α*-CLOSED SETS IN TOPOLOGICAL SPACES

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Abstract : In this research paper we introduce a new class of sets namely, α* -closed sets. Applying these sets, the authors introduce four new classes of space namely, T_α^* spaces, ${}_{\alpha g}T_\alpha^*$ spaces, ${}_{gs}T_\alpha^*$ spaces and ${}_{g}T_\alpha$ spaces. Further the authors introduce α*–continuous maps and α *–irresolute maps.

Key words : α* -closed sets; α*–continuous maps; α *–irresolute maps; T_α^* spaces; ${}_{\alpha g}T_\alpha^*$ spaces; ${}_{gs}T_\alpha^*$ spaces and ${}_{g}T_\alpha$ spaces.

1.INTRODUCTION

In 1970, Levine **[10]** first introduced the concept of generalized closed (briefly,g-closed)sets as a generalization of closed sets in topological spaces. S.P.Arya and T.Nour **[3]** defined gs-closed sets in 1990. Dontchev **[9]** introduced gsp-closed sets. Maki et.al.**[13]** defined αg-closed sets in1994. Levine **[11]**, Mashhour et.al. **[16]** introduced semi-open sets, preopen sets respectively. Maki et.al. **[13]** introduced gα-closed sets. S.N.Maheshwari and P.C.Jain **[12]** introduced and investigated α-closed sets.

The purpose of this paper is to introduce the concept of α*-closed sets, T_α^* spaces, ${}_{\alpha g}T_\alpha^*$ spaces, ${}_{gs}T_\alpha^*$ spaces and ${}_{g}T_\alpha$ spaces and investigate some of their properties.

2. PRELIMINARIES

Definition: 2.1 - A subset A of a topological space (X, τ) is called,

i. a pre open set **[16]** if $A \subseteq int (Cl (A))$ and a pre closed set if $Cl (int(A)) \subseteq A$.
ii. a semi-open set **[10]** if $A \subseteq Cl (int (A))$ and a semi-closed set if $int (Cl (A)) \subseteq A$.
iii. an α -open set **[17]** if $A \subseteq int (Cl (int (A)))$ and an α -closed if $Cl (int (Cl (A))) \subseteq A$.
iv. a semi-preopen set **[1]** if $A \subseteq Cl (int(Cl (A)))$ and a semi-preclosed set if $int (Cl (int (A))) \subseteq A$.

The intersection of all semi-closed (resp. Pre-closed, semi-preclosed and α-closed) sets containing a subset A of (X, τ) is called the semi-closure (resp. pre-closure, semi-pre-closure and α-closure) of A and is denoted by Scl(A) (resp. Pcl(A), Spcl(A) and αcl(A)).

Definition: 2.2 - A subset A of a topological space (X, τ) is called,

i. a generalized closed set **[10]** (briefly g-closed) if Cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).
ii. a semi-generalized closed set **[5]** (briefly sg-closed) if Scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi-open in (X, τ).
iii. a generalized semi-closed set **[3]** (briefly gs-closed) if Scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).
iv. a generalized α-closed set **[6]** (briefly gα-closed) if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is α-open in (X, τ).
v. a α-generalized closed set **[13]** (briefly αg-closed) if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).
vi. a generalized semi-preclosed set **[9]** (briefly gsp-closed) if Spcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).
vii. a generalized preclosed set **[14]** (briefly gp-closed) if Pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).
viii. Strongly g-closed **[18]** if Cl(intA) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).

Definition: 2.3 - A topological space (X, τ) is said to be,

i. a $T_{½}$space **[10]** if every g-closed set in it is closed.
ii. a T_b space **[7]** if every gs –closed set in it is closed.
iii. a T_d space **[7]** if every gs –closed set in it is g-closed.
iv. a $_{\alpha}T_d$ space **[6]** if every αg –closed set in it is g-closed.
v. a $_{\alpha}T_b$ space **[6]** if every αg –closed set in it is closed.

Definition: 2.4 A function f : (X, τ) $\longrightarrow$ (Y, σ) is called

1. an α-continuous **[13]** if $f^{-1}(V)$ is an α-closed set of (X, τ) for every closed set V of (Y, σ).
2. a g-continuous **[4]** if $f^{-1}(V)$ is an g-closed set of (X, τ) for every closed set V of (Y, σ).
3. an αg-continuous **[10]** if $f^{-1}(V)$ is an αg-closed set of (X, τ) for every closed set V of (Y, σ).
4. a gs-continuous **[8]** if $f^{-1}(V)$ is a gs-closed set of (X, τ) for every closed set V of (Y, σ).
5. a gsp-continuous **[9]** if $f^{-1}(V)$ is a gsp-closed set of (X, τ) for every closed set V of (Y, σ).
6. a gp-continuous **[2]** if $f^{-1}(V)$ is a gp-closed set of (X, τ) for every closed set V of (Y, σ).
7. a strongly g -continuous **[18]** if $f^{-1}(V)$ is a strongly g -closed set of (X, τ) for every closed set V of (Y, σ).

8. a gs-irresolute **[8]** if $f^{-1}(V)$ is an gs-closed set of (X, τ) for every gs-closed set V of (Y, σ).
9. an αg-irresolute **[6]** if $f^{-1}(V)$ is an αg-closed set of (X, τ) for every αg-closed set V of (Y, σ).

3. BASIC PROPERTIES OF α*-CLOSED SETS

We introduce the following definition

Definition 3.1 - A subset A of a topological space (X,τ) is called a α*-closed set if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open in X.

Theorem 3.2 - Every closed set is a α*-closed set.

The following example supports that an α*-closed set need not be closed in general.

Example 3.3 - Let X= { a,b,c }, τ = { ∅, X, {a,b} }. Let A = {a,c}. A is a α*-closed set but not a closed set of (X,τ).

So the class of α*-closed sets properly contains the class of closed sets. Next we show that the class of α*-closed sets is properly contained in the class of g-closed sets.

Theorem 3.4 - Every α*-closed set is a g-closed set.

The converse of the above theorem is not true in general as it can be seen from the following example.

Example 3.5 - Let X= { a,b,c }, τ = { ∅, X, {a}, {a,c} }. B = {a,b} is g-closed but not α*-closed in (X,τ).

Theorem 3.6 - Every α*-closed set is a gs-closed set.

The reverse implication does not hold as it can be seen from the following example.

Example 3.7 - Let X= { a,b,c }, τ = { ∅, X, {a}, {a,c} }. C = {a,b} is gs-closed but not α*-closed in (X,τ).

Theorem 3.8 - Every α*-closed set is a gp-closed set.

The converse of the above theorem is not true in general as it can be seen from the following example.

Example 3.9 - Let X= { a,b,c }, τ = { ∅, X, {a}, {a,c} }. D = {a,b} is gp-closed but not α*-closed in (X,τ).

Theorem 3.10 - Every α*-closed set is a gα-closed set.

The converse of the above theorem is not true in general as it can be seen from the following example.

Example 3.11 - Let X= { a,b,c }, τ = { ∅, X, {a}, {a,c} }. E = {c} is gα-closed but not α*-closed in (X,τ).

Theorem 3.12 - Every α*-closed set is a αg -closed set.

The reverse implication does not hold as it can be seen from the following example

Example 3.13 - Let X= { a,b,c }, τ = { ∅, X, {a}, {a,c} }. F = {c} is αg -closed but not α*-closed in (X,τ).

Theorem 3.14 - Every α*-closed set is a gsp -closed set.

The converse of the above theorem is not true in general as it can be seen from the following example.

Example 3.15 - Let X= { a,b,c }, τ = { ∅, X, {a}, {a,c} }. G = {a,b} is gsp -closed but not α*-closed in (X,τ).

Theorem 3.16 - Every α*-closed set is a strongly g closed set.

The converse of the above theorem is not true in general as it can be seen from the following example

Example 3.17 - Let X= { a,b,c }, τ = { ∅, X, {a}, {a,c} }. H = {a,b} is strongly g-closed but not α*-closed in (X,τ).

Remark 3.18 - α*-closedness is independent from α-closedness

Let (X, τ) be as in the example 3.5. Let B = {c}.Then B is α-closed but not α*-closed. Let (X, τ) be as in the example 3.3. Let B = {a,c}. Then B is α*-closed but not α-closed.

Remark 3.19 - If A and B are α*-closed sets, then A∩B is also a α*-closed set.

The result follows from the fact that Cl(A∩B) = Cl(A) ∩ Cl(B).

Theorem 3.20 - If A is both α-open and α*-closed in (X, τ), then A is a closed set.

Theorem 3.21 - A is an α*-closed set of (X, τ) if and only if Cl(A)\A does not contain any non-empty α-closed set.

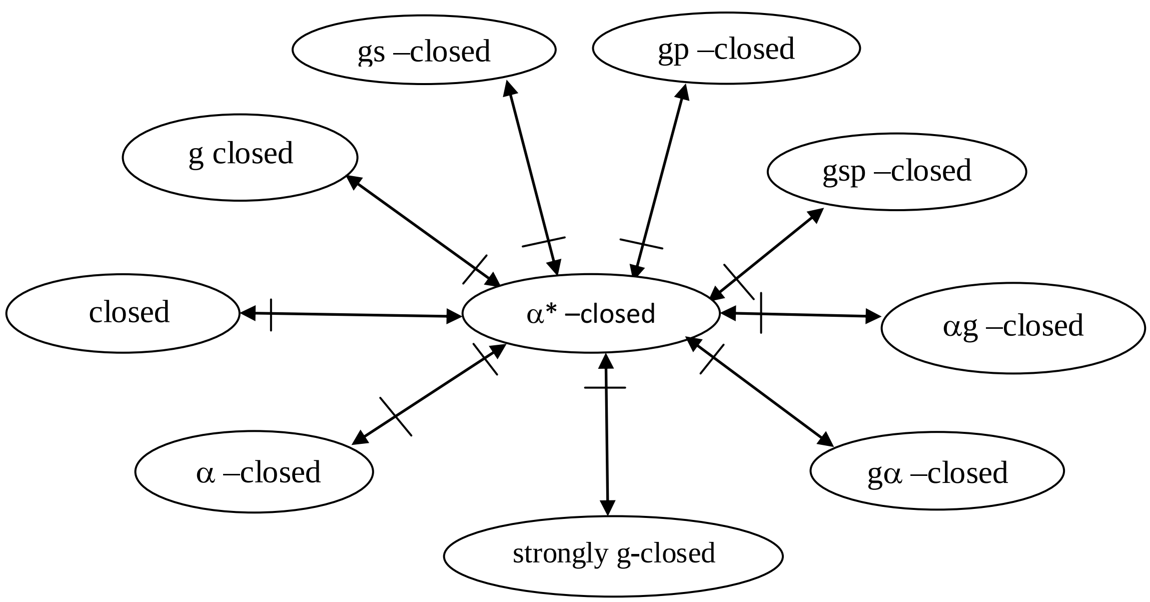
Proof : Necessity - Let F be a α-closed set of (X, τ) such that F ⊆ Cl(A)\A. Then A ⊆ X\F. Since A is an α*-closed set and X\F is α-open such that A ⊆ X\F. Then Cl(A) ⊆ X\F. This implies F ⊆ X\Cl(A). So F ⊆ (X\Cl(A) ∩ (Cl(A)\A). Therefore F = ∅

Sufficiency - Let A be a subset of (X, τ) such that Cl(A)\A does not contain any non-empty α-closed set. Let U be a α-open set of (X, τ) such that $A \subseteq U$. If $Cl(A) \nsubseteq U$, then $Cl(A) \cap U^C \neq \emptyset$ and $Cl(A) \cap U^C$ is α-closed. Then $\emptyset \neq Cl(A) \cap U^C \subseteq Cl(A)\backslash A$. So Cl(A)\A contains a non-empty α-closed set which is a contradiction. This implies $Cl(A) \subseteq U$. Therefore A is an α*-closed set **.**

Theorem 3.22 - If A is an α*-closed set of (X, τ) such that $A \subseteq B \subseteq Cl(A)$, then B is an α*-closed set of (X, τ).

PROOF : Let U be a α-open set such that $B \subseteq U$, then $A \subseteq B \subseteq U$. Since A is an α*-closed set, $Cl(A) \subseteq U$. Now $B \subseteq Cl(A)$ implies $Cl(B) \subseteq Cl(A) \subseteq U$. This implies $Cl(B) \subseteq U$. Therefore B is an α*-closed set of (X, τ).

3.23 – Thus we have the following diagram.



4.APPLICATIONS OF α*-CLOSED SETS

As applications of α*-closed sets, four new spaces namely T_α^*, ${}_{\alpha g}T_\alpha^*$, ${}_{gs}T_\alpha^*$ and ${}_{g}T_\alpha^*$ spaces are introduced.

We introduce the following definition.

Definition 4.1 - A space (X, τ) is called a T_α^* space if every α*-closed set is closed.

Theorem 4.2 - Every $T_{1/2}$ space is a T_α^* space.

A T_α^* space need not be a $T_{1/2}$ space in general as it can be seen from the following example.

Example 4.3 - Let X= { a,b,c }, τ = { ∅, X, {a}, {a,c} }. Then (X, τ) is a T_α^* space. A = {a,b} is a g-closed set but not a closed set. Therefore (X, τ) is not a $T_{1/2}$ space.

Theorem 4.4 - Every T_b space is a T_α^* space.

The converse of the above theorem is not true in general as it can be seen from the following example.

Example 4.5 - Let X= { a,b,c }, τ = { ∅, X, {a}, {a,c} }. α*-closed sets of τ are X, ∅, {b,c}, {b} and every α*-closed set is closed. Therefore (X, τ) is a T_α^* space. B = {a,b} is a gs-closed set but not a closed set of (X, τ). Hence (X, τ) is not a T_b space. Therefore a T_α^* space need not be a T_b space.

Theorem 4.6 - A space (X, τ) which is both $T_{1/2}$ and T_d is a T_α^* space.

Theorem 4.7 - A space (X, τ) which is both ${}_\alpha T_d$ and $T_{1/2}$ is a T_α^* space.

Theorem 4.8 - Every ${}_\alpha T_b$ space is a T_α^* space.

The converse of the above theorem is not true in general as it can be seen from the following example.

Example 4.9 – Let X= { a,b,c }, τ = { ∅, X, {a}, {a,c} }. Then D = {c} is a αg -closed set but not a closed set in (X, τ). Therefore (X,τ) is not a ${}_\alpha T_b$ space. α*-closed sets are ∅, X, {b}, {b,c} and all these are closed in (X,τ). Therefore (X,τ) is a T_α^* space. Hence (X, τ) is a T_α^* space but not a ${}_\alpha T_b$ space.

Definition 4.10 - A space (X, τ) is called a ${}_{gs}T_\alpha^*$ space if every gs-closed set is α*-closed.

Theorem 4.11 - Every T_b space is a ${}_{gs}T_\alpha^*$ space.

The converse of the above theorem is not true in general as it can be seen from the following example.

Example 4.12 - Let X={a,b,c}, τ = {∅, X, {a},{b,c}}. Then C = {b} is a gs-closed set but not a closed set in (X,τ). Therefore (X,τ) is not a T_b space. α*-closed sets are ∅, X, {a}, {b}, {c}, {a,b}, {a,c}, {b,c}. gs-closed sets are ∅,X,{a},{b},{c},{a,b},{a,c},{b,c}. Every gs-closed set is α*-closed in (X,τ). Therefore (X,τ) is a ${}_{gs}T_\alpha^*$ space. Hence (X,τ) is a ${}_{gs}T_\alpha^*$ space but not a T_b space.

Definition 4.13 - A space (X, τ) is called a ${}_{\alpha g}T_\alpha^*$ space if every αg-closed set is α*closed.

Theorem 4.14 - Every ${}_{\alpha}T_b$ space is a ${}_{\alpha g}T_{\alpha}^{*}$ space.

The converse of the above theorem is not true in general as it can be seen from the following example.

Example 4.15 - Let X={a,b,c}, τ = {∅, X, {a},{b,c}}. Then E={b} is a αg-closed set but not closed in (X,τ). Therefore (X,τ) is not a ${}_{\alpha}T_b$ space. α*-closed sets are ∅, X, {a}, {b}, {c}, {a,b}, {a,c}, {b,c}. The αg-closed sets are ∅,X,{a},{b},{c},{a,b},{a,c},{b,c}. Every αg-closed set is α*-closed in (X,τ). Therefore (X,τ) is a ${}_{\alpha g}T_{\alpha}^{*}$ space. Hence (X,τ) is a ${}_{\alpha g}T_{\alpha}^{*}$ space but not a ${}_{\alpha}T_b$ space.

Definition 4.16 - A space (X, τ) is called a ${}_{g}T_{\alpha}^{*}$ space if every g-closed set is α*-closed.

Theorem 4.17 - Every $T_{1/2}$ space is a ${}_{g}T_{\alpha}^{*}$ space.

The converse of the above theorem is not true in general as it can be seen from the following example.

Example 4.18 - Let X={a,b,c}, τ = {∅, X, {a},{b,c}}. Then F = {b} is a g-closed set but not closed set in (X, τ) . Therefore (X, τ) is not a $T_{1/2}$ space. α*-closed sets are ∅, X, {a}, {b}, {c}, {a,b}, {a,c}, {b,c}. g-closed sets are ∅,X,{a},{b},{c},{a,b},{a,c},{b,c}. Every g-closed set is α*-closed in (X, τ). Therefore (X, τ) is a ${}_{g}T_{\alpha}^{*}$ space. Hence (X, τ) is a ${}_{g}T_{\alpha}^{*}$ space but not a $T_{1/2}$ space.

Theorem 4.19 - A space (X,τ) is both ${}_{g}T_{\alpha}^{*}$ space and T_{α}^{*} space if and only if it is a $T_{1/2}$ space.

Proof : Necessity- Let (X,τ) be both a ${}_{g}T_{\alpha}^{*}$ space and a T_{α}^{*} space. Let A be a g-closed set. Then A is α*-closed, since (X,τ) is a ${}_{g}T_{\alpha}^{*}$ space. A is closed, since (X,τ) is a T_{α}^{*} space. Since every g-closed set is closed, (X,τ) is a $T_{1/2}$ space.

Sufficiency- Let (X,τ) be a $T_{1/2}$ space. Then by theorem 4.17, (X,τ) is a ${}_{g}T_{\alpha}^{*}$ space. By theorem 4.2, (X,τ) is a T_{α}^{*} space. Hence (X,τ) is both a ${}_{g}T_{\alpha}^{*}$ space and a T_{α}^{*} space.

Theorem 4.20 - A space (X,τ) is both a ${}_{gs}T_{\alpha}^{*}$ space and a T_{α}^{*} space if and only if it is a T_b space.

Proof : Necessity - Let (X,τ) be both a ${}_{gs}T_{\alpha}^{*}$ space and a T_{α}^{*} space. Let A be a gs-closed set. Then A is α*-closed, since (X,τ) is a ${}_{gs}T_{\alpha}^{*}$ space and A is closed, since (X,τ) is a T_{α}^{*}space. Therefore (X,τ) is a T_b space.

Sufficiency - Let (X,τ) be a T_b space. By theorem 4.11, (X,τ) is a ${}_{gs}T_{\alpha}^{*}$ space. By theorem 4.4, (X,τ) is a T_{α}^{*} space. Hence (X,τ) is both a ${}_{gs}T_{\alpha}^{*}$ space and a T_{α}^{*} space.

Theorem 4.21 - A space (X,τ) is both a ${}_{\alpha g}T_{\alpha}^{*}$ space and a T_{α}^{*} space if and only if it is a ${}_{\alpha}T_b$ space.

Proof : Necessity - Let (X,τ) be both a $_{\alpha g}T_\alpha^*$ space and a T_α^* space. Let A be a αg-closed set. Then A is α*-closed, since (X,τ) is a $_{\alpha g}T_\alpha^*$ space and A is closed, since (X,τ) is a T_α^* space. Therefore (X,τ) is a $_\alpha T_b$ space

Sufficiency - Let (X,τ) be a $_\alpha T_b$ space. By theorem 4.14, (X,τ) is a $_{\alpha g}T_\alpha^*$ space. By theorem 4.8, (X,τ) is a T_α^* space. Hence (X,τ) is both a $_{\alpha g}T_\alpha^*$ space and a T_α^* space.

Remark 4.22 - $_{gs}T_\alpha^*$ space and T_α^* space are independent as it can be seen from the next two examples.

Example 4.23 - Let X = {a,b,c}, τ = {∅ , X , {c} , {b,c} } (X, τ) is a T_α^* space, since every α*-closed set is closed. A = {b} is gs-closed but not α*-closed and hence (X, τ) is not a $_{gs}T_\alpha^*$ space. Therefore (X, τ) is a T_α^* space but not a $_{gs}T_\alpha^*$ space.

Example 4.24 - Let X={a,b,c}, τ = {∅, X, {a},{b,c}}. (X, τ) is a $_{gs}T_\alpha^*$ space, since every gs-closed set is α*-closed. But A = {b} is α*-closed and not closed and hence (X, τ) is not a T_α^* space. Therefore (X, τ) is a $_{gs}T_\alpha^*$ space but not a T_α^* space.

Remark 4.25 - $_{\alpha g}T_\alpha^*$ space and T_α^* space are independent as it can be seen from the next two examples.

Example 4.26 - Let (X,τ) be as in the example 4.23. (X, τ) is a T_α^* space since every α*-closed set in it is closed. But A = {b} is αg-closed but not α*-closed and hence (X, τ) is not a $_{\alpha g}T_\alpha^*$ space. Therefore (X, τ) is a T_α^* space but not a $_{\alpha g}T_\alpha^*$ space.

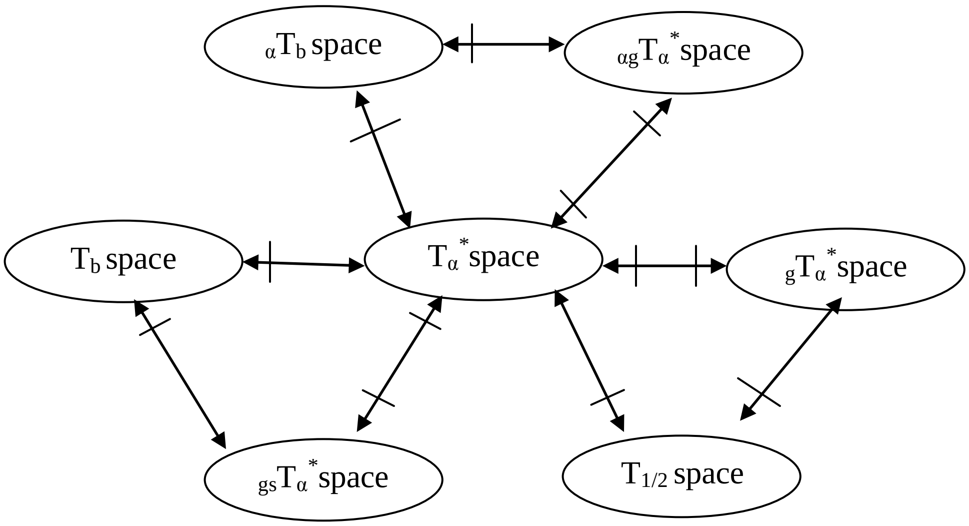
Example 4.27 - Let (X,τ) be as in the example 4.24. From example 4.12 and example 4.15, every αg-closed set is α*-closed. So (X, τ) is a $_{\alpha g}T_\alpha^*$ space. But A={b} is α*-closed but not closed. Hence (X, τ) is not a T_α^* space. Therefore (X, τ) is a $_{\alpha g}T_\alpha^*$ space but not a T_α^* space.

Remark 4.28 - $_gT_\alpha^*$ space and T_α^* space are independent as it can be seen from the next two examples.

Example 4.29 - Let (X,τ) be as in the example 4.23. (X, τ) is a T_α^* space since every α*-closed set in it is closed. But A = {a,c} is g-closed and not α*-closed and hence (X, τ) is not a $_gT_\alpha^*$ space. Therefore (X, τ) is a T_α^* space but not a $_gT_\alpha^*$ space.

Example 4.30 - Let (X,τ) be as in the example 4.24. From example 4.12 and example 4.18, every g-closed set is α*-closed. So (X, τ) is a $_gT_\alpha^*$ space. But A={b} is α*-closed but not closed. Hence (X, τ) is not a T_α^* space. Therefore (X, τ) is a $_gT_\alpha^*$ space but not a T_α^* space.

4.31 Thus we have the following diagram



5.α*-CONTINUOUS AND α*-IRRESOLUTE MAPS

We introduce the following definition

Definition 5.1 - A function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is called α*-continuous if $f^{-1}(V)$ is a α*-closed set of (X, τ) for every closed set V of (Y, σ).

Theorem 5.2 - Every continuous map is α*-continuous.

The following example supports that the converse of the above theorem is not true

Example 5.3 - Let X = {a,b,c} and τ = { ∅, X, {a}, {b,c} }.Define $g : (X, \tau) \longrightarrow (X, \tau)$ by g(a) = b, g(b) = a and g(c) = c. g is α*-continuous but not continuous since {b,c} is a closed set of (X, τ) but $g^{-1}(\{b,c\}) = \{a,c\}$ is not closed.

Theorem 5.4 - Every α*-continuous map is g-continuous, αg-continuous , gsp-continuous, gp-continuous, gs-continuous, gα-continuous and strongly g-continuous.

Proof : Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ be α*-continuous. Let $V \subseteq Y$ be a closed set in Y. Then $f^{-1}(V)$ is α*-closed since f is α*-continuous. Now by theorem 3.4 $f^{-1}(V)$ is g-closed, by theorem 3.12 $f^{-1}(V)$ is αg-closed, by theorem 3.14 $f^{-1}(V)$ is gsp-closed, by theorem 3.8 $f^{-1}(V)$ is gp-closed, by theorem 3.6 $f^{-1}(V)$ is gs-closed, by theorem 3.10 $f^{-1}(V)$ is gα-closed, by theorem 3.16 $f^{-1}(V)$ is strongly g-closed.

Example 5.5 - Let X = Y = {a,b,c}, τ = {∅ , X , {c} , {b,c} } and σ = {∅ , X , {a} , {a,b} }. Define $g : (X, \tau) \longrightarrow (Y, \sigma)$ by g(a) = c, g(b) = a, g(c) = b. The mapping g is g-continuous since $f^{-1}(V)$ is a g-closed set of (X, τ) for every closed set V of (Y, σ). g is not α*-continuous since {b,c} is a closed set of (Y, σ) but $g^{-1}(\{b,c\})$ = {a,c} is not α*-closed.

Example 5.6 - Let X = Y = {a,b,c}, τ = {∅ , X , {c} , {b,c} } and σ = {∅ , X , {a} , {a,b} }. Define g : (X, τ) $\longrightarrow$ (Y, σ) by g(a) = b, g(b) = c, g(c) = a. The mapping g is αg-continuous since $f^{-1}(V)$ is a αg-closed set of (X, τ) for every closed set V of (Y, σ). g is not α*-continuous since {c} is a closed set of (Y, σ) but $g^{-1}(\{c\})$ = {b} is not α*-closed.

Example 5.7 - Let X = Y = {a,b,c}, τ = {∅ , X , {c} , {b,c} } and σ = {∅ , X , {a} , {a,b} }. Define g : (X, τ) $\longrightarrow$ (Y, σ) by g(a) = c, g(b) = a, g(c) = b. The mapping g is gsp-continuous since $f^{-1}(V)$ is a gsp-closed set of (X, τ) for every closed set V of (Y, σ). g is not α*-continuous since {b,c} is a closed set of (Y, σ) but $g^{-1}(\{b,c\})$ = {a,c} is not α*-closed.

Example 5.8 - Let X = Y = {a,b,c}, τ = {∅ , X , {c} , {b,c} } and σ = {∅ , X , {a} , {a,b} }. Define g : (X, τ) $\longrightarrow$ (Y, σ) by g(a) = c, g(b) = a, g(c) = b. The mapping g is gp-continuous since $f^{-1}(V)$ is a gp-closed set of (X, τ) for every closed set V of (Y, σ). g is not α*-continuous since {b,c} is a closed set of (Y, σ) but $g^{-1}(\{b,c\})$ = {a,c} is not α*-closed.

Example 5.9 - Let X = Y = {a,b,c}, τ = {∅ , X , {c} , {b,c} } and σ = {∅ , X , {a} , {a,b} }. Define g : (X, τ) $\longrightarrow$ (Y, σ) by g(a) = b, g(b) = c, g(c) = a. The mapping g is gs-continuous since $f^{-1}(V)$ is a gs-closed set of (X, τ) for every closed set V of (Y, σ). g is not α*-continuous since {c} is a closed set of (Y, σ) but $g^{-1}(\{c\})$ = {b} is not α*-closed.

Example 5.10 - Let X = Y = {a,b,c}, τ = {∅ , X , {c} , {b,c} } and σ = {∅ , X , {a} , {a,b} }. Define g : (X, τ) $\longrightarrow$ (Y, σ) by g(a) = b, g(b) = c, g(c) = a. The mapping g is gα-continuous since $f^{-1}(V)$ is a gα-closed set of (X, τ) for every closed set V of (Y, σ). g is not α*-continuous since {c} is a closed set of (Y, σ) but $g^{-1}(\{c\})$ = {b} is not α*-closed.

Example 5.11 - Let X = Y = {a,b,c}, τ = {∅ , X , {c} , {b,c} } and σ = {∅ , X , {a} , {a,b} }. Define g : (X, τ) $\longrightarrow$ (Y, σ) by g(a) = c, g(b) = a, g(c) = b. The mapping g is strongly g -continuous since $f^{-1}(V)$ is a strongly g-closed set of (X, τ) for every closed set V of (Y, σ). g is not α*-continuous since {b,c} is a closed set of (Y, σ) but $g^{-1}(\{b,c\})$ = {a,c} is not α*-closed.

Definition 5.12 - A function f : (X, τ) $\longrightarrow$ (Y, σ) is called α*-irresolute if $f^{-1}(V)$ is a α*-closed set of (X, τ) for every α*-closed set V of (Y, σ).

Theorem 5.13 - Every α*-irresolute function is α*-continuous.

Proof: Let f : (X, τ) $\longrightarrow$ (Y, σ) be an α*-irresolute. Let V be a closed set of (Y, σ).Every closed set is α*-closed. Therefore V is α*-closed. Then $f^{-1}(V)$ is α*-closed since f is α*-irresolute and hence f is α*-continuous

The converse of the above theorem is not true in general as it can be seen from the following example.

Example 5.14 - Let X = Y = {a,b,c}, $\tau = \{\emptyset , X , \{c\} , \{b,c\} \}$ and $\sigma = \{\emptyset , X , \{a,b\} \}$.Define $g : (X, \tau) \longrightarrow (Y, \sigma)$ by g(a) = c, g(b) = a, g(c) = b. $g^{-1}(\{c\}) = \{a\}$ is α^*-closed in (X, τ) and hence g is α^*-continuous

$g^{-1}(\{b,c\}) = \{a,c\}$ is not α^*-closed in (X, τ) and hence g is not α^*-irresolute.

Theorem 5.15 - Let (X, τ) be a $T_\alpha{}^*$ space and $f : (X, \tau) \longrightarrow (Y, \sigma)$ be α^*-irresolute, then f is continuous.

Proof: Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ be an α^*-irresolute. Let V be a closed set of (Y, σ). Every closed set is α^*-closed. $f^{-1}(V)$ is α^*-closed since f is α^*-irresolute. Every α^*-closed set is closed in X, since (X, τ) is a $T_\alpha{}^*$ space. Therefore $f^{-1}(v)$ is closed and hence f is continuous.

Theorem 5.16 - Let (Y, σ) be a $T_\alpha{}^*$ space and $f : (X, \tau) \longrightarrow (Y, \sigma)$ be continuous , then f is α^*-irresolute.

Proof: Let A be a α^*-closed set in Y. Then A is closed, since (Y, σ) is a $T_\alpha{}^*$ space. $f^{-1}(A)$ is closed, since f is continuous. Every closed set is α^*-closed. Therefore f is α^*-irresolute.

Theorem 5.17 - Let (Y, σ) be a ${}_{\alpha g}T_{\alpha^*}$ space and $f : (X, \tau) \longrightarrow (Y, \sigma)$ be α^*-irresolute , then f is αg-irresolute.

Proof: Let A be a αg-closed set in Y.Then A is α^*-closed, since (Y, σ) is a ${}_{\alpha g}T_{\alpha^*}$ space. $f^{-1}(A)$ is α^*-closed, since f is α^*-irresolute. Every α^*-closed set is αg-closed. Therefore f is αg-irresolute.

Theorem 5.18 - Let (Y, σ) be a ${}_{gs}T_\alpha{}^*$ space and $f : (X, \tau) \longrightarrow (Y, \sigma)$ be α^*-irresolute , then f is gs-irresolute.

Proof: Let A be a gs-closed set in Y. Then A is α^*-closed, since (Y, σ) is a ${}_{gs}T_{\alpha^*}$ space. $f^{-1}(A)$ is α^*-closed, since f is α^*-irresolute. Every α^*-closed set is gs-closed. Hence $f^{-1}(A)$ is gs-closed and therefore f is gs-irresolute.

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