DRI of a vertex, Self Centered Graphs and Almost Self Centered Graphs - An algorithmic approach

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ABSTRACT

A graph is self centered if every node is in the center. Thus, in a self centered graph G all the nodes have the same eccentricity, so, $rad(G) = diam(G)$. The graphs in which there are exactly two diametral vertices and the remaining all the vertices are central, are called almost self centered graphs of type $(r+1,\ r)$. The graphs that have only one central vertex and all other vertices are diametral are also called as almost self centered graphs of type $(r, r+1)$. Using these definitions we developed an algorithm is developed to decide the type of graph. We also gave the relation between DRI of vertex, self centered graphs and almost self centered graphs.

Key words: DRI, Self centered graphs, Almost self centered graphs, Diameter

INTRODUCTOIN:

In this paper we are going to study about Self Centered and Almost self centered graphs and their relation with diametral reachable index of a vertex. Also we have developed a simple algorithm to decide whether the graph is self centered or almost self centered. Let us understand the basic terminologies needed for this.

For a vertex v in a graph G, the eccentricity $e(v)$ of v is the distance between v and the farthest vertex from v in G. The minimum of eccentricity among the vertices of G is its radius, $rad(G)$, and the maximum eccentricity is the diameter, $diam(G)$. A vertex of G is a central vertex if $e(v) = rad(G)$.

Diametral Reachable Index of a vertex:

The total number of diametral paths reachable from a vertex v is called the *Diametral Reachable Index* of that vertex, denoted $DRI(v)$[1].

For any vertex v, the $DRI(v) = 0$, if there are no diametral paths reachable from v. We write $DRI(v) = t$, when there are diametral paths reachable from vertex v, where t is the total number of diametral paths reachable from vertex v. In other words, the DRI of each vertex gives the maximum number of diametral paths reachable from that vertex.

Example 1: The diameter of the graph in figure 1(a) is 2. The diametral reachable index of each vertex is given in figure 1(b). In this graph the diametral vertices are 1, 3 and 4. The diametral paths from each vertex are as below:

From vertex 1:

(1) $1, 2, 3$ (2) $1, 2, 4$

From vertex 2: No diametral paths.

From vertex 3:

(1) $3, 2, 1$

From vertex 4:

(1) $4, 2, 1$

Therefore,

$DRI(1) = 2,$ $DRI(2) = 0,$ $DRI(3) = 1$ and $DRI(4) = 1$

The readers can refer [1] for the detailed study of DRI of a vertex and the algorithm to find the DRI of each vertex for a given graph.

Self centered and almost self centered graphs:

Some graphs have the property that each node of G is a central vertex. A graph is **self centered** if every node is in the center. Thus, in a self centered graph G all the nodes have the same eccentricity, so, $rad(G) = diam(G)$. Example: Cycles, Petersen Graph.

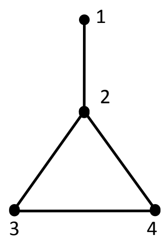

Figure 1(a)

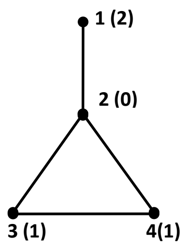

Figure 1(b)

For every graph there will be at least two diametral vertices. Based on this fact the graphs are also classified as almost self centered graphs. The graphs in which there are exactly two diametral vertices and the remaining all the vertices are central, are called **almost self centered graphs,** denoted $(r + 1,\ r)$ graphs [2]. With this detail it is clear that the graph contains only central and diametral vertices and no other values of eccentricity.

Example2: The graph given in figure 2 has two diametral vertices and remaining all are the central vertices. Hence it is almost self centered graph of type $(r + 1, r)$.

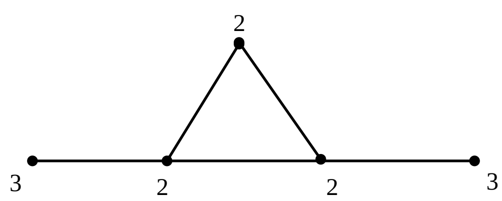

Figure 2: Almost self centered graph $(r + 1,\ r)$

If the graphs are not self centered then the graph contains at least one central vertex. The graphs that have only one central vertex and all other vertices are diametral are also called as **almost self centered graph of type $(\boldsymbol{r}, \boldsymbol{r}+\mathbf{1})$ [2].**

Example 3: The graph given in figure 3 has only one central vertex and remaining are all diametral vertices. Therefore the graph is almost self centered of type $(r, r+1)$. The eccentricity of each vertex is specified in graph.

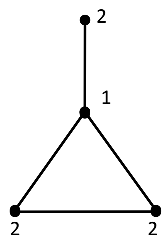

Figure 3: Almost self centered graph $(\boldsymbol{r},\ \boldsymbol{r}+\mathbf{1})$

In the next section we are going to discuss the algorithm using which we can decide whether a graph is self centered, almost self centered or none of these two. The algorithm takes the adjacency matrix and distance matrix as input and outputs the result.

ALGORITHM TO FIND WHETHER THE GRAPH IS SELF CENTERED OR ALMOST SELF CENTERED:

In this section we have explained the algorithm. The algorithm needs a distance matrix of the graph which can be obtained by Floyd's algorithm or Dijkstra's algorithm. The algorithm takes $O(n^2)$ time to find the eccentricity of each vertex and $O(n)$ time for finding radius and diameter and as well as for finding out whether the graph is self centered or almost self centered. The steps are given as follows:

Algorithm:

1. Read the values for number of vertices, adjacency matrix of graph and related distance matrix.
2. Find the radius and diameter of the graph using distance matrix.
3. Count the number of central vertices and diametral vertices.
4. If all the vertices are diametral vertices or central vertices, i.e., radius=diameter, then output that, the graph is self centered.
5. If the number of central vertices equals $n-2$ and number of diametral vertices equals to 2 then output that the graph is almost self centered of type $(r+1,\ r)$.
6. If the number of central vertices equals 1 and number of diametral vertices equals to $n-1$ then output that the graph is almost self centered of type $(r,\ r+1)$.
7. If any of the conditions specified in steps 4 or 5 or 6 are not true then output that the graph is neither self centered nor almost self centered.

THE RELATION BETWEEN DRI, SELF CENTERED GRAPHS AND ALMOST SELF CENTERED GRAPHS:

Proposition 1: For any graph G, if $DRI(u) \neq 0$, for all vertices u in G, then G is self centered graph.

Proof:

Let G be a graph and u be an arbitrary vertex in G with $DRI(u) \neq 0$. Then by the definition of DRI of vertex, there exists a vertex u in G such that $d(u, v) = diam(G)$. Then, $e(u) = diam(G)$. Since u is arbitrary vertex, hence $e(u) = diam(G)$, for all vertices in G. Thus all vertices of G are equi eccentric and hence G is self centered graph.

Proposition 2: If G contains only one vertex u with DRI value 0, then u is a central vertex.

Proof:

Let u be the only vertex with $DRI(u) = 0$, then $DRI(v) \neq 0$ for all $v \neq u$. This implies that $e(v) = diam(G)$, $\forall v \neq u$. Then the eccentricity of u is less than $diam(G)$. Since all other vertices have same eccentricity except u. Hence $e(u) = radius(G)$. Then u is a central vertex.

Remarks:

By the above proposition, if u is the only vertex with $DRI(u) = 0$, then $e(u) = r$ where r is radius of G. Further, all other vertices $v(\neq u)$ whose eccentricity is equal to $diam(G)$ and hence $diam(G) = r + 1$. Hence G is almost self-centered graph of the type $(r, r + 1) -$ graph.

A graph G is said to be Equi-DRI graph, if $DRI(u) = k$, for all vertices u, in G and for some fixed non zero integer k. For example, the Petersen graph, cycle C_n and complete graph K_n and complete bipartite graph $K_{n,n}$. From the proposition proved above, it is evident that Equi-DRI graphs are self centered graphs; but the converse need not be true

CONCLUSIONS:

The paper deals with many concepts related to self centered and almost self centered graphs. The algorithm is designed to find out the type of graph here.

We can further study the class of graph that satisfy the conditions for self centered and almost self centered graphs depending on the number of vertices present in the graph. For example let us consider paths. A path P_2 is self centered graph. A path P_3 is almost self centered graph of type $(r, r + 1)$. A path P_4 is almost self centered of type (r+1, r). The paths of order $n \geq 5$ are neither self centered nor almost self centered. Further the books [3, 4, 5 ,6] are referred for the basics of algorithms and graph theory.

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