

ANALYSIS OF SINGLE SERVER FIXED BATCH SERVICE QUEUEING SYSTEM UNDER MULTIPLE VACATIONS WITH UNRELIABLE SERVER

G. Ayyappan^{#1}, G.Devipriya^{#2}, A. Muthu Ganapathi Subramanian^{#3}

#1 Associate Professor, Pondicherry Engineering College, Pondicherry

#2 Assistant Professor, Sri Ganesh College of Engineering & Technology, Pondicherry

#3 Associate Professor, Kanchi Mamunivar Centre for Post Graduate Studies,
Pondicherry

ABSTRACT

Consider a single server fixed batch service queueing system under multiple vacations with unreliable server in which the arrival rate λ follows a Poisson process, the service time follows an exponential distribution with parameter μ . Further we assume that the length of time the server in vacation follows an exponential distribution with parameter η . Assume that the system initially contains k customers when the server enters in to the system and starts the service immediately in a batch of size k . After completion of a service, if he finds less than k customers in the queue, then the server goes for a multiple vacation of a length η . The concepts of breakdown and repair state of server are incorporated in this model. Further we assume that the breakdown may occur only if the server is busy (active breakdown). The time between breakdowns follows an exponential distribution with parameter α . Once the breakdown occurs, the repair starts. The time to the end of repair is also exponentially distributed with parameter β . If there is a breakdown in service for a batch of k customers (active breakdown) then the server goes to the state of breakdown and the k customers in this batch with incomplete service will stay in the queue in front of the service station and service will be given to this batch of k customers once again if the server returns from the repair state. This model is completely solved by constructing the generating function and Rouche's theorem is applied and we have derived the closed form solutions for probability of number of customers in the queue during the server busy, vacation and breakdown. Further we are providing the analytical solution for mean number of customers and variance of the system. Numerical studies have been done for analysis of mean, variance and probability of number of customers in the queue during the server busy, vacation and breakdown for various values of λ , μ , α , β , η and k and also various particular cases of this model have been discussed.

Keywords: Single Server, Batch Service, Breakdown and repair, Multiple vacations, Steady state distribution.

Corresponding Author: G. Devipriya

1. INTRODUCTION

The subject of queueing systems wherein the service channel is subject to breakdowns from time to time is a popular subject that has received a lot of attention for the last fifty years. In most queueing systems, it is assumed that the server is available on a permanent basis. However, these assumptions are practically unrealistic. The server may well be subjected to lengthy and unpredictable breakdowns while serving a customer. For instance, in manufacturing systems, the machine may breakdown due to machine or job related problems

in computer systems, the machine may be subjected to scheduled backups and unpredictable failures. In these systems, server breakdown results in a period of unavailable time until it is repaired. Understanding the behaviour of the unreliable server and the effect of machine breakdowns and repairs in these systems is important as this affects not only the systems efficiency but also the queue length and the customer's waiting time in the queue.

Several authors have investigated queueing models with server breakdowns and vacations in different frame works in recent past. Gaver [5] first proposed an ordinary M/G/1 queueing system with interrupted service and priorities. Ke [6] investigated the operating characteristics of an M[x]/G/1 queue with single or multiple vacation policy, server breakdown, and startup/closedown times. Wang *et al.* [9] treated an M[x]/M/1 queueing system with multiple vacations and server breakdowns. Ke and Lin [7] used the maximum entropy approach to investigate an M[x]/G/1 queue with N policy, server breakdowns, and single vacation policy. Ayyappan et al [1, 2] have studied the effect of unreliable server for Multi server retrial queueing system and also for priority services.

For batch service queues with vacations, there have been a few related works. Dhas [3] considered Markovian batch service systems and obtained the queue length distributions by matrix-geometric methods. Lee et al. [8] obtained various performance measures for M/G^B/1 queue with single vacation. Dshalalow and Yellen [4] considered a non-exhaustive batch service system with multiple vacations in which the server starts a multiple vacation whenever the queue drops below a level r and resumes service at the end of a vacation segment when the queue accumulates to at least r. They called such a system (r, R)-quorum system. They applied the theory of the first excess level (Dshalalow [4]). Lee et al. [8] showed that for some batch service queues; mean queue length may even decrease in systems with server vacations. This has an implication that for some batch service queues, customers do not have to complain about unavailability of the server. Instead, they would rather force the server to take a vacation.

In this paper we are analyzing a special batch service queue called the fixed size batch service queue under multiple vacations with unreliable server. The model is described in Section 2. In Section 3, we have derived the system steady state equations and using these equations, the probability generating functions for number of customers in the queue when the server is busy or in vacation or in breakdown are derived and also obtained steady state probability distributions. Section 4 deals with stability condition of the system. Closed form solutions of System performance measures are obtained section 6. A numerical study is carried out in Section 7 to test the effectiveness of the system. We are providing the analytical solution for mean number of customers and variance of the system. Numerical studies have been done for analysis of mean and variance for various values of λ , μ , α , β , η and k and also various particular cases of this model have been discussed.

2. DESCRIPTION OF THE MODEL

Consider a single server fixed batch service queueing system under multiple vacations with unreliable server in which the arrival rate λ follows a Poisson process, the service time follows an exponential distribution with parameter μ . Further we assume that the length of time the server in vacation follows an exponential distribution with parameter η . Assume that the system initially contains k customers when the server enters in to the system and starts the service immediately in a batch of size k . After completion of a service, if he finds less than k

customers in the queue, then the server goes for a multiple vacation of a length η . The concepts of breakdown and repair state of server are incorporated in this model. Further we assume that the breakdown may occur only if the server is busy (active breakdown). The time between breakdowns follows an exponential distribution with parameter α . Once the breakdown occurs, the repair starts. The time to the end of repair is also exponentially distributed with parameter β . If there is a breakdown in service for a batch of k customers (active breakdown) then the server goes to the state of breakdown and the k customers in this batch with incomplete service will stay in the queue in front of the service station and service will be given to this batch of k customers once again if the server returns from the repair state. Let $\langle N(t); C(t) \rangle$ be a random process where $N(t)$ be the random variable which represents the number of customers in queue at time t and $C(t)$ be the random variable which represents the server status (busy/vacation/breakdown) at time t .

We define

$P_{n,1}(t)$ - Probability that the server is busy if there are n customers in the queue at time t .

$P_{n,2}(t)$ - probability that the server is in vacation if there are n customers in the queue at time t

$P_{n,3}(t)$ - probability that the server is in breakdown if there are n customers in the queue at time t

The Chapman- Kolmogorov equations are

$$\dot{P}_{0,1}(t) = -(\lambda + \mu + \alpha)P_{0,1}(t) + \mu P_{k,1}(t) + \eta P_{k,2}(t) + \beta P_{k,3}(t) \quad (1)$$

$$\dot{P}_{n,1}(t) = -(\lambda + \mu + \alpha)P_{n,1}(t) + \lambda P_{n-1,1}(t) + \eta P_{n+k,2}(t) + \mu P_{n+k,1}(t) + \beta P_{n+k,3}(t) ; n = 1, 2, 3, \dots \quad (2)$$

$$\dot{P}_{0,2}(t) = -\lambda P_{0,2}(t) + \mu P_{0,1}(t) \quad (3)$$

$$\dot{P}_{n,2}(t) = -\lambda P_{n,2}(t) + \lambda P_{n-1,2}(t) + \mu P_{n,1}(t) ; n = 1, 2, 3, \dots, k-1 \quad (4)$$

$$\dot{P}_{n,2}(t) = -(\lambda + \eta)P_{n,2}(t) + \lambda P_{n-1,2}(t) ; n \geq k \quad (5)$$

$$\dot{P}_{k,3}(t) = -(\lambda + \beta)P_{k,3}(t) + \alpha P_{0,1}(t) \quad (6)$$

$$\dot{P}_{n,3}(t) = -(\lambda + \beta)P_{n,3}(t) + \lambda P_{n-1,3}(t) + \alpha P_{n-k,1}(t) ; n \geq k+1 \quad (7)$$

3. EVALUATION OF STEADY STATE PROBABILITIES

In this section we are finding the closed form solutions for number of customers in the queue when the server is busy, in vacation, in breakdown state using various Generating function.

When steady state prevails, the equations (1) to (7) becomes

$$(\lambda + \mu + \alpha)P_{0,1} = \mu P_{k,1} + \eta P_{k,2} + \beta P_{k,3} \quad (8)$$

$$(\lambda + \mu + \alpha)P_{n,1} = \lambda P_{n-1,1} + \eta P_{n+k,2} + \mu P_{n+k,1} + \beta P_{n+k,3} ; n = 1, 2, \dots \quad (9)$$

$$\lambda P_{0,2} = \mu P_{0,1} \quad (10)$$

$$\lambda P_{n,2} = \lambda P_{n-1,2} + \mu P_{n,1} ; n = 1, 2, 3, \dots, k-1 \quad (11)$$

$$(\lambda + \eta)P_{n,2} = \lambda P_{n-1,2} ; n \geq k \quad (12)$$

$$(\lambda + \beta)P_{k,3} = \alpha P_{0,1} \quad (13)$$

$$(\lambda + \beta)P_{n,3} = \lambda P_{n-1,3} + \alpha P_{n-k,1} ; n \geq k+1 \quad (14)$$

Generating functions for the number of customers in the queue when the server is busy, in vacation and in breakdown are defined as

$$G(z) = \sum_{n=0}^{\infty} P_{n,1} z^n , H(z) = \sum_{n=0}^{\infty} P_{n,2} z^n \text{ and } I(z) = \sum_{n=k}^{\infty} P_{n,3} z^n$$

Multiply the equation (8) with 1 and (9) with z^n on both sides and summing over $n = 0$ to ∞ , we get

$$G(z)[\lambda z^{k+1} - (\lambda + \mu + \alpha)z^k + \mu] + \eta H(z) + \beta I(z) = \mu \sum_{n=0}^{k-1} P_{n,1} z^n + \eta \sum_{n=0}^{k-1} P_{n,2} z^n \quad (15)$$

Adding equation (10), (11) and (12) after multiply with 1, z^n and z^n ($n = 1, 2, 3, \dots$) respectively, we get

$$H(z)[\eta + \lambda(1-z)] = \mu \sum_{n=0}^{k-1} P_{n,1} z^n + \eta \sum_{n=0}^{k-1} P_{n,2} z^n \quad (16)$$

Multiply the equation (13) with z^k and (14) with z^n on both sides and summing over $n=k$ to ∞ , we get

$$I(z)[\beta + \lambda - \lambda z] = \alpha z^k G(z) \quad (17)$$

Substitute equation (16) and (17) in (15), we get

$$G(z) \left[\frac{\lambda^2 z^{k+1} - \lambda(\lambda + \mu + \alpha + \beta)z^k + \mu\beta(1+z+\dots+z^{k-1}) + \mu\lambda}{\beta + \lambda - \lambda z} \right] = \lambda H(z) \quad (18)$$

From the equation (16), we get

$$H(z) = \frac{\mu \sum_{n=0}^{k-1} P_{n,1} z^n + \eta \sum_{n=0}^{k-1} P_{n,2} z^n}{\eta + \lambda(1-z)} \quad (19)$$

Equation (19) represents the probability generating function for number of customers in the queue when the server is in vacation.

From (18) and (19), we get

$$G(z) = \frac{\lambda \left(\mu \sum_{n=0}^{k-1} P_{n,1} z^n + \eta \sum_{n=0}^{k-1} P_{n,2} z^n \right) (\beta + \lambda - \lambda z)}{(\eta + \lambda(1-z)) (\lambda^2 z^{k+1} - \lambda(\lambda + \mu + \alpha + \beta)z^k + \mu\beta(1+z+\dots+z^{k-1}) + \mu\lambda)} \quad (20)$$

Equation (20) represents the probability generating function for number of customers in the queue when the server is busy.

Put $z = 1$ in equations (17) and (18), we get

$$I(1) = \left(\frac{\alpha}{\beta} \right) G(1) \quad (21)$$

$$G(1) = \frac{\frac{\lambda}{k\mu} H(1)}{1 - \frac{\lambda}{k\mu} \left(1 + \frac{\alpha}{\beta} \right)} \quad (22)$$

$$\text{The normalized condition is } G(1) + H(1) + I(1) = 1 \quad (23)$$

Substituting (21) and (22) in (23), we get

$$H(1) = 1 - \frac{\lambda}{k\mu} \left(1 + \frac{\alpha}{\beta} \right) \quad (24)$$

From the equations (21), (22) and (24), we get

$$\text{Steady state probability that the server is busy} = G(1) = \frac{\lambda}{k\mu}$$

$$\text{Steady state probability that the server in vacation} = H(1) = 1 - \frac{\lambda}{k\mu} \left(1 + \frac{\alpha}{\beta} \right)$$

$$\text{Steady state probability that the server in breakdown} = I(1) = \left(\frac{\alpha}{\beta} \right) \left(\frac{\lambda}{k\mu} \right)$$

The generating function $G(z)$ in the equation (20) has the property that it must converge inside the unit circle $|z| < 1$. We notice that the expression in the denominator of $G(z)$, $\lambda^2 z^{k+1} - \lambda(\lambda + \mu + \alpha + \beta)z^k + \mu\beta(1 + z + \dots + z^{k-1}) + \mu\lambda$ has $k+1$ zeros. By Rouche's theorem, we notice that $k-1$ zeros of this expression lies inside the circle $|z| < 1$ and must coincide with $k-1$ zeros of numerator of $G(z)$ and two zero lies outside the circle $|z| < 1$. Let z_0 and z_1 be a zero which lies outside the circle $|z| < 1$.

As $G(z)$ converges, $k-1$ zeros of numerator and denominator will be cancelled, we get

$$G(z) = \frac{A(\beta + \lambda - \lambda z)}{(\eta + \lambda - \lambda z)\lambda^2(z - z_0)(z - z_1)} \quad (25)$$

When $z = 1$ in the equation (25), we get

$$G(1) = \frac{A\beta}{\eta\lambda^2(1 - z_0)(1 - z_1)} \quad (26)$$

Using (22) and (24) in (26), we obtain

$$A = \left(\frac{\lambda^3 \eta}{k \mu \beta} \right) (1-z_0)(1-z_1) \quad (27)$$

From the equation (25) and (27), we get

$$G(z) = \frac{\lambda \eta}{k \mu \beta} (1-z_0)(1-z_1) \frac{\beta + \lambda - \lambda z}{(\eta + \lambda - \lambda z)(z - z_0)(z - z_1)} \quad (28)$$

By applying partial fractions, we get

$$\begin{aligned} G(z) &= \frac{\lambda \eta}{k \mu \beta} (1-z_0)(1-z_1) \left(\frac{A_1}{(\eta + \lambda - \lambda z)} + \frac{A_2}{(z - z_0)} + \frac{A_3}{(z - z_1)} \right) \\ G(z) &= \frac{\lambda \eta}{k \mu \beta} \left(\frac{(r_1 - 1)(r_2 - 1)}{r_1 r_2} \right) \left[\left(\frac{A_1}{\eta + \lambda} \right) \sum_{n=0}^{\infty} s^n z^n - A_2 \sum_{n=0}^{\infty} r_1^{n+1} z^n - A_3 \sum_{n=0}^{\infty} r_2^{n+1} z^n \right] \end{aligned} \quad (29)$$

where

$$r_1 = \frac{1}{z_0}, r_2 = \frac{1}{z_1}, s = \frac{\lambda}{\lambda + \eta}, A_1 = \frac{r_1 r_2 s^2 (\beta - \eta)}{(r_1 - s)(r_2 - s)}, A_2 = \frac{[r_1(\beta + \lambda) - \lambda] r_1 r_2}{[r_1(\eta + \lambda) - \lambda](r_2 - r_1)} \text{ and } A_3 = \frac{[r_2(\beta + \lambda) - \lambda] r_1 r_2}{[r_2(\eta + \lambda) - \lambda](r_1 - r_2)}$$

Comparing the coefficient of z^n on both sides of the equation (29), we get

$$p_{n,1} = \frac{\lambda \eta}{k \mu \beta} (r_1 - 1)(r_2 - 1) \left[\frac{(\beta - \eta)s^{n+2}}{(r_1 - s)(r_2 - s)(\eta + \lambda)} - \frac{[r_1(\beta + \lambda) - \lambda]}{[r_1(\eta + \lambda) - \lambda](r_2 - r_1)} r_1^{n+1} - \frac{[r_2(\beta + \lambda) - \lambda]}{[r_2(\eta + \lambda) - \lambda](r_1 - r_2)} r_2^{n+1} \right] \quad (30)$$

Equation (30) represents that the steady state probabilities that there are n customers in the queue when the server is busy.

From the equation (10), we get

$$P_{0,2} = \frac{\mu}{\lambda} P_{0,1} \quad (31)$$

Apply the equation (11) recursively for $n = 1, 2, 3, \dots, k-1$, we get

$$P_{n,2} = \frac{\mu}{\lambda} \sum_{t=0}^n P_{t,1} \quad \text{for } n = 1, 2, 3, \dots, k-1 \quad (32)$$

From the equation (12), we get

$$P_{n,2} = \left(\frac{\lambda}{\lambda + \eta} \right)^{n-k+1} P_{k-1,2}; n \geq k \quad (33)$$

Equations (31), (32) and (33) represent the steady state probabilities that there are n customers in the queue when the server is in vacation.

From the equation (13), we get

$$P_{k,3} = \left(\frac{\alpha}{\lambda + \beta} \right) P_{0,1} \quad (34)$$

Using equation (34) in (14) and apply recursively for $n = k+1, k+2, \dots$ we get

$$P_{n,3} = \frac{\alpha}{\lambda + \beta} \left(\frac{\lambda}{\lambda + \beta} \right)^{n-k} \sum_{t=0}^{n-k} P_{t,1} \left(\frac{\lambda + \beta}{\lambda} \right)^t \text{ for } n \geq k \quad (35)$$

Equations (34) and (35) represent the steady state probabilities that there are n customers in the queue when the server is in breakdown.

4. STABILITY CONDITION

The necessary and sufficient condition for the system to be stable is $\frac{\lambda}{k\mu} \left(1 + \frac{\alpha}{\beta} \right) < 1$.

5. PARTICULAR CASES

If $\alpha \rightarrow 0$ and $\beta \rightarrow \infty$, then this model becomes Single server fixed batch service queueing system under multiple vacation with parameter η .

6. SYSTEM PERFORMANCE MEASURES

In this section, we will list some important performance measures along with their formulas. These measures are used to bring out the qualitative behavior of the queueing model under study. Numerical study has been dealt in very large scale to study the following measures.

1. $P_{n,1} = \frac{\lambda\eta}{k\mu\beta} (r_1 - 1)(r_2 - 1) \left[\frac{(\beta - \eta)s^{n+2}}{(r_1 - s)(r_2 - s)(\eta + \lambda)} - \frac{[r_1(\beta + \lambda) - \lambda]}{[r_1(\eta + \lambda) - \lambda](r_2 - r_1)} r_1^{n+1} - \frac{[r_2(\beta + \lambda) - \lambda]}{[r_2(\eta + \lambda) - \lambda](r_1 - r_2)} r_2^{n+1} \right]$
2. $P_{n,2} = \frac{\mu}{\lambda} \sum_{t=0}^n P_{t,1} \text{ for } n = 0, 1, 2, 3, \dots, k-1$
3. $P_{n,2} = \left(\frac{\lambda}{\lambda + \eta} \right)^{n-k+1} P_{k-1,2} \text{ for } n \geq k$
4. $P_{n,3} = \frac{\alpha}{\lambda + \beta} \left(\frac{\lambda}{\lambda + \beta} \right)^{n-k} \sum_{t=0}^{n-k} P_{t,1} \left(\frac{\lambda + \beta}{\lambda} \right)^t \text{ for } n \geq k$
5. $L_q = \sum_{n=0}^{\infty} n(P_{n,1} + P_{n,2}) + \sum_{n=k}^{\infty} nP_{n,3}$
6. $V(x) = \left(\sum_{n=0}^{\infty} n^2 P_{n,1} + \sum_{n=0}^{\infty} n^2 P_{n,2} + \sum_{n=k}^{\infty} n^2 P_{n,3} \right) - (L_q)^2$

7. NUMERICAL STUDIES

The values of parameters $\lambda, \mu, \alpha, \beta$ and k are chosen so that they satisfy the stability condition discussed in section 4. The system performance measures of this model have been done and expressed in the form of tables for various values $\lambda, \mu, \alpha, \beta, \eta$ and k .

Tables 4, 8 and 12 show the impact of arrival rate λ and k over Mean number of customers in the queue.

Tables 1, 5 and 9 show the steady probabilities distribution when server is busy for various values of λ and different batch size k .

Tables 2, 6 and 10 show the steady state probabilities distribution when the server is in vacation for various values of λ and different batch size k .

Tables 3, 7 and 11 show the steady state probabilities distribution when the server is in breakdown state for various values of λ and different batch size k .

Tables 13, 14 and 15 show the impact of breakdown rate α and k over Mean number of customers in the queue.

Further we conclude that if $\alpha \rightarrow 0$ then this model becomes Single server fixed batch service queueing system under multiple vacation with parameter η .

Table 1: Steady state probabilities distribution for various values of λ and $\mu = 10$, $\alpha = 5$, $\beta = 100$, $\eta = 5$ when the server is busy and batch size is $K=2$

λ	P01	P11	P21	P31	P41	P51	P61	P71	P81	P91
1	0.0377	0.0099	0.0020	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
1.5	0.0497	0.0183	0.0052	0.0013	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000
2	0.0587	0.0272	0.0096	0.0031	0.0009	0.0003	0.0001	0.0000	0.0000	0.0000
2.5	0.0654	0.0359	0.0150	0.0057	0.0020	0.0007	0.0002	0.0001	0.0000	0.0000
3	0.0702	0.0439	0.0209	0.0090	0.0036	0.0014	0.0006	0.0002	0.0001	0.0000
3.5	0.0735	0.0513	0.0271	0.0129	0.0058	0.0025	0.0011	0.0005	0.0002	0.0001
4	0.0757	0.0577	0.0333	0.0173	0.0085	0.0040	0.0019	0.0009	0.0004	0.0002
4.5	0.0770	0.0634	0.0394	0.0220	0.0116	0.0059	0.0029	0.0014	0.0007	0.0003
5	0.0775	0.0681	0.0453	0.0269	0.0151	0.0082	0.0043	0.0023	0.0012	0.0006
5.5	0.0773	0.0721	0.0507	0.0319	0.0189	0.0108	0.0060	0.0033	0.0018	0.0010
6	0.0767	0.0753	0.0558	0.0369	0.0230	0.0138	0.0081	0.0046	0.0026	0.0015
6.5	0.0756	0.0778	0.0603	0.0417	0.0272	0.0170	0.0104	0.0062	0.0037	0.0022
7	0.0741	0.0795	0.0643	0.0464	0.0314	0.0205	0.0130	0.0081	0.0050	0.0030
7.5	0.0723	0.0807	0.0678	0.0508	0.0357	0.0241	0.0159	0.0103	0.0066	0.0041
8	0.0702	0.0812	0.0707	0.0548	0.0399	0.0279	0.0190	0.0127	0.0084	0.0054
8.5	0.0679	0.0812	0.0731	0.0585	0.0440	0.0317	0.0223	0.0154	0.0104	0.0070
9	0.0655	0.0807	0.0749	0.0618	0.0478	0.0356	0.0257	0.0182	0.0127	0.0088
9.5	0.0628	0.0797	0.0761	0.0646	0.0514	0.0393	0.0292	0.0213	0.0153	0.0108

Table 2: Steady state probabilities distribution for various values of λ and $\mu = 10$, $\alpha = 5$, $\beta = 100$, $\eta = 5$ when the server is in vacation and batch size is K=2

λ	P02	P12	P22	P32	P42	P52	P62	P72	P82	P92
1	0.3768	0.4756	0.0793	0.0132	0.0022	0.0004	0.0001	0.0000	0.0000	0.0000
1.5	0.3314	0.4537	0.1047	0.0242	0.0056	0.0013	0.0003	0.0001	0.0000	0.0000
2	0.2935	0.4296	0.1228	0.0351	0.0100	0.0029	0.0008	0.0002	0.0001	0.0000
2.5	0.2614	0.4049	0.1350	0.0450	0.0150	0.0050	0.0017	0.0006	0.0002	0.0001
3	0.2339	0.3804	0.1426	0.0535	0.0201	0.0075	0.0028	0.0011	0.0004	0.0001
3.5	0.2101	0.3565	0.1468	0.0605	0.0249	0.0102	0.0042	0.0017	0.0007	0.0003
4	0.1894	0.3337	0.1483	0.0659	0.0293	0.0130	0.0058	0.0026	0.0011	0.0005
4.5	0.1711	0.3119	0.1477	0.0700	0.0332	0.0157	0.0074	0.0035	0.0017	0.0008
5	0.1550	0.2913	0.1456	0.0728	0.0364	0.0182	0.0091	0.0046	0.0023	0.0011
5.5	0.1406	0.2717	0.1423	0.0746	0.0391	0.0205	0.0107	0.0056	0.0029	0.0015
6	0.1278	0.2533	0.1382	0.0754	0.0411	0.0224	0.0122	0.0067	0.0036	0.0020
6.5	0.1162	0.2359	0.1333	0.0754	0.0426	0.0241	0.0136	0.0077	0.0043	0.0025
7	0.1058	0.2195	0.1280	0.0747	0.0436	0.0254	0.0148	0.0086	0.0050	0.0029
7.5	0.0964	0.2040	0.1224	0.0734	0.0441	0.0264	0.0159	0.0095	0.0057	0.0034
8	0.0878	0.1893	0.1165	0.0717	0.0441	0.0272	0.0167	0.0103	0.0063	0.0039
8.5	0.0799	0.1755	0.1105	0.0696	0.0438	0.0276	0.0174	0.0109	0.0069	0.0043
9	0.0727	0.1624	0.1044	0.0671	0.0432	0.0277	0.0178	0.0115	0.0074	0.0047
9.5	0.0661	0.1500	0.0983	0.0644	0.0422	0.0276	0.0181	0.0119	0.0078	0.0051

Table 3: Steady state probabilities distribution for various values of λ and $\mu = 10$, $\alpha = 5$, $\beta = 100$, $\eta = 5$ when the server is in breakdown and batch size is K=2

λ	P03	P13	P23	P33	P43	P53	P63	P73	P83	P93
1	0	0	0.0019	0.0005	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
1.5	0	0	0.0024	0.0009	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000
2	0	0	0.0029	0.0014	0.0005	0.0002	0.0000	0.0000	0.0000	0.0000
2.5	0	0	0.0032	0.0018	0.0008	0.0003	0.0001	0.0000	0.0000	0.0000
3	0	0	0.0034	0.0022	0.0011	0.0005	0.0002	0.0001	0.0000	0.0000
3.5	0	0	0.0036	0.0026	0.0014	0.0007	0.0003	0.0001	0.0001	0.0000
4	0	0	0.0036	0.0029	0.0017	0.0009	0.0004	0.0002	0.0001	0.0000
4.5	0	0	0.0037	0.0032	0.0020	0.0011	0.0006	0.0003	0.0002	0.0001
5	0	0	0.0037	0.0034	0.0023	0.0014	0.0008	0.0004	0.0002	0.0001
5.5	0	0	0.0037	0.0036	0.0026	0.0016	0.0010	0.0006	0.0003	0.0002
6	0	0	0.0036	0.0038	0.0028	0.0019	0.0012	0.0007	0.0004	0.0002
6.5	0	0	0.0035	0.0039	0.0031	0.0021	0.0014	0.0009	0.0005	0.0003
7	0	0	0.0035	0.0039	0.0033	0.0024	0.0016	0.0011	0.0007	0.0004
7.5	0	0	0.0034	0.0040	0.0034	0.0026	0.0018	0.0013	0.0008	0.0005
8	0	0	0.0033	0.0040	0.0036	0.0028	0.0021	0.0014	0.0010	0.0007
8.5	0	0	0.0031	0.0040	0.0037	0.0030	0.0023	0.0016	0.0012	0.0008
9	0	0	0.0030	0.0040	0.0038	0.0031	0.0025	0.0018	0.0013	0.0009
9.5	0	0	0.0029	0.0039	0.0038	0.0033	0.0026	0.0020	0.0015	0.0011

Table 4: Average number of customers in the queue and Variance for various values of λ and $\mu = 10$, $\alpha = 5$, $\beta = 100$, $\eta = 5$ and batch size is K=2

λ	μ	α	β	η	P1	P2	P3	MEAN	VARIANCE
1	10	5	100	5	0.0500	0.9475	0.0025	0.7060	0.4973
1.5	10	5	100	5	0.0750	0.9213	0.0038	0.8106	0.6541
2	10	5	100	5	0.1000	0.8950	0.0050	0.9169	0.8344
2.5	10	5	100	5	0.1250	0.8688	0.0063	1.0254	1.0395
3	10	5	100	5	0.1500	0.8425	0.0075	1.1365	1.2706
3.5	10	5	100	5	0.1750	0.8163	0.0088	1.2504	1.5296
4	10	5	100	5	0.2000	0.7900	0.0100	1.3678	1.8184
4.5	10	5	100	5	0.2250	0.7638	0.0113	1.4890	2.1393
5	10	5	100	5	0.2500	0.7375	0.0125	1.6144	2.4949
5.5	10	5	100	5	0.2750	0.7112	0.0137	1.7448	2.8888
6	10	5	100	5	0.3000	0.6850	0.0150	1.8807	3.3247
6.5	10	5	100	5	0.3250	0.6587	0.0162	2.0228	3.8076
7	10	5	100	5	0.3500	0.6325	0.0175	2.1720	4.3433
7.5	10	5	100	5	0.3750	0.6062	0.0187	2.3292	4.9392
8	10	5	100	5	0.4000	0.5800	0.0200	2.4955	5.6042
8.5	10	5	100	5	0.4250	0.5537	0.0212	2.6724	6.3498
9	10	5	100	5	0.4500	0.5275	0.0225	2.8614	7.1901
9.5	10	5	100	5	0.4750	0.5012	0.0237	3.0645	8.1437

Table 5: Steady state probabilities distribution for various values of λ and $\mu = 10$, $\alpha = 5$, $\beta = 100$, $\eta = 5$ when the server is busy and batch size is K = 4

λ	P01	P11	P21	P31	P41	P51	P61	P71	P81	P91
1	0.0189	0.0049	0.0010	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1.5	0.0249	0.0091	0.0026	0.0007	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0295	0.0136	0.0048	0.0015	0.0005	0.0001	0.0000	0.0000	0.0000	0.0000
2.5	0.0330	0.0179	0.0074	0.0028	0.0010	0.0003	0.0001	0.0000	0.0000	0.0000
3	0.0356	0.0219	0.0103	0.0043	0.0017	0.0007	0.0003	0.0001	0.0000	0.0000
3.5	0.0376	0.0256	0.0133	0.0062	0.0028	0.0012	0.0005	0.0002	0.0001	0.0000
4	0.0391	0.0289	0.0163	0.0083	0.0040	0.0019	0.0009	0.0004	0.0002	0.0001
4.5	0.0401	0.0319	0.0193	0.0105	0.0054	0.0027	0.0013	0.0006	0.0003	0.0001
5	0.0408	0.0345	0.0222	0.0128	0.0070	0.0037	0.0019	0.0010	0.0005	0.0003
5.5	0.0413	0.0368	0.0249	0.0152	0.0087	0.0049	0.0027	0.0014	0.0008	0.0004
6	0.0415	0.0388	0.0275	0.0175	0.0105	0.0061	0.0035	0.0020	0.0011	0.0006
6.5	0.0415	0.0405	0.0300	0.0199	0.0124	0.0075	0.0045	0.0026	0.0015	0.0009
7	0.0414	0.0420	0.0322	0.0222	0.0144	0.0090	0.0055	0.0033	0.0020	0.0012
7.5	0.0412	0.0432	0.0343	0.0244	0.0163	0.0106	0.0067	0.0042	0.0026	0.0016
8	0.0409	0.0443	0.0362	0.0265	0.0183	0.0122	0.0079	0.0051	0.0032	0.0020
8.5	0.0405	0.0451	0.0379	0.0285	0.0202	0.0138	0.0092	0.0061	0.0040	0.0026
9	0.0401	0.0458	0.0395	0.0305	0.0221	0.0155	0.0106	0.0072	0.0048	0.0031
9.5	0.0395	0.0463	0.0409	0.0323	0.0240	0.0172	0.0120	0.0083	0.0056	0.0038

Table 6: Steady state probabilities distribution for various values of λ and $\mu = 10$, $\alpha = 5$, $\beta = 100$, $\eta = 5$ when the server is in vacation and batch size is K=4

λ	P02	P12	P22	P32	P42	P52	P62	P72	P82	P92
1	0.1885	0.2379	0.2478	0.2496	0.0416	0.0069	0.0012	0.0002	0.0000	0.0000
1.5	0.1661	0.2271	0.2442	0.2486	0.0574	0.0132	0.0031	0.0007	0.0002	0.0000
2	0.1476	0.2153	0.2392	0.2467	0.0705	0.0201	0.0058	0.0016	0.0005	0.0001
2.5	0.1320	0.2034	0.2330	0.2440	0.0813	0.0271	0.0090	0.0030	0.0010	0.0003
3	0.1188	0.1918	0.2260	0.2405	0.0902	0.0338	0.0127	0.0048	0.0018	0.0007
3.5	0.1074	0.1806	0.2185	0.2362	0.0973	0.0401	0.0165	0.0068	0.0028	0.0012
4	0.0976	0.1700	0.2108	0.2315	0.1029	0.0457	0.0203	0.0090	0.0040	0.0018
4.5	0.0891	0.1600	0.2029	0.2262	0.1072	0.0508	0.0240	0.0114	0.0054	0.0026
5	0.0816	0.1507	0.1951	0.2207	0.1103	0.0552	0.0276	0.0138	0.0069	0.0034
5.5	0.0750	0.1420	0.1873	0.2149	0.1126	0.0590	0.0309	0.0162	0.0085	0.0044
6	0.0691	0.1338	0.1798	0.2090	0.1140	0.0622	0.0339	0.0185	0.0101	0.0055
6.5	0.0639	0.1263	0.1724	0.2030	0.1147	0.0648	0.0366	0.0207	0.0117	0.0066
7	0.0592	0.1192	0.1653	0.1969	0.1149	0.0670	0.0391	0.0228	0.0133	0.0078
7.5	0.0550	0.1126	0.1584	0.1909	0.1145	0.0687	0.0412	0.0247	0.0148	0.0089
8	0.0511	0.1065	0.1517	0.1849	0.1138	0.0700	0.0431	0.0265	0.0163	0.0100
8.5	0.0477	0.1007	0.1454	0.1789	0.1127	0.0709	0.0447	0.0281	0.0177	0.0111
9	0.0445	0.0954	0.1392	0.1731	0.1113	0.0715	0.0460	0.0296	0.0190	0.0122
9.5	0.0416	0.0903	0.1334	0.1673	0.1096	0.0718	0.0471	0.0308	0.0202	0.0132

Table 7: Steady state probabilities distribution for various values of λ and $\mu = 10$, $\alpha = 5$, $\beta = 100$, $\eta = 5$ when the server is in breakdown and batch size is K = 4

λ	P03	P13	P23	P33	P43	P53	P63	P73	P83	P93
1	0	0	0	0	0.0009	0.0003	0.0001	0.0000	0.0000	0.0000
1.5	0	0	0	0	0.0012	0.0005	0.0001	0.0000	0.0000	0.0000
2	0	0	0	0	0.0014	0.0007	0.0002	0.0001	0.0000	0.0000
2.5	0	0	0	0	0.0016	0.0009	0.0004	0.0001	0.0001	0.0000
3	0	0	0	0	0.0017	0.0011	0.0005	0.0002	0.0001	0.0000
3.5	0	0	0	0	0.0018	0.0013	0.0007	0.0003	0.0001	0.0001
4	0	0	0	0	0.0019	0.0015	0.0008	0.0004	0.0002	0.0001
4.5	0	0	0	0	0.0019	0.0016	0.0010	0.0005	0.0003	0.0001
5	0	0	0	0	0.0019	0.0017	0.0011	0.0007	0.0004	0.0002
5.5	0	0	0	0	0.0020	0.0018	0.0013	0.0008	0.0005	0.0003
6	0	0	0	0	0.0020	0.0019	0.0014	0.0009	0.0005	0.0003
6.5	0	0	0	0	0.0019	0.0020	0.0015	0.0010	0.0006	0.0004
7	0	0	0	0	0.0019	0.0021	0.0016	0.0011	0.0007	0.0005
7.5	0	0	0	0	0.0019	0.0021	0.0017	0.0013	0.0008	0.0006
8	0	0	0	0	0.0019	0.0022	0.0018	0.0014	0.0009	0.0006
8.5	0	0	0	0	0.0019	0.0022	0.0019	0.0015	0.0010	0.0007
9	0	0	0	0	0.0018	0.0023	0.0020	0.0016	0.0011	0.0008
9.5	0	0	0	0	0.0018	0.0023	0.0021	0.0017	0.0012	0.0009

Table 8: Average number of customers in the queue and Variance for various values of $\lambda, \mu = 10, \alpha = 5, \beta = 100, \eta = 5$ and batch size is $K = 4$

λ	μ	α	β	η	P1	P2	P3	MEAN	VARIANCE
1	10	5	100	5	0.0250	0.9738	0.0013	1.7050	1.4961
1.5	10	5	100	5	0.0375	0.9606	0.0019	1.8076	1.6501
2	10	5	100	5	0.0500	0.9475	0.0025	1.9102	1.8248
2.5	10	5	100	5	0.0625	0.9344	0.0031	2.0130	2.0203
3	10	5	100	5	0.0750	0.9213	0.0038	2.1161	2.2369
3.5	10	5	100	5	0.0875	0.9081	0.0044	2.2195	2.4747
4	10	5	100	5	0.1000	0.8950	0.0050	2.3234	2.7342
4.5	10	5	100	5	0.1125	0.8819	0.0056	2.4278	3.0156
5	10	5	100	5	0.1250	0.8687	0.0062	2.5329	3.3194
5.5	10	5	100	5	0.1375	0.8556	0.0069	2.6387	3.6459
6	10	5	100	5	0.1500	0.8425	0.0075	2.7455	3.9958
6.5	10	5	100	5	0.1625	0.8294	0.0081	2.8532	4.3694
7	10	5	100	5	0.1750	0.8162	0.0087	2.9619	4.7675
7.5	10	5	100	5	0.1875	0.8031	0.0094	3.0719	5.1908
8	10	5	100	5	0.2000	0.7900	0.0100	3.1831	5.6398
8.5	10	5	100	5	0.2125	0.7769	0.0106	3.2958	6.1155
9	10	5	100	5	0.2250	0.7637	0.0112	3.4100	6.6187
9.5	10	5	100	5	0.2375	0.7506	0.0119	3.5258	7.1503

Table 9: Steady state probabilities distribution for various values of λ and $\mu = 10, \alpha = 5, \beta = 100, \eta = 5$ when the server is busy and batch size is $K = 6$

λ	P01	P11	P21	P31	P41	P51	P61	P71	P81	P91
1	0.0126	0.0033	0.0007	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1.5	0.0166	0.0061	0.0017	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0197	0.0090	0.0032	0.0010	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000
2.5	0.0220	0.0119	0.0049	0.0018	0.0007	0.0002	0.0001	0.0000	0.0000	0.0000
3	0.0238	0.0146	0.0068	0.0029	0.0012	0.0005	0.0002	0.0001	0.0000	0.0000
3.5	0.0251	0.0171	0.0088	0.0041	0.0018	0.0008	0.0003	0.0001	0.0001	0.0000
4	0.0261	0.0193	0.0109	0.0055	0.0026	0.0012	0.0006	0.0003	0.0001	0.0001
4.5	0.0268	0.0213	0.0128	0.0070	0.0036	0.0018	0.0009	0.0004	0.0002	0.0001
5	0.0273	0.0230	0.0148	0.0085	0.0046	0.0025	0.0013	0.0007	0.0003	0.0002
5.5	0.0277	0.0246	0.0166	0.0101	0.0058	0.0032	0.0017	0.0009	0.0005	0.0003
6	0.0279	0.0260	0.0183	0.0116	0.0070	0.0040	0.0023	0.0013	0.0007	0.0004
6.5	0.0280	0.0271	0.0200	0.0132	0.0082	0.0050	0.0029	0.0017	0.0010	0.0006
7	0.0280	0.0282	0.0215	0.0147	0.0095	0.0059	0.0036	0.0022	0.0013	0.0008
7.5	0.0279	0.0290	0.0229	0.0161	0.0108	0.0069	0.0044	0.0027	0.0017	0.0010
8	0.0278	0.0298	0.0242	0.0176	0.0120	0.0080	0.0052	0.0033	0.0021	0.0013
8.5	0.0276	0.0304	0.0254	0.0189	0.0133	0.0090	0.0060	0.0039	0.0026	0.0016
9	0.0274	0.0310	0.0264	0.0202	0.0146	0.0101	0.0069	0.0046	0.0031	0.0020
9.5	0.0272	0.0314	0.0274	0.0214	0.0158	0.0112	0.0078	0.0053	0.0036	0.0024

Table 10: Steady state probabilities distribution for various values of λ and $\mu = 10$, $\alpha = 5$, $\beta = 100$, $\eta = 5$ when the server is in vacation and batch size is $K = 6$

λ	P02	P12	P22	P32	P42	P52	P62	P72	P82	P92
1	0.1257	0.1586	0.1652	0.1664	0.1666	0.1667	0.0278	0.0046	0.0008	0.0001
1.5	0.1108	0.1514	0.1628	0.1657	0.1664	0.1666	0.0384	0.0089	0.0020	0.0005
2	0.0984	0.1436	0.1594	0.1645	0.1660	0.1665	0.0476	0.0136	0.0039	0.0011
2.5	0.0880	0.1356	0.1553	0.1627	0.1653	0.1662	0.0554	0.0185	0.0062	0.0021
3	0.0792	0.1279	0.1507	0.1603	0.1642	0.1657	0.0621	0.0233	0.0087	0.0033
3.5	0.0717	0.1205	0.1457	0.1575	0.1628	0.1650	0.0680	0.0280	0.0115	0.0047
4	0.0652	0.1135	0.1406	0.1544	0.1610	0.1641	0.0729	0.0324	0.0144	0.0064
4.5	0.0596	0.1069	0.1354	0.1509	0.1589	0.1629	0.0772	0.0365	0.0173	0.0082
5	0.0547	0.1008	0.1303	0.1473	0.1566	0.1615	0.0807	0.0404	0.0202	0.0101
5.5	0.0503	0.0950	0.1252	0.1435	0.1540	0.1598	0.0837	0.0439	0.0230	0.0120
6	0.0465	0.0897	0.1203	0.1396	0.1513	0.1580	0.0862	0.0470	0.0256	0.0140
6.5	0.0430	0.0848	0.1155	0.1357	0.1484	0.1560	0.0882	0.0498	0.0282	0.0159
7	0.0400	0.0802	0.1109	0.1318	0.1454	0.1538	0.0897	0.0523	0.0305	0.0178
7.5	0.0372	0.0759	0.1064	0.1280	0.1423	0.1515	0.0909	0.0546	0.0327	0.0196
8	0.0347	0.0720	0.1022	0.1241	0.1392	0.1491	0.0918	0.0565	0.0348	0.0214
8.5	0.0325	0.0683	0.0981	0.1204	0.1360	0.1467	0.0923	0.0581	0.0366	0.0230
9	0.0305	0.0648	0.0942	0.1167	0.1328	0.1441	0.0926	0.0595	0.0383	0.0246
9.5	0.0286	0.0616	0.0905	0.1131	0.1297	0.1415	0.0927	0.0607	0.0398	0.0261

Table 11: Steady state probabilities distribution for various values of λ and $\mu = 10$, $\alpha = 5$, $\beta = 100$, $\eta = 5$ when the server is in breakdown and batch size is $K = 6$

λ	P03	P13	P23	P33	P43	P53	P63	P73	P83	P93	P103
1	0	0	0	0	0	0	0.0006	0.0002	0.0000	0.0000	0.0000
1.5	0	0	0	0	0	0	0.0008	0.0003	0.0001	0.0000	0.0000
2	0	0	0	0	0	0	0.0010	0.0005	0.0002	0.0001	0.0000
2.5	0	0	0	0	0	0	0.0011	0.0006	0.0003	0.0001	0.0000
3	0	0	0	0	0	0	0.0012	0.0007	0.0004	0.0002	0.0001
3.5	0	0	0	0	0	0	0.0012	0.0009	0.0005	0.0002	0.0001
4	0	0	0	0	0	0	0.0013	0.0010	0.0006	0.0003	0.0001
4.5	0	0	0	0	0	0	0.0013	0.0011	0.0007	0.0004	0.0002
5	0	0	0	0	0	0	0.0013	0.0012	0.0008	0.0004	0.0002
5.5	0	0	0	0	0	0	0.0013	0.0012	0.0009	0.0005	0.0003
6	0	0	0	0	0	0	0.0013	0.0013	0.0009	0.0006	0.0004
6.5	0	0	0	0	0	0	0.0013	0.0014	0.0010	0.0007	0.0004
7	0	0	0	0	0	0	0.0013	0.0014	0.0011	0.0008	0.0005
7.5	0	0	0	0	0	0	0.0013	0.0014	0.0012	0.0008	0.0006
8	0	0	0	0	0	0	0.0013	0.0015	0.0012	0.0009	0.0006
8.5	0	0	0	0	0	0	0.0013	0.0015	0.0013	0.0010	0.0007
9	0	0	0	0	0	0	0.0013	0.0015	0.0013	0.0010	0.0008
9.5	0	0	0	0	0	0	0.0012	0.0015	0.0014	0.0011	0.0008

Table 12: Average number of customers in the queue and Variance for various values of λ and $\mu = 10$, $\alpha = 5$, $\beta = 100$, $\eta = 5$ and batch size is K=6

λ	μ	α	β	η	P1	P2	P3	MEAN	VARIANCE
1	10	5	100	5	0.0167	0.9825	0.0008	2.7050	3.1628
1.5	10	5	100	5	0.0250	0.9737	0.0013	2.8075	3.3167
2	10	5	100	5	0.0333	0.9650	0.0017	2.9100	3.4912
2.5	10	5	100	5	0.0417	0.9563	0.0021	3.0125	3.6862
3	10	5	100	5	0.0500	0.9475	0.0025	3.1151	3.9019
3.5	10	5	100	5	0.0583	0.9388	0.0029	3.2176	4.1382
4	10	5	100	5	0.0667	0.9300	0.0033	3.3203	4.3952
4.5	10	5	100	5	0.0750	0.9213	0.0037	3.4230	4.6730
5	10	5	100	5	0.0833	0.9125	0.0042	3.5259	4.9716
5.5	10	5	100	5	0.0917	0.9037	0.0046	3.6289	5.2913
6	10	5	100	5	0.1000	0.8950	0.0050	3.7322	5.6321
6.5	10	5	100	5	0.1083	0.8862	0.0054	3.8357	5.9942
7	10	5	100	5	0.1167	0.8775	0.0058	3.9394	6.3778
7.5	10	5	100	5	0.1250	0.8687	0.0062	4.0435	6.7831
8	10	5	100	5	0.1333	0.8600	0.0067	4.1480	7.2104
8.5	10	5	100	5	0.1417	0.8512	0.0071	4.2530	7.6598
9	10	5	100	5	0.1500	0.8425	0.0075	4.3584	8.1317
9.5	10	5	100	5	0.1583	0.8337	0.0079	4.4643	8.6264

Table 13: Average number of customers in the queue and Variance for various values of α and $\mu = 10$, $\lambda = 5$, $\beta = 100$, $\eta = 5$ and batch size is K=2

α	λ	μ	β	η	p1	p2	p3	Mean	variance
10.0000	5	10	100	5	0.2500	0.7250	0.0250	1.6527	2.5847
5.0000	5	10	100	5	0.2500	0.7375	0.0125	1.6144	2.4949
2.5000	5	10	100	5	0.2500	0.7438	0.0062	1.5958	2.4522
1.2500	5	10	100	5	0.2500	0.7469	0.0031	1.5865	2.4313
0.6250	5	10	100	5	0.2500	0.7484	0.0016	1.5819	2.4209
0.3125	5	10	100	5	0.2500	0.7492	0.0008	1.5796	2.4158
0.1563	5	10	100	5	0.2500	0.7496	0.0004	1.5785	2.4132
0.0781	5	10	100	5	0.2500	0.7498	0.0002	1.5779	2.4120
0.0391	5	10	100	5	0.2500	0.7499	0.0001	1.5776	2.4113
0.0195	5	10	100	5	0.2500	0.7500	0.0000	1.5775	2.4110
0.0098	5	10	100	5	0.2500	0.7500	0.0000	1.5774	2.4108
0.0049	5	10	100	5	0.2500	0.7500	0.0000	1.5774	2.4108
0.0024	5	10	100	5	0.2500	0.7500	0.0000	1.5774	2.4107
0.0012	5	10	100	5	0.2500	0.7500	0.0000	1.5774	2.4107
0.0006	5	10	100	5	0.2500	0.7500	0.0000	1.5774	2.4107
0.0003	5	10	100	5	0.2500	0.7500	0.0000	1.5774	2.4107
0.0002	5	10	100	5	0.2500	0.7500	0.0000	1.5774	2.4107

Table 14: Average number of customers in the queue and Variance for various values of α and $\mu = 10, \lambda = 5, \beta = 100, \eta = 5$ and batch size is K=4

α	λ	μ	β	η	p1	p2	p3	Mean	variance
10.0000	5	10	100	5	0.1250	0.8625	0.0125	2.5595	3.3776
5.0000	5	10	100	5	0.1250	0.8687	0.0062	2.5329	3.3194
2.5000	5	10	100	5	0.1250	0.8719	0.0031	2.5197	3.2910
1.2500	5	10	100	5	0.1250	0.8734	0.0016	2.5131	3.2769
0.6250	5	10	100	5	0.1250	0.8742	0.0008	2.5098	3.2699
0.3125	5	10	100	5	0.1250	0.8746	0.0004	2.5081	3.2665
0.1563	5	10	100	5	0.1250	0.8748	0.0002	2.5073	3.2647
0.0781	5	10	100	5	0.1250	0.8749	0.0001	2.5069	3.2639
0.0391	5	10	100	5	0.1250	0.8750	0.0000	2.5067	3.2634
0.0195	5	10	100	5	0.1250	0.8750	0.0000	2.5066	3.2632
0.0098	5	10	100	5	0.1250	0.8750	0.0000	2.5065	3.2631
0.0049	5	10	100	5	0.1250	0.8750	0.0000	2.5065	3.2630
0.0024	5	10	100	5	0.1250	0.8750	0.0000	2.5065	3.2630
0.0012	5	10	100	5	0.1250	0.8750	0.0000	2.5065	3.2630
0.0006	5	10	100	5	0.1250	0.8750	0.0000	2.5065	3.2630
0.0003	5	10	100	5	0.1250	0.8750	0.0000	2.5065	3.2630
0.0002	5	10	100	5	0.1250	0.8750	0.0000	2.5065	3.2630

Table 15: Average number of customers in the queue and Variance for various values of α and $\mu = 10, \lambda = 5, \beta = 100, \eta = 5$ and batch size is K=6

α	λ	μ	β	η	p1	p2	p3	mean	variance
10.0000	5	10	100	5	0.0833	0.9083	0.0083	3.5511	5.0266
5.0000	5	10	100	5	0.0833	0.9125	0.0042	3.5259	4.9716
2.5000	5	10	100	5	0.0833	0.9146	0.0021	3.5133	4.9447
1.2500	5	10	100	5	0.0833	0.9156	0.0010	3.5070	4.9313
0.6250	5	10	100	5	0.0833	0.9161	0.0005	3.5038	4.9247
0.3125	5	10	100	5	0.0833	0.9164	0.0003	3.5023	4.9214
0.1563	5	10	100	5	0.0833	0.9165	0.0001	3.5015	4.9197
0.0781	5	10	100	5	0.0833	0.9166	0.0001	3.5011	4.9189
0.0391	5	10	100	5	0.0833	0.9166	0.0000	3.5009	4.9185
0.0195	5	10	100	5	0.0833	0.9167	0.0000	3.5008	4.9183
0.0098	5	10	100	5	0.0833	0.9167	0.0000	3.5007	4.9182
0.0049	5	10	100	5	0.0833	0.9167	0.0000	3.5007	4.9181
0.0024	5	10	100	5	0.0833	0.9167	0.0000	3.5007	4.9181
0.0012	5	10	100	5	0.0833	0.9167	0.0000	3.5007	4.9181
0.0006	5	10	100	5	0.0833	0.9167	0.0000	3.5007	4.9181
0.0003	5	10	100	5	0.0833	0.9167	0.0000	3.5007	4.9181
0.0002	5	10	100	5	0.0833	0.9167	0.0000	3.5007	4.9180

8. CONCLUSION:

The Numerical studies show the changes in the system due to impact of batch size, arrival rate, breakdown rate and repair rate. The mean number of customers in the queue increases as arrival rate increase. The mean number of customers in the system decreases as repair rate and vacation rate increases. Various special cases have been discussed, which are particular cases of this research work. This research work can be extended further by introducing various concepts like second optional service etc.

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