g**compact space and g**compact modulo *I* space

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Abstract: In this paper, g**-isolated point, g**-compact, g**-locally compact, g**-compact modulo *I*, g**- sequentially compact, g**- sequentially compact modulo *I*, g**-countably compact, g**-countably compact modulo *I* spaces are introduced and the relationship between these concepts are studied.
Key words: g**-isolated point, g**-compact, g**-locally compact, g**-compact modulo *I*, g**-sequentially compact, g**-sequentially compact modulo *I*, g**-countably compact, g**-countably compact modulo *I*.

1. Introduction

Levine **[1]** introduced the class of g-closed sets in 1970 and M.K.R.S. Veerakumar**[5]** introduced g*-closed sets in 1991. Ideal topological spaces have been first introduced by K. Kuratowski **[2]** in 1930. In this paper g**-compact spaces, g**-locally compact spaces, g**-compact modulo *I* spaces, g**- sequentially compact spaces, g**- sequentially compact modulo *I* spaces, g**-countably compact spaces, g**-countably compact modulo *I* spaces are defined and their properties are investigated.

2. Preliminaries

Definition 2.1: A subset A of a topological space(X, τ) is called

1) *generalized closed* (briefly *g-closed*)**[1]** if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).
2) *generalized star closed* (briefly *g*-closed*)**[7]** if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is g-open in (X, τ).

3) *generalized star star closed* (briefly *g**-closed*)**[4]** if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is g*- open in (X, τ).

Definition 2.2: A function f **:** (X, τ) $\rightarrow$ (Y, σ) is called

1) g**-irresolute **[4]** if $f^{-1}(V)$ is a g**-closed set of (X, τ) for every g**-closed set V of (Y, σ).
2) g**-continuous **[4]** if $f^{-1}(V)$ is a g**-closed set of (X,τ) for every closed set V of (Y,σ).
3) g**-resolute **[6]** if $f(U)$ is g**-open in Y whenever U is g**-open in X.

Definition 2.3: An ideal**[2]** I on a non empty set X is a collection of subsets of X which satisfies the following properties.(i) $A \in I$, $B \in I \Rightarrow A \cup B \in I$ (ii) $A \in I$, $B \subset A \Rightarrow B \in I$ .A topological space (X,τ) with an ideal I on X is called an ideal topological space and is denoted by (X,τ,I).

Definition 2.4:[6] Let (X,τ) be a topological space and $x \in X$. Every $g^{**}-open$ set containing x is said to be a $g^{**}-neighbourhood$ of x.

Definition 2.5:[6] Let A be a subset of X. A point $x \in X$ is said to be a $g^{**}-$ limit point of A if every $g^{**}-neighbourhood$ of x contains a point of A other than x.

Definition 2.6:[6] Let A be a subset of a topological space (X,τ). $g^{**}cl(A)$ is defined to be the intersection of all $g^{**}-closed$ sets containing A.

Note: **[6]** $g^{**}cl(A)$ need not be $g^{**}-closed$, since intersection of $g^{**}-closed$ sets need not be $g^{**}-closed$. But if A is $g^{**}-closed$ then $g^{**}cl(A) = A$.

Definition 2.8:[6] A topological space (X,τ) is said to be $g^{**}-multiplicative$ if arbitrary intersection of $g^{**}-closed$ sets is $g^{**}-closed$. Equivalently arbitrary union of $g^{**}-open$ sets is $g^{**}-open$.

Note: If (X,τ) is $g^{**}-multiplicative$ then $A = g^{**}cl(A)$ if and only if A is $g^{**}-closed$.

Definition 2.9:[2] A collection $\mathcal{C}$ of subsets of X is said to have finite intersection property if for every sub collection $\{C_1, C_2, ...C_n\}$ of $\mathcal{C}$ the intersection $C_1 \cap C_2 \cap .... \cap C_n$ is non empty.

Definition 2.10:[5] An ideal topological space (X, τ, I) is called $g^{**}I - compact$ if for every $g^{**}I - open$ cover $\{A_\alpha / \alpha \in \Omega\}$ in (X, τ, I) there exists a finite subset Ω_0 of Ω such that $X = \bigcup_{\alpha \in \Omega} A_\alpha$.

Definition 2.11:[6] A topological space (X, τ) is said to be a g**- T_2 space if for every pair of distinct points x, y in X there exists disjoint g**-open sets U and V in X such that $x \in U$ and $y \in V$.

3. g-compact space**

Definition 3.1: A collection $\{U_\alpha\}_{\alpha \in \Delta}$ of g**-open sets in X is said to be g**-open cover of X if $X = \cup_{\alpha \in \Delta} U_\alpha$

Definition 3.2: A topological space (X, τ) is said to be g**-compact if every g**-open covering of X contains a finite sub collection that also covers X. A subset A of X is said to be g**-compact if every g**-open covering of A contains a finite sub collection that also covers A

Remark 3.3: An ideal topological space (X, τ, I) is

(1) $g^{**}I - compact \Rightarrow g^{**} - compact \Rightarrow compact$

Proof: Since every open set is g**-open and every g**-open set is $g^{**}I$ -open.

(2) Any topological space having only finitely many points is necessarily $g^{**}I - compact, g^{**} - compact$ and $compact$.

The inverse implications of (1) of remark (3.3) are not true as seen in the following example.

Example 3.4: Let (X, τ) be an infinite indiscrete topological space. In this space all subsets are g**-open. Obviously it is compact. But $\{x\}_{x \in X}$ is a g**-open cover which has no finite sub cover. Hence it is not g**-compact and hence not g**I compact.

Example 3.5: Let (X, τ) be an infinite cofinite topological space. Then G**IO(X) = $\{\varphi, X, A / A^c$ is finite$\}$ = G**O(X). Let $\{U_\alpha\}_{\alpha \in \Delta}$ be an arbitrary g**-open cover for X. Let U_{α_0} be one g**-open (g**I –open) set in the open cover $\{U_\alpha\}_{\alpha \in \Delta}$. Then $X - U_{\alpha_0}$ is finite, say

$\{x_1, x_2, x_3, ...x_n\}$. Choose U_{α_i} such that $x_{\alpha_i} \in U_{\alpha_i}$ for $i = 1,2,.....n.$ Then $X = U_{\alpha_0} \cup U_{\alpha_1} \cup .... \cup U_{\alpha_n}$. The space is g**-compact (g**I –compact) and hence compact.

Theorem 3.6: A g**-closed subset of g**-compact space is g**-compact.

Proof: Let A be a g**-closed subset of a g**-compact space (X, τ) and $\{U_\alpha\}_{\alpha\in\Delta}$ be a g**-open cover for A. Then $\{\{U_\alpha\}_{\alpha\in\Delta}, (X - A)\}$ is a g**-open cover for X. Since X is g**-compact, there exists $\alpha_1, \alpha_2, ...\alpha_n \in \Delta$ such that $X = U_{\alpha_1} \cup .... \cup U_{\alpha_n} \cup (X - A)$. $\therefore A \subseteq U_{\alpha_1} \cup U_{\alpha_2} \cup .... \cup U_{\alpha_n}$ which proves A is g**-compact.

Remark 3.7: The converse of the above theorem need not be true as seen in the following example.

Example 3.8: Let $X = \{a, b, c, d\}, \tau = \{\varphi, \{a\}, X\}$. Here $G^{**}O(X) = \{\varphi, \{a\}, X\}$. (X, τ) is g**-compact. $Y = \{b, c\}$ is g**-compact but not g**-closed.

Theorem 3.9: Let (X, τ) be a g**-multiplicative g** T_2-space. Then every g**-compact subset of X is g**-closed.

Proof: Let Y be a g**-compact subset of g** T_2-space X. Let $x_0 \in X - Y$. For each point $y \in Y$, there exists disjoint g**-open sets U_y and V_y containing y and x_0 respectively. $\therefore \{U_y / y \in Y\}$ is a g**-open cover for Y. Now there exists $\{y_1, y_2, ...y_n\} \in Y$ such that $Y \subseteq \bigcup_{i=1}^{n} U_{y_i} = U(say)$. Let $V = \bigcap_{i=1}^{n} V_{y_i}$. Then V is g**-open. Since X is g**-multiplicative, U is g**-open. Obviously $U \cap Y = \varphi$. $\therefore V$ is a g**-neighbourhood of x_0 contained in X – Y. Therefore X – Y is g**-open and hence Y is g**-closed.

Note: The converse of theorem (3.9) is true if (X, τ) is g**-multiplicative and g** T_2.

Remark 3.10:

(1) In theorem (3.9), the condition g**-T_2 is necessary. An infinite cofinite topological space is g**-multiplicative but not g**-T_2. In this space all subsets are g**-compact but only finite sets are g**-closed.

Theorem 3.11: Let Y be a g**-compact subset of a g** T_2-space X and $x_0 \notin Y$. Then there exists disjoint g**-open sets U and V of X containing x_0 and Y respectively.

Proof: The g**-open sets U and V discussed in the proof of theorem (3.9) are disjoint g**-open sets containing Y and x_0 respectively.

Theorem 3.12: Let (X,τ) and (Y,σ) be two topological spaces and $f:(X,\tau)\to(Y,\sigma)$ be a function. Then

1. f is g**-irresolute and A is a g**-compact subset of X $\Rightarrow f(A)$ is a g**-compact subset of Y.
2. f is one to one,g**-resolute and B is a g**-compact subset of Y $\Rightarrow f^{-1}(B)$ is a g**-compact subset of X.
3. f is g**-irresolute , X is g**-compact, Y is g**-multiplicative and g**-$T_2 \Rightarrow f$ is a g**- resolute function.
4. f is g**-resolute and Y is g**-compact and X is g**-multiplicative and g**-$T_2 \Rightarrow f$ is a g**- irresolute function.

Proof: (1) & (2) Obvious from the definitions.

(3) Proof follows from (1) and theorem (3.9).

(4) Proof follows from (2) and theorem (3.9).

Theorem 3.13: A topological space (X,τ) is g**-compact if and only if for every collection $\mathcal{C}$ of g**-closed sets in X having finite intersection property, $\cap_{C\in C} C$ of all elements of $\mathcal{C}$ is non-empty.

Proof: Let (X,τ) be g**-compact and $\mathcal{C}$ be a collection of g**-closed sets with finite intersection property. Suppose $\cap_{C\in C} C=\varphi$ then $\cap_{C\in C} (X-C)=X$.$\therefore \{X-C\}_{C\in C}$ is a g**-open cover for X. Then there exists $C_1, C_2,........C_n \in \mathcal{C}$ such that $\bigcup_{i=1}^{n}(X-C_i)=X$. $\therefore \bigcap_{i=1}^{n} C_i=\varphi$ which is a contradiction. $\therefore \bigcap_{C\in C} C\neq\varphi.$ Conversely, assume the hypothesis given in the statement. To prove X is g**-compact. Let $\{U_\alpha\}_{\alpha\in\Delta}$ be a g**-open cover for X. Then $\bigcup_{\alpha\in\Delta} U_\alpha = X \Rightarrow \bigcap_{\alpha\in\Delta}(X-U_\alpha)=\varphi.$ By the hypothesis there exists $\alpha_1,\alpha_2,.........\alpha_n$ such that $\bigcap_{i=1}^{n} X-U_{\alpha_i}=\varphi.$ $\therefore \bigcup_{i=1}^{n} U_{\alpha_i}=X$. $\therefore X$ is g**-compact.

Corollary 3.14: Let (X,τ) be a g**-compact space and let $C_1 \supseteq C_2 \supseteq .... \supseteq C_n \supseteq C_{n+1} \supseteq ....$ be a nested sequence of non-empty g**-closed sets in X. Then $\bigcap_{n\in Z^+} C_n$ is non-empty.

Proof: Obviously $\{C_n\}_{n\in Z^+}$ has finite intersection property. $\therefore$ By theorem (3.13) $\bigcap_{n\in Z^+} C_n$ is non-empty.

Theorem 3.15: Let $f:(X,\tau)\to(Y,\sigma)$ be a funtion, then

(1) f is g**-continuous, onto and X is g**-compact $\Rightarrow$ Y is compact.

(2) f is continuous, onto and X is g**-compact $\Rightarrow$ Y is compact.

(3) f is g**-irresolute, onto and X is g**-compact $\Rightarrow$ Y is g**-compact.

(4) f is strongly g**-irresolute, onto and X is compact $\Rightarrow$ Y is g**-compact.

(5) f is g**-open, bijection and Y is g**-compact $\Rightarrow$ X is compact.

(6) f is open, bijection and Y is g**-compact $\Rightarrow$ X is compact.

(7) f is g**-resolute, bijection and Y is g**-compact $\Rightarrow$ X is g**-compact.

Proof:(1): Let $\{U_\alpha\}_{\alpha\in\Delta}$ be an open cover for Y. Then $\{f^{-1}(U_\alpha)\}_{\alpha\in\Delta}$ is a g**-open cover for X. Since X is g**-compact, there exists $\alpha_1,\alpha_2,......\alpha_n$ such that $X \subseteq \bigcup_{i=1}^{n} f^{-1}(U_{\alpha_i})$. $\therefore Y = f(X) \subseteq \bigcup_{i=1}^{n} U_{\alpha_i}$. Therefore Y is compact.

Proof for (2) to (7) are similar to the above.

4. g**-countably compact space

Definition 4.1: A subset A of a topological space (X,τ) is said to be g**-countably compact if every countable g**-open covering of A has a finite sub cover.

Example 4.2: An infinite cofinite topological space is g**-countably compact.

Example 4.3: A countably infinite indiscrete topological space is not g**-countably compact.

Remark 4.4: Every g**-compact space is g**-countably compact.

Theorem 4.5: In a g**-countably compact topological space every infinite subset has a g**-limit point.

Proof: Let (X,τ) be g**-countably compact. Suppose that there exists an infinite subset which has no g**-limit point. Let $B=\{a_n / n\in N\}$ be a countable subset of A. Since B has no g**-limit point of B, there exists a g**-neighbourhood U_n of a_n such that $B\cap U_n=\{a_n\}$. Now $\{U_n\}$ is a g**-open cover for B. Since B^c is g**-open, $\{B^c,\{U_n\}_{n\in Z^+}\}$ is a countable g**-open cover for X. But it has no finite sub cover which is a contradiction, since X is g**-countably compact. Therefore every infinite subset of X has a g**-limit point.

Corollary 4.6: In a g**-compact topological space every infinite subset has a g**-limit point.
Proof follows from theorem (4.5), since every g**-compact space is g**-countably compact.

Theorem 4.7: A g**-closed subset of g**-countably compact space is g**-countably compact.
Proof is similar to theorem (3.6)

Definition 4.8: In a topological space (X,τ) a point $x\in X$ is said to be a g**-isolated point of A if every g**-open set containing x contains no point of A other than x.

Theorem 4.9: Let X be a non empty g**-compact g**-T_2 space. If X has no g**-isolated points then X is uncountable.

Proof: Let $x_1\in X$. Choose a point y of X different from x. This is possible since $\{x_1\}$ is not a g**-isolated point. Since X is g**-T_2, there exists g**-open sets U_1 and V_1 such that $U_1\cap V_1=\varphi; x\in U_1, y\in V_1$. Therefore V_1 is g**-open and $x_1\notin g^{**}cl(V_1)$. By repeating the same process with V_1 in the place of X and x_1 in the place of y we get a point $x\neq x_1$ and a g**-open set V_2 such that V_2 is g**-open and $x_2\notin g^{**}cl(V_2)$. In general, given V_{n-1} which is g**-open and non empty, choose V_n to be a non empty g**-open set such that $V_n\subseteq V_{n-1}$ and $x_n\notin g^{**}cl(V_n)$. Hence we get a nested sequence of g**-closed sets such that $g^{**}cl(V_n)\supseteq g^{**}cl(V_{n+1})\supseteq$ .... Since X is g**-compact $\cap g^{**}cl(V_n)\neq\varphi$. Therefore there exists $x\in\cap g^{**}cl(V_n)$. But $x\neq x_n$ for every n, since $x_n\notin g^{**}cl(V_n)$ and $x\in g^{**}cl(V_n)$. Define $f: Z_+\to X$ such that $f(n)=x_n$. Then $x\in X$ has no pre image. Therefore f is not onto and hence X is uncountable.

Note:The converse of Theorem 4.5 is true in a g**-T_1 space.

Theorem 4.10: In a g**-T_1 space X ,if every infinite subset has a g**-limit point then X is g**-countably compact .

Proof: Let every infinite subset has a g**-limit point. To prove X is g**-countably compact. If not there exists a countable g**-open cover $\{U_n\}$ such that it has no finite sub cover .Since $U_1 \neq X$. there exists $x_1 \notin U_1; Since X \neq U_1 \cup U_2$. there exists $x_2 \notin U_1 \cup U_2$. Proceeding like this there exists $x_n \notin U_1 \cup U_2 \cup ...... \cup U_n$ for all n. $A = \{x_n\}$ is an infinite set. If $x \in X$ then $x \in U_n$ for some n. But $x_k \notin U_n$ for all $k \geq n$. $U_n - \{x_1, x_2, \ldots, x_{n-1}\}$ is a g**-open set (since X is g**-T_1) containing x which does not have a point of A other than x.Therefore x is not a limit point of A which is a contradiction.

Theorem 4.11: A topological space (X, τ) is g**- countably compact if and only if for every countable collection $\mathcal{C}$ of g**-closed sets in X having finite intersection property, $\cap_{C \in C} C$ of all elements of $\mathcal{C}$ is non-empty.

Proof: Similar to the proof of Theorem 3.13

Corollary 4.12: X is g**-countably compact if and only if every nested sequence of g**-closed non empty sets $C_1 \supset C_2 \supset ....$ has a non empty intersection.

Proof: Obviously $\{C_n\}_{n \in Z^+}$ has finite intersection property. $\therefore$ By theorem (4.11) $\bigcap_{n \in Z^+} C_n$ is non-empty.

5. Sequentially g**-compact space

Definition 5.1: A subset A of a topological space (X, τ) is said to be sequentially g**- compact if every sequence in A contains a subsequence which g**-converges to some point in A.

Example 5.2: Any finite topological space is sequentially g**-compact.

Example 5.3: An infinite indiscrete topological space is not sequentially g**-compact.

Theorem 5.4: A finite subset A of a topological space (X, τ) is sequentially g**-compact.

Proof: Let $\{x_n\}$ be an arbitrary sequence in X. Since A is finite, at least one element of the sequence say x_0 must be repeated infinite number of times. So the constant subsequence x_0, x_0 ,…….must g**-converges to x_0.

Remark 5.5: Sequentially g**-compactness implies sequentially compactness but the inverse implication is not true as seen in the following example.

Example 5.6: Any infinite indiscrete space is sequentially compact but not sequentially g**-compact.

Theorem 5.7: Every sequentially g**-compact space is g**-countably compact.

Proof: Let (X,τ) be sequentially g**-compact. Suppose X is not g**-countably compact. Then there exists countable g**-open cover $\{U_n\}_{n\in Z^+}$ which has no finite sub cover.Then $X = \underset{n\in Z^+}{\cup} U_n$. Choose $x_1 \in U_1, x_2 \in U_2 - U_1, x_3 \in U_3 - \underset{i=1,2}{\cup} U_i$ ....$x_n \in U_n - \overset{n-1}{\underset{i=1}{\cup}} U_i$. This is possible since $\{U_n\}$ has no finite sub cover. Now $\{x_n\}$ is a sequence in X. Let $x \in X$ be arbitrary. Then $x \in U_k$ for some k. By our choice of $\{x_n\}$, $x_i \notin U_k$ for all i greater than k. Hence there is no subsequence of $\{x_n\}$ which can g**-converge to x. Since x is arbitrary the sequence $\{x_n\}$ has no convergent subsequence which is a contradiction. Therefore X is g**-countably compact.

Theorem 5.8: Let $f:(X,\tau)\to(Y,\sigma)$ be a function, then

(1) f is g**-resolute, bijection and Y is sequentially g**-compact $\Rightarrow$ X is sequentially g**-compact.

(2) f is onto, g**-irresolute and X is sequentially g**-compact $\Rightarrow$ Y sequentially g**-compact.

(3) f is onto, g**-irresolute and X is sequentially g**-compact $\Rightarrow$ Y is sequentially g**-compact.

(4) f is onto, continuous and X is sequentially g**-compact $\Rightarrow$ Y is sequentially compact.

(5) f is onto, strongly g**-continuous and X is sequentially g**-compact $\Rightarrow$ Y is sequentially g**- compact.

Proof: (1): Let $\{x_n\}$ be a sequence in X. Then $\{f(x_n)\}$ is a sequence in Y. It has a g**convergent subsequence $\{f(x_{n_k})\}$ such that $f(x_{n_k}) \xrightarrow{g^{**}} y_0$ in Y. Then there exists $x_0 \in X$ such that $f(x_0) = y_0$. Let U be a g**-open set containing x_0. Then $f(U)$ is a g**-open set containing y_0 . Then there exists N such that $f(x_{n_k}) \in f(U)$ for all $k \geq N$. $\therefore f^{-1}\circ f(x_{n_k}) \in f^{-1}\circ f(U). \therefore x_{n_k} \in U$ for all $k \geq N$. This proves that X is sequentially g**-compact.

Proof for (2) to (5) is similar to the above.

6. g**-locally compact space

Definition 6.1: A topological space (X,τ) is said to be g**-locally compact if every point of x is contained in a g**-neighbourhood whose g**closure is g**-compact.

Remark 6.2: Any g**-compact space is g**-locally compact but the converse need not be true as seen in the following example.

Example 6.3: Let (X,τ) be an infinite indiscrete topological space. It is not g**-compact. But for every $x \in X, \{x\}$ is a g**-neighbourhood and $\overline{\{x\}} = \{x\}$ is g**-compact. Therefore it is g**-locally compact.

Theorem 6.4: Let (X,τ) be g**-multiplicative g**-T_2 space. Then X is g**-locally compact if and only if each of its points is a g**-interior point of some g**-compact subset of X.

Proof: Let X be g**-locally compact and $x \in X$. Then x has a g**-neighbourhood N such that g**cl(N) is g**-compact. Conversely, let every point $x \in X$ be a g**- interior point of some g**compact subset of X. Given $x \in X$, there exists g**-compact subset N such that $x \in$ g**int(N). So, N is a g**-neighbourhood of $x.$ By the hypothesis and theorem (3.9), N is g**-closed. Therefore X is g**-locally compact.

7. g**-compact modulo *I*

Definition 7.1: An ideal topological space (X,τ,I) is said to be g**-compact modulo I if for every g**-open covering $\{U_\alpha\}_{\alpha\in\Delta}$ of X, there exists a finite subset Δ_0 of Δ such that $X - \bigcup_{\alpha\in\Delta_0} U_\alpha \in I.$

Remark 7.2: g**-compactness implies g**-compact modulo *I* for any ideal *I* but not conversely.

Example 7.3: Let (X,τ,I) be an indiscrete infinite topological space where $I = \wp(X)$. Let $\{U_\alpha\}_{\alpha\in\Delta}$ be a g**-open cover for X. Let $\alpha_0 \in \Delta.$ Then $X - U_{\alpha_0} \in I.$ Therefore (X,τ,I) is g**-compact modulo *I* but not g**-compact.

Note: When $I = \{\varphi\}$ the concepts "g**-compact modulo *I*" and "g**-compact" coincide.

Remark 7.4: $g^{**}I-$ compact modulo I implies g**-compact modulo I and g**-compact modulo I implies compact modulo I

Proof: Obvious, since $\tau \subseteq G^{**}O(X) \subseteq G^{**}IO(X)$.

Example 7.5: An indiscrete space $(X,\tau,\{\varphi\})$ is compact modulo *I* but not g**-compact modulo {φ}.

Theorem 7.6: If $I \subseteq J$, then (X,τ,I) is g**-compact modulo *I* implies (X,τ,J) is g**-compact modulo *J*.

Proof is obvious.

Theorem 7.7: Let I_F denote the ideal of all finite subsets of X. Then (X,τ) is compact if and only if (X,τ,I_F) is compact modulo I_F.

Proof: Necessity: Follows since $\{\varphi\} \in I_F$.

Sufficiency: Let $\{U_\alpha\}_{\alpha\in\Delta}$ be a g**-open cover for X. then there exists a finite subset Δ_0 of Δ such that $X - \bigcup_{\alpha\in\Delta_0} U_\alpha \in I_F$. $\therefore X - \bigcup_{\alpha\in\Delta_0} U_\alpha = \{x_1, x_2, ...x_n\}$. Choose α_i such that $x_i \in U_{\alpha_i}$ for i = 1,2,……..n. Then $X = \{\bigcup_{\alpha\in\Delta_0} U_\alpha\} \cup \{\bigcup_{i=1}^{n} U_{\alpha_i}\}$. Therefore X is g**-compact modulo I_F.

8. g**-countably compact modulo *I*

Definition 8.1: An ideal topological (X,τ,I) is said to be g**-countably compact modulo *I* if for every countable g**-open covering $\{U_\alpha\}_{\alpha\in\Delta}$ of X, there exists a finite subset Δ_0 of Δ such that $X - \bigcup_{\alpha\in\Delta_0} U_\alpha \in I$.

All the results, from Remark (7.2) to theorem (7.7) are true in the case when (X,τ,I) is g**-countably compact modulo *I*.

References

[1] N.Levine, Rend. Cire. Math. Palermo, 19 (1970), 89 – 96.

[2] K.Kuratowski,Topologie,I.Warszawa,1933

[3] James R. Munkres, Topology, Ed.2.,PHI Learning Pvt.Ltd. New Delhi, 2010.

[4] Pauline Mary Helen. M, Veronica Vijayan, Ponnuthai Selvarani, g**-closed sets in topological spaces, IJM A 3(5), (2012),1-15.

[5] Pauline Mary Helen. M, Veronica Vijayan, Ponnuthai Selvarani, g***I*-continuous functions, IJM A .

[6] Pauline Mary Helen. M, Veronica Vijayan, Ponnuthai Selvarani, Seperation axioms via g**-closed sets in topological spaces and in ideal topological spaces, IJCA

[7] M.K.R.S. Veera Kumar, Mem. Fac. Sci.Kochi Univ. (Math.), 21 (2000), 1 – 19.