A STUDY ON $(i,j)-\Psi^*, (i,j)-\overline{\Psi}$ and $(i,j)-\overline{\Psi}$*-CLOSED SETS IN BITOPOLOGICAL SPACES

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ABSTRACT

In this paper we introduce new classes of sets $(i,j)-\Psi^*, (i,j)-\overline{\Psi}$ and $(i,j)-\overline{\Psi}$*- Closed Sets in Bitopological Spaces. Properties of these sets are investigated and we introduce nine new spaces namely, (i,j)-$T_{\overline{\Psi}}$-space, (i,j)- $T_{\overline{\Psi}^*}$-space, $(i,j)-T_{\Psi^*}$-space, (i,j)- $T_{\Psi}{}^*$-space, $(i,j)-T_{\overline{\Psi}}{}^*$-space, (i,j)-$T_{\Psi^*}{}^*$-space, (i,j)-${}_gT_{\overline{\Psi}^*}$-space, (i,j)-${}_gT_{\overline{\Psi}}$-space and (i,j)-${}_\alpha T_{\overline{\Psi}}$ –space.

Keywords: $(i,j)-\Psi^*, (i,j)-\overline{\Psi}$ and $(i,j)-\overline{\Psi}$*- closed Sets, (i,j)-$T_{\overline{\Psi}}$, (i,j)- $T_{\overline{\Psi}^*}$, $(i,j)-T_{\Psi^*}$, (i,j)-$T_{\Psi}{}^*$, $(i,j)-T_{\overline{\Psi}}{}^*$, (i,j)- $T_{\Psi^*}{}^*$, (i,j)-${}_gT_{\overline{\Psi}^*}$, (i,j)-${}_gT_{\overline{\Psi}}$ *and* (i,j)-${}_\alpha T_{\overline{\Psi}}$ spaces.

1. INTRODUCTION

A triple (X, τ_1, τ_2) where X is a non empty set and τ_1 and τ_2 are topologies in X is called a bitopological space and Levine[10] introduced the class of generalized closed sets, a super class of closed sets in 1970.Purpose of this paper is to introduce the concept of $(i,j)-\Psi^*$ *closed sets*, $(i,j)-\overline{\Psi}$ *closed sets* and $(i,j)-\overline{\Psi}$*- Closed Sets, (i,j)-$T_{\overline{\Psi}}$, (i,j)- $T_{\overline{\Psi}^*}$, $(i,j)-T_{\Psi^*}$, (i,j)- $T_{\Psi}{}^*$, $(i,j)-T_{\overline{\Psi}}{}^*$, (i,j)- $T_{\Psi^*}{}^*$, (i,j)-${}_gT_{\overline{\Psi}^*}$ (i,j)-${}_gT_{\overline{\Psi}}$ *and* (i,j)-${}_\alpha T_{\overline{\Psi}}$ spaces in bitopological spaces and investigate some of their properties.

2. PRELIMINARIES

Definition 2.1: A subset A of a topological space (X,τ) is called

1. a semi open set **[9]** if A⊆cl(int(A)) and a semi closed set if int(cl(A))⊆A
2. a generalized closed set **[10]** (briefly g-closed) if cl(A) ⊆U whenever A⊆U and U is open in (X,τ).
3. a generalized semi closed set**[2]**(briefly gs-closed) if scl(A) ⊆Uwhenever A⊆U and U is open in (X,τ).
4. a α –open set**[13]** if A ⊆int(cl(int(A))) and a α-closed set if cl(int(cl(A))) ⊆A
5. a generalized α closed set**[12]**(briefly gα-closed) if αcl(A) ⊆Uwhenever A⊆U and U is α-open in (X,τ).

6. a semi generalized closed set**[3]**(briefly sg-closed) if scl(A) ⊆U whenever A⊆U and U is semiopen in (X,τ) .
7. A α generalised closed set**[11]** (briefly αg-closed) if αcl(A) ⊆ U whenever A⊆U and U is open in (X,τ).
8. a g^*-closed set**[15]** if cl(A) ⊆ U whenever A⊆U and U is g-open in (X,τ).
9. a ψ-closed set**[16]** if scl(A) ⊆ U whenever A⊆U and U is sg-open in (X,τ).
10. a g^*s-closed set**[4]** if scl(A) ⊆ U whenever A⊆U and U is gs-open in (X,τ).
11. a Ψ^*-closed set **[17]** if scl(A) ⊆ U whenever A⊆U and U is ψ-open in (X,τ).
12. a $\overline{\Psi}$-closed set **[17]** if cl(A) ⊆ U whenever A⊆U and U is sg-open in (X,τ).
13. a $\overline{\Psi}^*$-closed set **[17]** if cl(A) ⊆ U whenever A⊆U and U is $\overline{\Psi}$-open in (X,τ).

If A is a subset of X with topology τ, then the closure of A is denoted by τ-cl(A) or cl(A). The interior of A is denoted by τ-int (A) or int (A) and the complement of A in X is denoted by A^c

Definition 2.2: A subset A of a bitopological space (X,$\tau_{i,}\tau_j$) is called

1. (i,j)-g-closed **[7]** if τ_j-cl (A) ⊆ U whenever A⊆U and $U\epsilon\tau_i$.
2. (i,j)-ω-closed **[8]** if τ_j-cl (A) ⊆ U whenever A⊆U and U is semi open in τ_i.
3. (i,j)-g^*s-closed **[14]** if τ_j-scl (A) ⊆ U whenever A⊆U and U is gs-open in τ_i.
4. (i,j)-sg-closed**[5]** if τ_j-scl(A) ⊆ U whenever A⊆U and U is semi-open in τ_i.
5. (i,j)-αg-closed**[6]** if τ_j-αcl(A) ⊆ U whenever A⊆U and $U\epsilon\tau_i$.
6. (i,j)-ψ-closed **[1]** if τ_j-scl (A) ⊆ U whenever A⊆U and U is sg-open in τ_i.

Defnition 2.3: A bitopological space (X,$\tau_{i,}\tau_j$) is called

1. an (i,j)-$T_{1/2}$ space**[7]** if every (i,j)-g-closed set in it is τ_j-closed.
2. an (i,j)-T_b-space**[6]** if every (i,j)-gs-closed set in it is τ_j-closed.
3. an (i,j)-T_d-space**[6]** if every (i,j)-gs-closed set in it is (i,j)-g-closed.
4. an (i,j)-${}_{\alpha}T_b$-space**[6]** if every (i,j)-αg-closed set in it is τ_j closed.
5. an (i,j)-${}_{\alpha}T_d$-space**[7]** if every (i,j)-αg-closed set in it is (i,j)- g-closed

3. $(i,j)-\Psi^*$, $(i,j)-\overline{\Psi}$ and $(i,j)-\overline{\Psi}$*-Closed sets.

We introduce the following definitions.

Definition 3.1: A subset A of a bitopological space (X,$\tau_{1,}\tau_2$) is called (i,j)-Ψ^*-closed if τ_j-scl(A) ⊆ U whenever A⊆U and U is τ_i-Ψ-open.

Definition 3.2: A subset A of a bitopological space (X,$\tau_{1,}\tau_2$) is called (i,j)-$\overline{\Psi}$ -closed if τ_j-cl(A) ⊆ U whenever A⊆U and U is τ_i-sg-open.

Definition 3.3: A subset A of a bitopological space (X,$\tau_{1,}\tau_2$) is called (i,j)-$\overline{\Psi}^*$ -closed if τ_j-cl(A) ⊆ U whenever A⊆U and U is $\tau_i - \overline{\Psi}$-open.

Remark 3.4: By setting $\tau_1 = \tau_2$ in definitions 3.1, 3.2 and 3.3, a (i,j)-Ψ^*-closed set is Ψ^*-closed,

a (i,j)-$\overline{\Psi}$ –closed set is $\overline{\Psi}$ –closed &

a (i,j)-$\overline{\Psi}^*$ -closed set is $\overline{\Psi}^*$ -closed.

Proposition 3.5: Every τ_j -closed subset of a bitopological space (X,τ_1,τ_2) is

(i) (i,j)- Ψ^*-closed,
(ii) (i,j)-$\overline{\Psi}$ –closed &
(iii) (i,j)-$\overline{\Psi}^*$-closed.

The following example shows that the converse of the above proposition is not true.

Example 3.6: Let X={a,b,c}, τ_1 = {X,ϕ,{a}}, τ_2 = {X,ϕ,{b,c}}**.** Then the set {b,c} is (1,2)-ψ^*-closed, (1,2)- $\overline{\Psi}$-closed & (1,2)- $\overline{\Psi}^*$-closed but not τ_2-closed.

Proposition 3.7: Every (i,j)- Ψ^*-closed set is (i,j)- gs-closed .

Converse of the above proposition is not true as seen in the following example.

Example 3.8: LetX={a,b,c}, τ_1={X,ϕ,{a}} ,τ_2={X,ϕ,{b,c}}**.** Then the set {a,b} is (1,2)-gs-closed but not (1,2)-Ψ^*-closed. Hence every (1,2)-gs-closed set need not be (1,2)-Ψ^*-closed.

Proposition 3.9: Every (i,j)- $\overline{\Psi}$-closed set is (i,j)- gs-closed .

The converse of the above proposition is not true as seen in the following example.

Example 3.10: Let X={a,b,c}, τ_1 = {X,ϕ,{a}}, τ_2 = {X,ϕ,{b,c}}. Then the set {b} is (1,2)- gs-closed but not (1,2)-$\overline{\Psi}$-closed.

Proposition 3.11: Every (i,j)- $\overline{\Psi}^*$-closed set is (i,j)- gs-closed .

The converse of the above proposition is not true.

Example 3.12: Let X={a,b,c}**,** τ_1= {X,ϕ,{a,b},{a}}**,** τ_2= {X,ϕ,{b}}**.** Then the set{a} is (1,2) gs-closed but not (1,2)-$\overline{\Psi}^*$-closed.

Proposition 3.13: **I**n a bitopological space (X,τ_1,τ_2) every (i,j)- $\overline{\Psi}$-closed set is (i) (i,j)- sg-closed,(ii) (i,j)- g-closed, (iii) (i,j)- αg-closed, (iv) (i,j)- ψ-closed, (v) (i,j)- $\overline{\Psi}^*$-closed,(vi) (i,j)- ω-closed and (vii) (i,j)-g^*s-closed.

The following examples support that the converse of the above proposition is not true.

Example 3.14: Let X={a,b,c}, τ_1 = {X,ϕ,{a,b},{a}},τ_2 = {X,ϕ,{b}}. The set {a} is (1,2)- sg-closed but not (1,2)-$\overline{\Psi}$-closed.

Example 3.15: Let X={a,b,c}**,** τ_1= {X,ϕ,{a}}, τ_2= {X,ϕ,{b,c}}**.** The set {b} is (1,2)- g closed but not (1,2)-$\overline{\Psi}$-closed.

.**Example 3.16:** Let X={a,b,c}, τ_1 = {X,ϕ,{a}}, τ_2 = {X,ϕ,{b,c}}. Then the set {a,c} is (1,2)- αg-closed but not (1,2)-$\overline{\Psi}$-closed.

Example 3.17: Let X={a,b,c}, τ_1 = {X,ϕ,{a,b},{a}}, τ_2 = {X,ϕ,{b}}. Then set {a} is (1,2)-ψ -closed but not (1,2)-$\overline{\Psi}$-closed.

Example 3.18: Let X={a,b,c}, τ_1 = {X,ϕ,{a}}, τ_2 = {X,ϕ,{b,c}. Then the set {a,b} is (1,2)-$\overline{\Psi}^*$-closed but not (1,2)-$\overline{\Psi}$-closed.

Example 3.19: Let X={a,b,c}, τ_1 = {X,ϕ,{a},{b,c}}, τ_2 = {X,ϕ,{b}}. Then the set {a,b} is (1,2)- ω-closed but not (1,2)-$\overline{\Psi}$-closed.

Example 3.20: Let X={a,b,c}, τ_1 = {X,ϕ,{a},{b,c}}, τ_2 = {X,ϕ,{b}}. Then the set {c} is (1,2)- g^*s-closed but not (1,2)-$\overline{\Psi}$ closed.

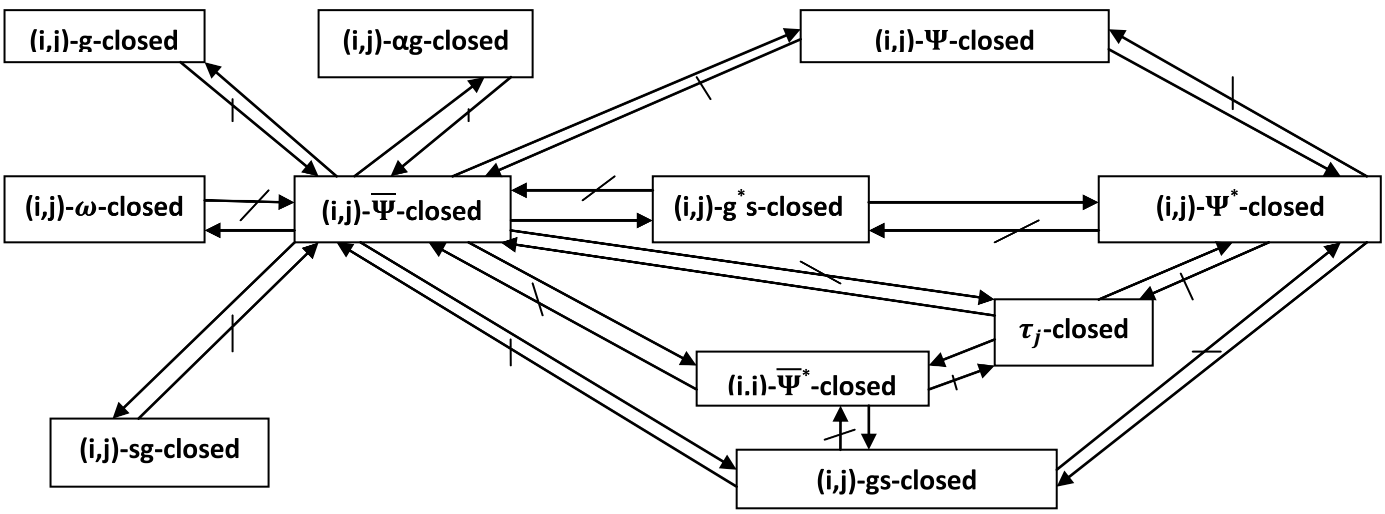
Proposition 3.21: Every (i,j)- g*s-closed set is (i,j)- ψ^*-closed .But the converse is not true.

Example 3.22: Let X={a,b,c}, τ_1 = {X,ϕ,{a},{b,c}}, τ_2 = {X,ϕ,{c}}. Then the set {a,c} is (1,2)- ψ^*-closed but not (1,2)- g^*s-closed.

Proposition3.23: Every (i,j)- ψ-closed set is (i,j)- ψ^*-closed . Converse of this proposition is not true.

Example 3.24: Let X={a,b,c}, τ_1 = {X,ϕ,{a},{b,c}}, τ_2 = {X,ϕ,{c}}. Then the set {a,c} is (1,2)-ψ^* -closed but not (1,2)-ψ-closed.

All the above results can be represented by following diagram.



where A $\longrightarrow$ B represents A implies B and A $\not\longrightarrow$ B represents A does not imply B

Theorem 3.25: If A and B are (i,j)- ψ^*-closed then A∪B is (i,j)- ψ^*-closed.

Proof:.Let U be ψ-open in τ_i such that A∪B⊆ U.Then A⊆Uand B⊆U.Since A and B are (i,j)- ψ^*-closed sets τ_j-scl(A) ⊆ U and τ_j-scl(B)⊆ U.Hence τ_j-scl(A∪B)⊆ U.Therefore τ_j-scl(A∪B)⊆ U whenever A∪B⊆ U and U is ψ-open in τ_i. Hence A∪B is (i,j)- ψ^*-closed.

Theorem 3.26: If A and B are (i,j)- $\overline{\Psi}$-closed then A∪B is (i,j)- $\overline{\Psi}$-closed.

Proof: Let U be sg-open in τ_i such that A∪B⊆ U.Then A⊆U and B⊆U.Since A and B are (i,j)- $\overline{\Psi}$-closed sets τ_j-cl(A) ⊆ U and τ_j-cl(B) ⊆ U.Hence τ_j-cl(A∪B)⊆ U.Therefore τ_j-cl(A∪B)⊆ U whenever A∪B⊆ U and U is sg-open in τ_i.Hence A∪B is (i,j)- $\overline{\Psi}$-closed.

Theorem 3.27: If A and B are(i,j)- $\overline{\Psi}^*$-closed then A∪B is (i,j)- $\overline{\Psi}^*$-closed.

Proof: Since τ_j-cl(A∪B)⊆ τ_j-cl(A) ∪ τ_j-cl(B) , τ_j-cl(A∪B)⊆ U whenever A∪B⊆ U and U is $\overline{\Psi}$-open in τ_i.Hence A∪B is (i,j)- $\overline{\Psi}^*$-closed.

Definition 3.28

A subset A of a bitopological space (X,$\tau_{1,}\tau_2$) is called (i,j)-Ψ^*-open if A^c is (i,j)-Ψ^*-closed.

Theorem 3.29: If A and B are (i,j)- Ψ^*-open sets in (X,$\tau_{1,}\tau_2$), then A∩B is also an (i,j)-Ψ^*-open- set in (X,$\tau_{1,}\tau_2$).

Proof: Let A and B are (i,j)- Ψ^*-open sets in (X,$\tau_{1,}\tau_2$).Then A^c and B^c are (i,j)-Ψ^*-closed. By theorem 3.25,$A^c \cup B^c$ is (i,j)-Ψ^*-closed in (X,$\tau_{1,}\tau_2$).(ie) $(A \cap B)^c$ is(i,j)-Ψ^*-closed in (X,$\tau_{1,}\tau_2$) and hence A∩B is (i,j)-Ψ^*-open set in (X,$\tau_{1,}\tau_2$).

Definition 3.30: A subset A of a bitopological space (X,$\tau_{1,}\tau_2$) is called (i,j)-$\overline{\Psi}$ –open if A^c is (i,j)- $\overline{\Psi}$ –closed.

Theorem 3.31: If A and B are (i,j)- $\overline{\Psi}$ -open sets in (X,$\tau_{1,}\tau_2$) then A∩B is also an (i,j)- $\overline{\Psi}$ -open set in (X,$\tau_{1,}\tau_2$).

Proof follows from theorem 3.26.

Definition 3.32:A subset A of a bitopological space (X$\tau_{1,}\tau_2$,) is called (i,j)-$\overline{\Psi}^*$ -open if A^c is (i,j)-$\overline{\Psi}^*$-closed set in (X,$\tau_{1,}\tau_2$).

Theorem 3.33: If A and B are (i,j)- $\overline{\Psi}^*$-open sets in (X,$\tau_{1,}\tau_2$), then A∩B is also an (i,j)- $\overline{\Psi}^*$-open set in (X,$\tau_{1,}\tau_2$).

Proof follows from theorem 3.27.

Theorem 3.34: For each element x of (X,$\tau_{1,}\tau_2$),

(i) {x} is either τ_i ψ-closed or $\{x\}^c$ is (i,j)-ψ^*-closed,
(ii) {x} is either τ_i- sg-closed or $\{x\}^c$ is (i,j)- $\overline{\Psi}$-closed &
(iii) {x} is either τ_i-$\overline{\Psi}$-closed or $\{x\}^c$ is (i,j)- $\overline{\Psi}^*$-closed

Proof (i): If {x} is not τ_i ψ-closed, then $\{x\}^c$ is not τ_i -ψ-open and the only τ_i -ψ-open set containing $\{x\}^c$ is X. Also τ_j-scl($\{x\}^c$) ⊆ X. Therefore $\{x\}^c$ is (i,j)-ψ^*-closed.

Proof (ii): If {x} is not τ_i-sg-closed, then $\{x\}^c$ is not τ_i -sg-open and the only τ_i -sg-open set containing $\{x\}^c$ is X. Also τ_j-cl($\{x\}^c$) ⊆ X. Therefore $\{x\}^c$ is (i,j)- $\overline{\Psi}$-closed.

Proof (iii): If {x} is not τ_i-$\overline{\Psi}$ -closed, then $\{x\}^c$ is not τ_i -$\overline{\Psi}$-open and the only τ_i -$\overline{\Psi}$-open set containing $\{x\}^c$ is X. Also τ_j-cl($\{x\}^c$) ⊆ X. Therefore $\{x\}^c$ is (i,j)- $\overline{\Psi}^*$-closed.

Theorem 3.35: If A is an (i,j)- Ψ^*-closed set of (X,$\tau_{1,}\tau_2$) such that A⊆ B⊆ τ_j-scl(A) then B is also (i,j)- Ψ^*-closed in (X,$\tau_{1,}\tau_2$)

Proof: Let U be τ_i ψ-open such that B⊆U. Then A⊆U. Since A is an (i,j)- Ψ^*-closed, τ_j-scl(A)⊆U. We have τ_j-scl(B)⊆ τ_j-scl(A)⊆U. ∴ τ_j-scl(B)⊆U,whenever B⊆U and U is τ_i ψ-open. Thus B is also (i,j)- Ψ^*-closed in (X,$\tau_{1,}\tau_2$) .

Theorem 3.36: If A is an (i,j)-$\overline{\Psi}$ -closed set of (X,$\tau_{1,}\tau_2$) such that A⊆ B⊆ τ_j-cl(A), then B is also (i,j)- $\overline{\Psi}$-closed in (X,$\tau_{1,}\tau_2$)

Proof: Let U be τ_i-sg -open set such that B⊆U. Then A⊆U.Since A is (i,j)- $\overline{\Psi}$-closed, τ_j-cl(A)⊆U. We have τ_j-cl(B)⊆ τ_j-cl(A)⊆U.Therefore τ_j-cl(B)⊆U,whenever B⊆U and U is τ_i-sg- open. Thus B is also (i,j)- $\overline{\Psi}$-closed in (X,$\tau_{1,}\tau_2$).

Theorem 3.37: If A is an (i,j)-$\overline{\Psi}^*$-closed set of (X,$\tau_{1,}\tau_2$) such that A⊆ B⊆ τ_j-cl(A), then B is also (i,j)- $\overline{\Psi}^*$-closed in (X,$\tau_{1,}\tau_2$)

Proof: Let U be τ_i-$\overline{\Psi}$-open set such that B⊆U. Then A⊆U.Since A is (i,j)- $\overline{\Psi}^*$-closed, τ_j-cl(A)⊆U.We have τ_j-cl(B)⊆ τ_j-cl(τ_j-cl(A)) ⊆U.Therefore τ_j-cl(B)⊆U,whenever B⊆U and U is τ_i-$\overline{\Psi}$- open.Thus B is also (i,j)- $\overline{\Psi}^*$-closed in (X,$\tau_{1,}\tau_2$)

Theorem 3.38: If A is (i,j)- Ψ^*-closed set, $then$ τ_j-scl(A)-A contains no non empty τ_i ψ-closed set.

Proof: let A be (i,j)- Ψ^*-closed set and F be a τ_i ψ-closed subset of τ_j-scl(A)-A. Now F ⊆ τ_j-scl(A)-A.Then F ⊆ (τ_j-scl(A))∩ A^c. Therefore F ⊆ τ_j-scl(A) and F⊆A^c. Then A ⊆ F^c .Since F^c is τ_i ψ-open set and A is (i,j)- Ψ^*-closed set,τ_j-scl(A) ⊆F^c.Then F⊆(τ_j-scl(A))^c.Hence F ⊆ (τ_j-scl(A))∩ F⊆(τ_j-scl(A))^c= Φ, (ie) F= Φ. Thus τ_j-scl(A)-A contains no non empty τ_i ψ-closed set.

Theorem 3.39: If A is (i,j)-$\overline{\Psi}$ -closed set, $then$ τ_j-cl(A)-A contains no non empty τ_i-sg-closed set.

Proof: let A be (i,j)- $\overline{\Psi}$-closed set and F be a τ_i-sg -closed subset of τ_j-cl(A)-A. Now F ⊆ τ_j-cl(A)-A.Then F ⊆ (τ_j-cl(A))∩ A^c.Therefore F ⊆ τ_j-cl(A) and F⊆A^c. Then A ⊆ F^c .Since F^c is τ_i-sg-open set and A is (i,j)- $\overline{\Psi}$-closed set,τ_j-cl(A) ⊆F^c (ie) F⊆(τ_j-cl(A))^c.Hence F ⊆ (τ_j-cl(A))∩ F⊆(τ_j-cl(A))^c= Φ, Therefore F= Φ.Hence τ_j-cl(A)-A contains no non empty τ_i-sg-closed set.

Theorem 3.40: If A is (i,j)-$\overline{\Psi}^*$ -closed set, $then$ τ_j-cl(A)-A contains no non empty τ_i-$\overline{\Psi}$-closed set.

Proof: let A be (i,j)- $\overline{\Psi}^*$-closed set and F be a τ_i-$\overline{\Psi}$ -closed subset of τ_j-cl(A)-A. Now F ⊆ τ_j-cl(A)-A.Then F ⊆ (τ_j-cl(A))∩ A^c.Therefore F ⊆ τ_j-cl(A) and F⊆A^c .Then A ⊆ F^c .Since F^c is τ_i-$\overline{\Psi}$-open set and A is (i,j)- $\overline{\Psi}^*$-closed set,τ_j-cl(A) ⊆F^c (ie) F⊆(τ_j-cl(A))^c.Hence F ⊆ (τ_j-cl(A))∩ F⊆(τ_j-cl(A))^c = Φ, Therefore F= Φ. Hence τ_j-cl (A)-A contains no non empty τ_i-$\overline{\Psi}$-closed set.

4. Applications of $(i,j)-\Psi^*, (i,j)-\overline{\Psi}$ and $(i,j)-\overline{\Psi}$*- Closed Sets.

As Applications of $(i,j)-\Psi^*, (i,j)-\overline{\Psi}$ and $(i,j)-\overline{\Psi}$*- Closed Sets, nine new space namely (i,j)-$T_{\overline{\Psi}}$-space, (i,j)- $T_{\overline{\Psi}^*}$-space, (i, j) $-T_{\Psi^*}$-space, (i,j)- $T_{\Psi}{}^*$-space, (i, j) $-$ $T_{\overline{\Psi}}{}^*$-space, (i,j)- $T_{\Psi^*}{}^*$-space, (i,j)- ${}_gT_{\overline{\Psi}^*}$-space, (i,j)- ${}_gT_{\overline{\Psi}}$-space, (i,j)- ${}_\alpha T_{\overline{\Psi}}$ –space are introduced.

We introduce the following definitions:

Defnition 4.1: A bitopological space (X,$\tau_{1,}\tau_2$) is said to be an (i,j)- T_{Ψ^*}-space if every (i,j)-ψ*-closed set is τ_j-closed.

Defnition 4.2: A bitopological space (X,$\tau_{1,}\tau_2$) is said to be an (i,j)- $T_{\overline{\Psi}}$-space if every (i,j)-$\overline{\Psi}$-closed set is τ_j-closed

Defnition 4.3: A bitopological space (X,$\tau_{1,}\tau_2$) is said to be an (i,j)- $T_{\overline{\Psi}^*}$-space if every (i,j)-$\overline{\Psi}^*$ closed set is τ_j-closed

Defnition 4.4: A bitopological space (X,$\tau_{1,}\tau_2$) is said to be an (i,j)- $T_{\overline{\Psi}}{}^*$-space if every (i,j)-$\overline{\Psi}^*$-closed set is (i,j)-$\overline{\Psi}$-closed.

Defnition 4.5: A bitopological space (X,$\tau_{1,}\tau_2$) is said to be an (i,j)-$T_{\Psi^*}{}^*$-space if every (i,j)-ψ*-closed set is (i,j)-$\overline{\Psi}$-closed.

Defnition 4.6: A bitopological space (X,$\tau_{1,}\tau_2$) is said to be an (i,j)-${}_gT_{\overline{\Psi}}$-space if every (i,j)-g-closed set is (i,j)-$\overline{\Psi}$-closed.

Defnition 4.7: A bitopological space (X,$\tau_{1,}\tau_2$) is said to be an (i,j)-$T_{\Psi}{}^*$-space if every (i,j)-ψ^*-closed set is (i,j)-ψ -closed.

Defnition 4.8:A bitopological space (X,$\tau_{1,}\tau_2$) is said to be an (i,j)-${}_gT_{\overline{\Psi}^*}$-space if every (i,j)-gs-closed set is-(i,j)-$\overline{\Psi}^*$-closed

Defnition 4.9:A bitopological space (X,$\tau_{1,}\tau_2$) is said to be an (i,j)-${}_\alpha T_{\overline{\Psi}}$ -space if every (i,j)-αg-closed set is (i,j)-$\overline{\Psi}$-closed.

Theorem 4.10: Every (i,j)- T_{Ψ^*}-space is (i,j)-$T_{\Psi}{}^*$-space but not conversely.

Example 4.11: Let X={a,b,c}**,** τ_1 = {X,ϕ,{a,b},{a}}**,** τ_2 = {X,ϕ,{b,c},{b}}**.** Then (X**,**$\tau_{1,}\tau_2$) is a (i,j)-$T_{\Psi}{}^*$-space. But it is not a (i,j)- T_{Ψ^*}-space since A={c} is (1,2)-ψ^* closed but not τ_2-closed.

Theorem 4.12: Every (i,j)-$T_{\overline{\Psi}^*}$ space is (i,j)- $T_{\overline{\Psi}}{}^*$--space but not conversely.

Example 4.13: Let X={a,b,c}**,** τ_1 = {X,ϕ,{a,c}}, τ_2 = {X,ϕ,{a,b}}. Then (X,$\tau_{1,}\tau_2$) is (i,j)-$T_{\overline{\Psi}}{}^*$-space. But it is not a (i,j)-$T_{\overline{\Psi}^*}$ space since {b,c} is (1,2)- $\overline{\Psi}^*$ closed but not τ_2-closed.

Theorem 4.14: Every (i,j)-$T_{\Psi^*}{}^*$ space is (i,j)-$T_{\Psi}{}^*$-space but not conversely.

Example 4.15: Let X={a,b,c}, τ_1 = {X,ϕ,{a,c}}, τ_2 = {X,ϕ,{b},{a,b},{b}}. Then (X,$\tau_{1,}\tau_2$) is (i,j)-$T_{\Psi}{}^*$-space. But it is not a (i,j)-$T_{\Psi^*}{}^*$ space since {b,c} is (1,2)- Ψ^*-closed but not $\overline{\Psi}$-closed.

Theorem 4.16: Every (i,j)-T_b- space is (i,j)- ${}_gT_{\overline{\Psi}^*}$-space but not conversely.

Example 4.17: Let X={a,b,c}, τ_1 = {X,ϕ,{a,b},{a}} , τ_2 = {X,ϕ,{b,c},{b}}.Then (X,$\tau_{1,}\tau_2$) is (i,j)- ${}_gT_{\overline{\Psi}^*}$ space. But it is not a (i,j)- T_b –space, since the set {c} is (1,2)- gs closed but not τ_2-closed.

Theorem 4.18: Every (i,j)- $T_{1/2}$- space is (i,j)- ${}_gT_{\overline{\Psi}}$ -space but not conversely.

Example 4.19: Let X={a,b,c}, τ_1 = {X,ϕ,{a,b},{a}}, τ_2 = {X,ϕ,{b,c},{b}}. Then (X,$\tau_{1,}\tau_2$) is (i,j)- ${}_gT_{\overline{\Psi}}$ space.But it is not a (i,j)- $T_{1/2}$ –space, since the set {b,c} is (1,2)- g closed but not τ_2-closed.

Theorem 4.20: Every (i,j)- ${}_\alpha T_b$- space is (i,j)- ${}_\alpha T_{\overline{\Psi}}$ -space but not conversely.

Example 4.21: Let X={a,b,c}, τ_1 = {X,ϕ,{a,b},{a}}, τ_2 = {X,ϕ,{b,c},{b}}.Then (X,$\tau_{1,}\tau_2$) is (i,j)- ${}_\alpha T_{\overline{\Psi}}$ space.But it is not a (i,j)- ${}_\alpha T_b$–space, since the set{b,c} is (1,2)- αg closed but not τ_2-closed.

Theorem 4.22: Every (i,j)-${}_\alpha T_{\overline{\Psi}}$- space is (i,j)- ${}_\alpha T_d$ -space but not conversely.

Example 4.23:

Let X={a,b,c}, τ_1 = {X,ϕ,{a}}, τ_2 = {X,ϕ,{b,c}}. Then (X,$\tau_{1,}\tau_2$) is (i,j)- ${}_\alpha T_d$ space.But it is not a (i,j)- ${}_\alpha T_{\overline{\Psi}}$ –space, since the set {a,b} is (1,2)- αg closed but not (1,2)- $\overline{\Psi}$closed.

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