cl}(A) \setminus A$. Since $A \in \alpha^*C(i,j)$, we have $\tau_j \text{-Cl}(A) \subseteq F^c \therefore F \subseteq (\tau_j \text{-Cl}(A))^c$ and hence $F \subseteq (\tau_j \text{-Cl}(A) \cap ((\tau_j \text{-Cl}(A))^c) = \emptyset \therefore F = \emptyset$

The converse of the above proposition is not true as seen in the following example.

Example 3.31 Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, \{b\}, \{c\}, \{b, c\}, \{a, c\}, X\}$ and $\tau_2 = \{\emptyset, \{a\}, \{b, c\}, X\}$

Let $A = \{b\}$. Then $\tau_2 \text{-cl}(A) \setminus A = \{c\}$ is not τ_1 - α -closed. i.e. $\tau_2 \text{-cl}(A) \setminus A$ contains no nonempty

τ_1 - α -closed set but $A = \{b\}$ is not $(1,2)$ - α^* -closed.

Theorem 3.32 If A is (i,j) - α^* -closed in (X, τ_i, τ_j) , then A is τ_j -closed if and only if $\tau_j \text{-Cl}(A) \setminus A$ is τ_i - α -closed.

Proof: Necessity: If A is τ_j -closed then $\tau_j \text{-Cl}(A) = A$ (i.e) $\tau_j \text{-Cl}(A) \setminus A = \emptyset$ which is τ_i - α -closed.

Sufficiency: If $\tau_j \text{-Cl}(A) \setminus A$ is τ_i - α -closed, since A is (i,j) - α^* -closed by proposition 3.29, $\tau_j \text{-Cl}(A) \setminus A$ contains no non empty τ_i - α -closed set. $\therefore \tau_j \text{-Cl}(A) \setminus A = \emptyset$ and hence $A = \tau_j \text{-Cl}(A)$ and A is τ_j -closed.

Theorem 3.33 If A is an (i,j) - α^* -closed set of (X, τ_i, τ_j) such that $A \subseteq B \subseteq \tau_j \text{-Cl}(A)$, then B is also an (i,j) - α^* -closed set of (X, τ_i, τ_j) .

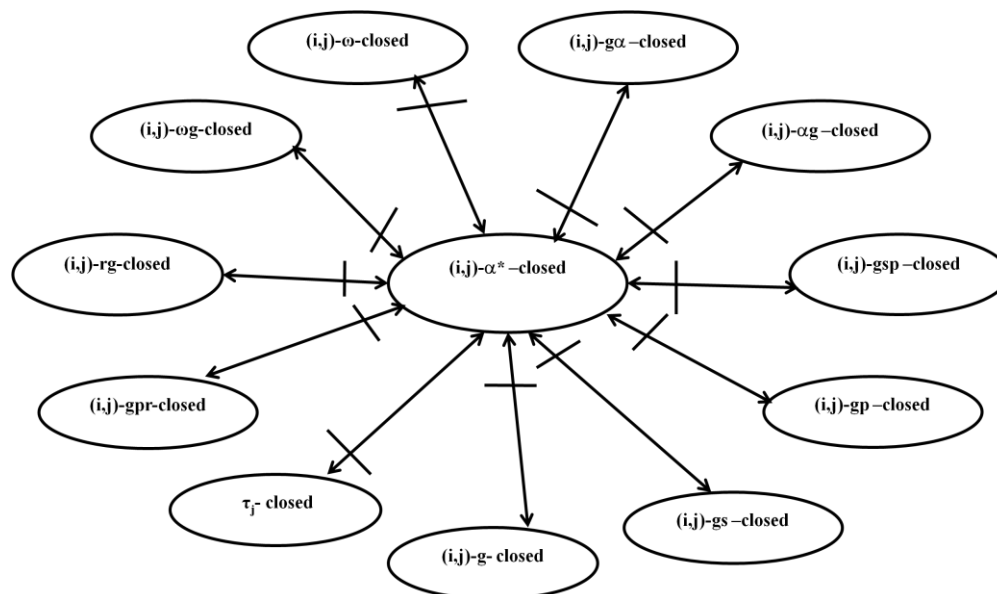
Proof : Let A be a (i,j) - α^* -closed set. Let U be a τ_i - α -open set such that $B \subseteq U$ then $A \subseteq B \subseteq U$. Then

$\tau_j \text{-Cl}(A) \subseteq U$, since A is (i,j) - α^* -closed. $B \subseteq \tau_j \text{-Cl}(A) \Rightarrow \tau_j \text{-Cl}(B) \subseteq \tau_j \text{-Cl}(A) \subseteq U \therefore \tau_j \text{-Cl}(B) \subseteq U$

$\therefore B$ is a (i,j) - α^* -closed set.

Proposition 3.34 For each element x of (X, τ_1, τ_2) , $\{x\}$ is either τ_i - α -closed or $X - \{x\}$ is (i,j) - α^* -closed.

The following figure gives all the above results we have proved.



where $A \longrightarrow B$ represents A implies B and $A \not\longrightarrow B$ represents A does not imply B .

4. Applications of (i,j) - α^* -closed sets

As applications of (i,j) - α^* -closed sets, we introduce seven new bitopological spaces, (i,j) - T_α^* space, (i,j) - $_{\alpha g}T_\alpha^*$ space, (i,j) - $_{gs}T_\alpha^*$ space, (i,j) - $_gT_\alpha^*$ space, (i,j) - $_{gsp}T_\alpha^*$ space, (i,j) - $_{gp}T_\alpha^*$ space and (i,j) - $_{gpr}T_\alpha^*$ space.

We introduce the following definitions.

Definition 4.1 A bitopological space (X, τ_1, τ_2) is said to be an (i,j) - T_α^* space, if every (i,j) - α^* -closed set is τ_j -closed.

Definition 4.2 A bitopological space (X, τ_1, τ_2) is said to be an (i,j) - $_{\alpha g}T_\alpha^*$ space, if every (i,j) - αg -closed set is (i,j) - α^* -closed.

Definition 4.3 A bitopological space (X, τ_1, τ_2) is said to be an (i,j) - $_{gs}T_\alpha^*$ space, if every (i,j) - gs -closed set is (i,j) - α^* -closed.

Definition 4.4 A bitopological space (X, τ_1, τ_2) is said to be an (i,j) - T_α^* space, if every (i,j) -g-closed set is (i,j) - α^* -closed.

Definition 4.5 A bitopological space (X, τ_1, τ_2) is said to be an (i,j) - $_{gsp}T_\alpha^*$ space, if every (i,j) -gsp-closed set is (i,j) - α^* -closed.

Definition 4.6 A bitopological space (X, τ_1, τ_2) is said to be an (i,j) - $_{gp}T_\alpha^*$ space, if every (i,j) -gp-closed set is (i,j) - α^* -closed.

Definition 4.7 A bitopological space (X, τ_1, τ_2) is said to be an (i,j) - $_{gpr}T_\alpha^*$ space, if every (i,j) -gpr-closed set is (i,j) - α^* -closed.

Theorem 4.8 Every (i,j) - $T_{1/2}$ space is a (i,j) - T_α^* space.

The converse of the above theorem is not true.

Example 4.9 In example (3.8), (X, τ_1, τ_2) is a $(1,2)$ - T_α^* space, since all the (i,j) - α^* -closed sets are τ_j -closed. Since $A=\{a,b\}$ is $(1,2)$ -g-closed but not τ_2 -closed, it is not a $(1,2)$ - $T_{1/2}$ space.

Theorem 4.10 Every (i,j) - T_b space is a (i,j) - T_α^* space.

The following example shows that the converse of the above theorem is not true.

Example 4.11 In example (3.8), we have proved that the (i,j) - α^* -closed sets are $X, \emptyset, \{b,c\}$ which are τ_j -closed. Hence (X, τ_1, τ_2) is a $(1,2)$ - T_α^* space. Since $A=\{a,b\}$ is $(1,2)$ -gs-closed but not τ_2 -closed. Hence (X, τ_1, τ_2) is not a $(1,2)$ - T_b space.

Theorem 4.12 A space which is both (i,j) - T_d and (i,j) - $T_{1/2}$ is a (i,j) - T_α^* space.

Theorem 4.13 Every (i,j) - T_b space is a (i,j) - T_α^* space.

The following example shows that the converse of the above theorem is not true.

Example 4.14 In example (3.8), (X, τ_1, τ_2) is a $(1,2)$ - T_α^* space. Since $A=\{b\}$ is a $(1,2)$ - α -g-closed but not τ_2 -closed. Hence (X, τ_1, τ_2) is not a $(1,2)$ - T_b space.

Theorem 4.15 Every (i,j) - T_b space is a (i,j) - $_{gs}T_\alpha^*$ space.

The converse of the above theorem is not true as seen in the following example.

Theorem 4.16 In example (3.31), (X, τ_1, τ_2) is a (i,j) - $_{gs}T_a^*$ space. It is not a (i,j) - T_b space, since $A=\{a,b\}$ is $(1,2)$ - gs -closed but not τ_2 -closed.

Theorem 4.17 Every (i,j) - $_aT_b$ space is a (i,j) - $_{ag}T_a^*$ space.

The converse of the above theorem is not true as seen in the following example.

Example 4.18 In example (3.31), (X, τ_1, τ_2) is a $(1,2)$ - $_{ag}T_a^*$ space, since every (i,j) - ag -closed set in it is (i,j) - α^* -closed. Since $A=\{a,b\}$ is a $(1,2)$ - ag -closed but not τ_2 -closed, (X, τ_1, τ_2) is not a $(1,2)$ - $_aT_b$ space.

Theorem 4.19 Every (i,j) - $T_{1/2}$ space is a (i,j) - $_gT_a^*$ space.

The following example shows that the converse of the above theorem is not true.

Example 4.20 In example (3.4), we have proved that the (i,j) - g -closed sets are $X, \emptyset, \{b\}, \{a,b\}, \{b,c\}$ which are (i,j) - α^* -closed in (X, τ_1, τ_2) , where $X=\{a,b,c\}$, $\tau_1=\{\emptyset, \{c\}, \{a,c\}, X\}$ and $\tau_2=\{\emptyset, \{a\}, X\}$. $A=\{b\}$ is $(1,2)$ - g -closed but not τ_2 -closed. Hence (X, τ_1, τ_2) is not a $(1,2)$ - $T_{1/2}$ space.

Theorem 4.21 A space is both (i,j) - $_gT_a^*$ space and (i,j) - T_a^* space if and only if it is a (i,j) - $T_{1/2}$ space.

Theorem 4.22 A space (X, τ_i, τ_j) which is both (i,j) - $_aT_d$ and (i,j) - $T_{1/2}$ is a (i,j) - T_a^* space.

Theorem 4.23 A space (X, τ_i, τ_j) which is both (i,j) - $_{gs}T_a^*$ space and (i,j) - T_a^* space is a (i,j) - T_b space.

Theorem 4.24 A space (X, τ_i, τ_j) which is both (i,j) - $_{ag}T_a^*$ space and (i,j) - T_a^* space is a (i,j) - $_aT_b$ space.

Theorem 4.25 Every (i,j) - $_{ag}T_a^*$ space is a (i,j) - $_gT_a^*$ space.

The following example shows that the converse of the above theorem is not true.

Example 4.26 Let $X=\{a,b,c\}$, $\tau_1=\{\emptyset, \{a\}, \{b,c\}, X\}$ and $\tau_2=\{\emptyset, \{c\}, \{a,c\}, X\}$. (X, τ_1, τ_2) is a $(1,2)$ - $_gT_a^*$ space, since all its (i,j) - g -closed sets, $X, \emptyset, \{b\}, \{a,b\}, \{a,c\}$ are (i,j) - α^* -closed. $A=\{a\}$ is $(1,2)$ - ag -closed but not $(1,2)$ - α^* -closed. Hence (X, τ_1, τ_2) is not a $(1,2)$ - $_{ag}T_b$ space.

Theorem 4.27 Every (i,j) - $_{gs}T_a^*$ space is a (i,j) - $_gT_a^*$ space.

The converse of the above theorem is not true as seen in the following example.

Example 4.28 Consider example 4.26, (X, τ_1, τ_2) is a $(1,2)$ - $_gT_a^*$ space. But it is not a $(1,2)$ - $_{gs}T_a^*$ space, since $A=\{a\}$ is $(1,2)$ - gs -closed but not $(1,2)$ - α^* -closed.

Theorem 4.29 Every $(i,j)\text{-}_{gp}T_{\alpha}^*$ space is a $(i,j)\text{-}_gT_{\alpha}^*$ space.

The converse of the above theorem is not true as seen in the following example.

Example 4.30 In example 3.15, (X, τ_1, τ_2) is a $(1,2)\text{-}_gT_{\alpha}^*$ space. But it is not a $(1,2)\text{-}_{gp}T_{\alpha}^*$ space, since $A=\{b\}$ is $(1,2)\text{-gp-closed}$ but not $(1,2)\text{-}\alpha^*\text{-closed}$.

Theorem 4.31 Every $(i,j)\text{-}_{gsp}T_{\alpha}^*$ space is a $(i,j)\text{-}_gT_{\alpha}^*$ space.

The converse of the above theorem is not true as seen in the following example.

Example 4.32 Consider example 3.10, where $X=\{a,b,c\}$, $\tau_1=\{\emptyset, \{c\}, \{a,b\}, X\}$ and $\tau_2=\{\emptyset, \{a\}, X\}$.

(X, τ_1, τ_2) is a $(1,2)\text{-}_gT_{\alpha}^*$ space. But it is not a $(1,2)\text{-}_{gsp}T_{\alpha}^*$ space, since $A=\{b\}$ is $(1,2)\text{-gsp-closed}$ but not $(1,2)\text{-}\alpha^*\text{-closed}$.

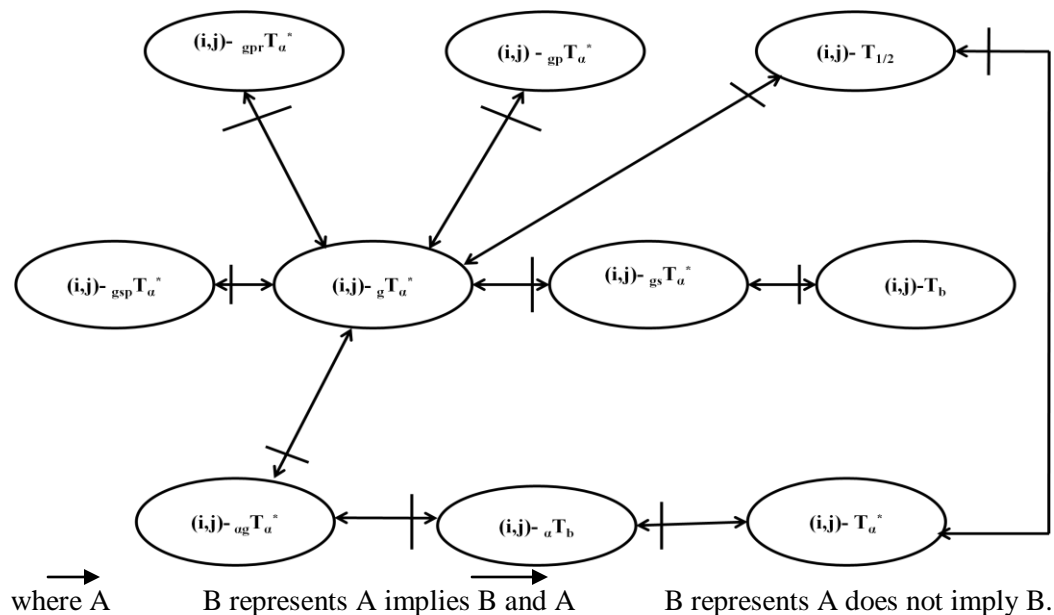
Theorem 4.33 Every $(i,j)\text{-}_{gpr}T_{\alpha}^*$ space is a $(i,j)\text{-}_gT_{\alpha}^*$ space.

The following example shows that the converse of the above theorem is not true.

Example 4.34 Consider example 3.10, where $X=\{a,b,c\}$, $\tau_1=\{\emptyset, \{c\}, \{a,b\}, X\}$ and $\tau_2=\{\emptyset, \{a\}, X\}$.

(X, τ_1, τ_2) is a $(1,2)\text{-}_gT_{\alpha}^*$ space. But it is not a $(1,2)\text{-}_{gpr}T_{\alpha}^*$ space, since $A=\{c\}$ is $(1,2)\text{-gpr-closed}$ set but not $(1,2)\text{-}\alpha^*\text{-closed}$.

All the results we have proved in this section can be represented by the following figure:



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