A note on the weaker form of bI sets and its generalization on SEITS

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ABSTRACT

The focus of this paper is to introduce a new class of sets known as bI^+ open sets, defined in the light of simple expansion topology and ideal topology. This set is investigated and found to be a weaker form of bI open sets.We have also generalized this concept and studied its properties.

1.INTRODUCTION

Levine [9] , in 1963 defined simple expansion of topology τ by a non open set B where B $\in\tau$ as $\tau[B]=\{ OU (O' \cap B) / O,O' \in \tau\}$. In 1990, Jankovic and Hamlett [8] introduced the notion of I open sets in ideal topological space. M.E Abd.El-Monsef et al [2] further investigated I open sets and I continuous function.

In 1999 Dontchev [6] introduced the notion of pre I open sets which is a combination of pre open set and an ideal and found that to be weaker than that of I open sets. The concept of pre open set was introduced by Corson and Micheal [4] who used the term "locally dense". This set defined by Corson was redefined by the name "pre- open set" by A. S. Mashhour. M.E Abd Ed.

The other notions of αopen set, Semi-open set, β open set, t set, b open and * perfect sets were introduced and studied by many topologists in [12],[10], [1], [13], [5]

These sets defined above were idealized as αI open, semiI-open and βI -open by Hatir and Noiri [7]. Caksu Guler and Aslim[3] have introduced the notion of bI open sets and bI continuous functions.

The prerequisites of the paper are defined as follows:

A set A of a ideal topological space is said to be

1.1 I open [8] if $A \subseteq int(A^*)$
1.2 $\propto$I open [12] if $A \subseteq int(cl^*(int(A)))$
1.3 PreI-open[6] if $A \subseteq int(cl^*(A))$
1.4 SemiI-open[7] if $A \subseteq cl^*(int (A))$
1.5 bI open[3] if $A \subseteq int(cl^* (A)) \cup cl^*(int (A))$
1.6 βI open[7] if $A \subseteq cl^* (int(cl^*(A))$
1.7 * perfect[5] if $A=A^*$

In this paper we have made an attempt to extend these concepts of I openness, ∝I openness, pre-openness, semi-openness,tI openness, βI openness and bI openness in simple expansion topology.

2.bI$^+$ OPEN SETS

Definitions:

Let A be a subset of a SEITS, then A is said to be

1. b$^+$ open if $A \subseteq int(cl^+(A)) \cup cl^+(int(A))$
2. I$^+$ open if $A \subseteq int(A^{+*})$
3. αI$^+$ open if $A \subseteq int(cl^{+*}(int(A^{+*})))$
4. PreI$^+$open if $A \subseteq int(cl^{+*}(A))$
5. SemiI$^+$open if $A \subseteq cl^{+*}(int(A))$
6. tI$^+$ open if $int(cl^{+*}(A)) = int(A)$
7. βI$^+$ open if $A \subseteq cl^{+*}(int(cl^{+*}(A)))$
8. bI$^+$ open if $A \subseteq int(cl^{+*}(A)) \cup cl^{+*}(int(A))$.

In all the above definitions the interior refers to the interior in usual topology and $cl^{+*}(A)$ denotes the closure with respect to the ideal topological space under simple expansion.

Here a new local function is defined on the simple expansion ideal topological space (SEITS) and it is denoted as $A^{+*} = \{x \in X / U \cap A \notin I$ for each neighbourhood U of x in $\tau^+(B)\}$ and known as extended local function with respect to τ^+ and I. Also we define the closure operator as

$cl^{+*}(A) = A \cup A^{+*}$

A subset A of (X, τ^+, I) is called *+ perfect if $A = A^{+*}$

Theorem 2.1:

i)Every open set is bI$^+$ open.

ii)Every bI$^+$ open set is bI open.

iii)Every I$^+$ open set is bI$^+$ open

Proof:

i)Let A be any subset of (X, τ^+, I) if A is open in τ, we have,

$A = int(A)$

ie.,$A \subseteq int(cl^{+*}(A))$

ie., $A \subseteq int(cl^{+*}(A)) \cup cl^{+*}(int(A))$

$\Rightarrow$ A is bI$^+$ open

ii) By the definitions of bI$^+$ open and bI open sets and the condition that $cl^{+*}(A) \subseteq cl^*(A)$, every bI^+ open set is bI open.

iii) Proof is obvious.

Remark 2.2:

From the above theorem we note that the class of bI^+ open sets is properly placed between an open set and a bI open set.

But the converses of the above theorem are not true .

Example 2.3:

X={a,b,c} τ = {ϕ,X,{a},{a,b}}; I={ϕ,{b}}; B={b};τ^+(B)= {ϕ,X,{a},{b},{a,b}}.

Here {a,c} is bI^+ open but not open in the topology τ and τ^+(B).

Example 2.4:

X={a,b,c} τ = {ϕ,X,{a},{b},{a,b}}I={ϕ,{a}}; B={b,c};τ^+(B)= {ϕ,X,{a},{b},{a,b},{b,c}}.

Here {a},{b,c},{a,b} are bI^+ open but not I^+ open.

Example 2.5:

X={a,b,c} τ = {ϕ,X,{a},{b},{a,b}}I={ϕ,{c}}; B={b,c};τ^+(B)= {ϕ,X,{a},{b},{a,b},{b,c}}.

Here {a,c} is bI open but not bI^+ open.

Theorem 2.6:

For an SEITS (X , τ^+,I) and $A \subseteq X$ we have the following:

i) If I = ϕ then A is bI^+ open if and only if A is b^+ open

ii) If I = P(X) then A is bI^+ open if and only if A is open in τ

iii) If I = N then A is bI^+ open if and only if A is b^+ open

Proof:

i)If I= ϕ then $A^{+*}= cl^+(A)$ for any subset A of X and hence

$cl^{+*}(A)=A^{+*}\cup A=cl^+(A)$.Hence we have $A^{+*}= cl^+(A)= cl^{+*}(A)$.

Thus (i) follows immediately.

ii) If I= P(X) then $A^{+*}= \phi$ for any subset A of X.

Since A is bI^+ open we have , $A \subseteq int(cl^{+*}(A)) \cup cl^{+*}(int (A))$

ie., $A \subseteq int(A^{+*}\cup A) \cup[(int (A)^{+*})\cup int(A)]$

$A \subseteq int(\phi \cup A) \cup[\phi \cup int(A)]$

$A \subseteq int (A) \Rightarrow$A is open in τ.

iii)Every bI^+ open set is b^+ open .

Let A be a bI^+ open set then,

$A \subseteq int (cl^{+*}(A)) \cup cl^{+*}(int (A))$

$A \subseteq int(A^{+*} \cup A) \cup [(int(A)^{+*}) \cup int(A)]$

$A \subseteq int(cl^{+}(A) \cup A) \cup [cl^{+}(int(A) \cup int(A)]$

$A \subseteq int(cl^{+}(A)) \cup cl^{+}(int(A))$

$\Rightarrow$ A is b^{+} open .

Hence (iii) is proved.

Now let us consider I = N and A is b^{+} open

If I=N , then $A^{+*} = cl^{+*}(int(cl^{+*}(A))$

Since A is b^{+} open $\Rightarrow$ $A \subseteq int(cl^{+}(A) \cup cl^{+}(int(A)$

Then $A \subseteq int(A \cup cl^{+}(int(cl^{+}(A))) \cup cl^{+}(int(A)$

$\subseteq int(A \cup cl^{+}(int(cl^{+*}(A))) \cup cl^{+*}(int(A)$

$\subseteq int(A \cup A+*) \cup cl^{+*}(int(A)$

$A \subseteq int(cl^{+*}(A)) \cup cl^{+*}(int(A))$

$\Rightarrow$ A is bI^{+} open. Hence the proof.

Theorem 2.7:

Let A be a subset of a SEITS (X , τ^{+},I) then the following properties are true

a)Every semi I^{+} open set is bI^{+} open

b)Every pre I^{+} open set is bI^{+} open

c) Every b I^{+} open set is βI^{+} open.

d)Every αI^{+} open set is βI^{+} open.

Proof:

(a) & (b) are obvious from the definition of bI^{+} open set

c) Let A be a b I^{+} open set then we have,

$A \subseteq int(cl^{+*}(A)) \cup cl^{+*}(int(A))$

ie., $A \subseteq cl^{+*}\{(int(cl^{+*}(A)) \cup [(int(A)^{+*} \cup int(A)]\}$

$A \subseteq cl^{+*}\{(int(cl^{+*}(A)) \cup cl^{+*}[(int(A)^{+*} \cup int(A)]\}$

$A \subseteq cl^{+*}(int(cl^{+*}(A)) \cup cl^{+*}[(int(A)^{+*}]$

$A \subseteq cl^{+*}(int(cl^{+*}(A))$

ie., A is βI^{+} open.

d)Proof is obvious.

Remark:2.8:

Open in τ $\rightarrow$ αI^{+} open $\rightarrow$ semiI^{+}open

$\downarrow$ $\downarrow$

I^+ open $\rightarrow$ preI^+open $\rightarrow$ bI^+ open $\rightarrow$ βI^+ open

Some of the reverse implications are not ture as shown by the following examples.

Example 2.9:

X={a,b,c} τ = {ϕ,X,{a},{b},{a,b}}I={ϕ,{a}}; B={b,c};τ^+(B)= {ϕ,X,{a},{b},{a,b}{b,c}}.

Here {b,c} is bI^+ open but not preI^+open .

Example 2.10:

X={a,b,c} τ = {ϕ,X,{c};I={ϕ,{c}}; B={a};τ^+(B)= {ϕ,X,{a},{c},{a ,c}}.

Here {a, c} is bI^+ open but not semi I^+ open.

Example 2.11

X={a,b,c} τ = {ϕ,X,{a},{b},{a,b}}I={ϕ,{a}}; B={b,c};τ^+(B)= {ϕ,X,{a},{b},{a,b}{b,c}}.

Here {b,c} is bI^+ open but not αI^+open .

Theorem 2.12:

Let (X , τ^+,I) be a SEITS with I and J as ideals on X and let A & B be subsets of X then we have the following

a) $A\subseteq B \Rightarrow A^{+*} \subseteq B^{+*}$
b) $I\subseteq J \Rightarrow A^{+*}(I) \subseteq A^{+*}(J)$
c) $A^{+*}= cl(A^{+*}) \subseteq cl(A)$
d) $(A^{+*})^{+*} \subseteq A^{+*}$
e) $(A\cup B)^{+*}= A^{+*} \cup B^{+*}$
f) $U\in\tau \Rightarrow U\cap A^{+*}= U \cap(U\cap A)^{+*} \subseteq (U\cap A)^{+*}$
g) $I\in I\Rightarrow(A\cup I)^{+*}= A^{+*}=(A\backslash I)^{+*}$

Proof:

Obvious using the definition of A^{+*}

Theorem 2.13:

Let (X , τ^+,I) be a SEITS and let A,U $\in$ X .If A is a bI^+ open set and U $\in$ τ, then $A\cap U$ is a bI^+ open set.

Proof:

By assumption let A be a bI^+ open set then, $A \subseteq int(cl^{+*}(A)) \cup cl^{+*}(int (A))$ and $U\subseteq intU$

By theorem 2.12 (f) we have

$A\cap U \subseteq [int(cl^{+*}(A)) \cup cl^{+*}(int (A))] \cap intU$

$\subseteq [int(cl^{+*}(A)) \cap intU] \cup [cl^{+*}(int (A)) \cap intU]$

$=[int(A^{+*} \cap U)\cup(A\cap U\_] \cup [int (A)^{+*} \cap intU] \cup [int (A)\cap intU]$

$\subseteq$ int[(A$\cap$ U) $^{+*}$ $\cup$(A$\cap$U)_] $\cup$ [int (A$\cap$U) $^{+*}$ $\cup$int(A$\cap$U)]

A $\subseteq$ int(cl^{+*}(A$\cap$U)) $\cup$ cl^{+*}(int (A$\cap$U))

$\Rightarrow$ A is is U$\cap$A is is bI$^+$ open.

Theorem 2.14:

Let (X , τ^+,I) be a SEITS .Then the following hold

a) Union of arbitrary family of bI$^+$ open sets is bI$^+$ open.

b) Intersection of arbitrary family of bI$^+$ closed sets is bI$^+$ closed.

c) If A $\in$ BI$^+$O(X , τ^+, I) and B$\in$ τ ,then A$\cap$B $\in$ BI$^+$O(X , τ^+, I)

Proof:

a)Let $\{A_\alpha/\alpha\in\Delta\}$ be a family of bI$^+$ open sets then,

$A_\alpha \subseteq int(cl^{+*}(A_\alpha)) \cup cl^{+*}(int (A_\alpha))$

Hence $\cup_\alpha A_\alpha \subseteq \cup_\alpha [int(cl^{+*}(A_\alpha)) \cup cl^{+*}(int (A_\alpha))]$

$\subseteq \cup_\alpha [int(cl^{+*}(A_\alpha))] \cup [\cup_\alpha (cl^{+*}(int (A_\alpha))]$

$\subseteq int(\cup_\alpha (cl^{+*}(A_\alpha)) \cup cl^{+*}(\cup_\alpha (int (A_\alpha))$

$\subseteq int((cl^{+*}(\cup_\alpha A_\alpha)) \cup cl^{+*}((int (\cup_\alpha A_\alpha))$

$\Rightarrow \cup_\alpha A_\alpha$ is bI$^+$ open.

b) Let $\{B_\alpha/\alpha\in\Delta\}$ be a family of bI$^+$ closed sets.

Then $\{B_\alpha^c/\alpha\in\Delta\}$ be a family of bI$^+$ open sets. By (a) $\cup_\alpha B_\alpha^c$ is bI$^+$ open.

Hence $(\cap_\alpha B_\alpha)^c =(\cup_\alpha B_\alpha)^c$ is bI$^+$ open.

$\Rightarrow(\cap_\alpha B_\alpha)$ is bI$^+$ closed set. Hence the proof.

c) Let A $\in$ BI$^+$O(X , τ^+, I) and B $\in$ τ .Then A $\subseteq$ int(cl^{+*}(A)) $\cup$ cl^{+*}(int (A)) and

A $\cap$ B $\subseteq$ [int(cl^{+*}(A)) $\cup$ cl^{+*}(int (A))] $\cap$ B

= [(int(cl^{+*}(A)) $\cap$ B)] $\cup$ [cl^{+*}(int (A)) $\cap$ B]

= [int (A$\cup$ A^{+*}) $\cap$ B] $\cup$ [(int(A) $\cup$ (int(A) $^{+*}$) $\cap$ B]

$\subseteq$ [int[(A$\cap$B) $\cup$ (A^{+*}$\cap$ B)]] $\cup$ [(int(A $\cap$ B) $\cup$ int(A$\cap$ B) $^{+*}$] (using theorem 3.9)

$\subseteq$[int cl^{+*} (A$\cap$B)] $\cup$ [cl^{+*}(int(A $\cap$ B)]

Hence A $\cap$ B $\in$ BI$^+$O(X , τ^+, I) .Hence the proof.

If (X , τ^+,I) be a SEITS and A is a subset of X, we denote by $\tau^+/_A$, the relative topology on A and $I/_A$ =$\{A\cap I : I\in I\}$ is clearly an ideal on A.

Lemma 2.15:

Let (X, τ^+, I) be a SEITS and A,B are subsets of X such that $B \subseteq A$.

Then $B^{+*}(\tau^+/_A, I/_A) = B^{+*}(\tau^+, I) \cap A$.

Theorem 2.16:

Let (X, τ^+, I) be a SEITS and if $U \in \tau$ and $V \in BI^+O(X, \tau^+, I)$ then

$U \cap V \in BI^+O(U, \tau^+/_A, I/_A)$.

Proof:

Since U is open, we have $int_u A = int\ A$ for any subset A of U. By using this fact and theorem 2.15 we have,

$U \cap V \subseteq U \cap (cl^{+*}(int(V)) \cup int(cl^{+*}(V)))$

$\subseteq [U \cap [(int(V) \cup (int(V))^{+*})] \cup [U \cap int(V \cup V^{+*})]]$

$\subseteq \{U \cap [U \cap int(V) \cup U \cap (int(V))^{+*}]\} \cup \{U \cap [U \cap [int(V \cup V^{+*})]]\}$

$\subseteq \{U \cap [U \cap int(V) \cup (U \cap intV)^{+*}] \cup \{U \cap [U \cap [int(V \cup V^{+*})]]\}$

$\subseteq \{U \cap [int_U(U \cap V) \cup (U \cap int_U(U \cap V)^{+*}] \cup \{U \cap [[int(U \cap V) \cup (U \cap V)^{+*}]]\}$

$= \{[int_U(U \cap V)] \cup [int_U(U \cap V)]^{+*}{}_{(\tau^+/U, I/U)}]\} \cup \{U \cap [[int(U \cap V) \cup (U \cap V)^{+*}]]\}$

$= cl^{+*}[int_U(U \cap V)] \cup [int_U(cl^{+*}(U \cap V))]$

This shows that $U \cap V \in BI^+O(U, \tau^+/_A, I/_A)$.

Definition 2.17:

A point $x \in X$ is said to be an I^+ limit point of A if for every I^+ open set U in X,$U \cap (A \backslash x) \neq \phi$.

The set of all I^+ limit point of A is called the I^+ derived set of A denoted by $D_I^+(A)$

Definition 2.18:

Let A be a subset of (X, τ^+, I). A point $x \in X$ is said to be an bI^+ limit point of A if for every bI^+ open set U in X, $U \cap (A \backslash x) \neq \phi$.

The set of all bI^+ limit point of A is called the bI^+ derived set of A denoted by $D_{bI}^+(A)$

Since every open set is preI^+open and every preI^+open is bI^+ open we have

$D_{bI}^+(A) \subseteq D(A)$ for any subset $A \subseteq X$. Moreover ,since every closed set is bI^+ open we have

$A \subseteq bI^+cl(A) \subseteq cl(A)$

Lemma 2.19:

If $D(A) = D_{bI}^+(A)$,then we have $cl(A) = bI^+cl(A)$

Proof: Straightforward

Corollary 2.20:

If D (A) $\subseteq D_{bI}^{+}(A)$, for every subset A of X. Then for any subset F and B of X , we have $bI^{+}cl(F \cup B) = bI^{+}cl(F) \cup bI^{+}cl(B)$.

Theorem 2.21:

If A be a subset of (X , τ^{+},I), then $x \in bI^{+}cl(A)$ if and only if every bI^{+} open set U containing x intersects A.

Proof:

Let us prove that $x \notin bI^{+}cl(A)$ if and only if there exists a bI^{+} open set U containing x which does not intersect A.

ie.$x \notin bI^{+}cl(A) \Rightarrow x \in X \setminus bI^{+}cl(A)$ which does not intersect A.

Conversely, let U be a bI^{+} open set U containing x which does not intersect A.Then (X\U) is a bI^{+} open set U containing A and $x \notin ((X\setminus U)$ but $bI^{+}cl(A) \subseteq X\setminus U$.

Therefore ,$x \in bI^{+}cl(A)$.

Theorem 2.22:

$bI^{+}cl(A) = A \cup D_{bI}^{+}(A)$.

Proof: If $x \in D_{bI}^{+}(A)$.

Then for every bI+ open set U containing x ,we have $U \cap \{A \setminus \{x\}\} \neq \phi$.Therefore $x \in bI^{+}cl(A)$

ie., $A \cup D_{bI}^{+}(A) \subseteq bI^{+}cl(A)$ ⟶ (1)

Conversely ,let $x \in bI^{+}cl(A)$

If $x \in A$,then $x \in A \cup D_{bI}^{+}(A)$.Let $x \notin A$,since $x \in bI^{+}cl(A)$ every bI^{+} open set U containing x intersect A. But $x \notin A \Rightarrow U \cap \{A \setminus \{x\}\} \neq \phi$. Therefore $x \in D_{bI}^{+}(A)$

ie., $bI^{+}cl(A) \subseteq A \cup D_{bI}^{+}(A)$ ⟶ (2)

From (1) and (2) we get $bI^{+}cl(A) = A \cup D_{bI}^{+}(A)$.Hence the proof.

Theorem 2.23:

In a SEITS (X , τ^{+},I) then $D_{bI}^{+}(A) \subseteq D_{I}^{+}(A)$ for every subset A of X.

Proof:

Let $x \in D_{bI}^{+}(A)$ and let U be an open set in A containing x, then U is bI^{+} open.

Therefore ,U is bI^{+} open set containing x.

Hence $U \cap \{A \setminus \{x\}\} \neq \phi$.

ie., $x \in D_{I}^{+}(A)$. Hence $D_{bI}^{+}(A) \subseteq D_{I}^{+}(A)$.

3.GENERALISED bI^+ CLOSED SETS

Definition3.1: A subset A of a SEITS (X , τ^+,I) is said to be a gbI^+ closed if $bI^+cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in τ^+.

The collection of all gbI^+ closed sets of X is denoted as $GBI^+C(X)$

Example 3.2:

Let X= {a,b,c} τ = {ϕ,X,{a},{a,b}}I={ϕ,{b}}; B={b};τ^+(B)= {ϕ,X,{a},{b},{a,b}}.

Here bI^+ open sets are {ϕ,X,{a},{a,b},{a,c}}and gbI^+ closed sets are {ϕ,X,{b},{c},{b,c},{a,c}}.

Note:

Since every I^+ closed set is bI^+ closed we have $bI^+cl(A) \subseteq I^+cl(A)$

Theorem 3.3:

i)Every I^+ closed set is gbI^+ closed.

ii)Every bI^+ closed set is gbI^+ closed.

Proof:

Let $A \subseteq U$ and U is open in τ^+.

Since A is I^+ closed we have $A= I^+cl(A) \subseteq U$ .By the above note we have $bI^+cl(A) \subseteq I^+cl(A)$

ie., $bI^+cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in τ^+. Hence the proof.

ii) Let A be a bI^+ closed set. Then $bI^+cl(A) =A \subseteq U$. Hence A is a gbI^+ closed.

But the converse need not be true.

Example 3.4:

Let X= {a,b,c} τ = {ϕ,X,{a},{a,b}}I={ϕ,{b}}; B={b};τ^+(B)= {ϕ,X,{a},{b},{a,b}}.

Here {a,c} is gbI^+ closed but not bI^+ closed.

Theorem 3.5:

If A is a gbI^+ closed set of a SEITS (X , τ^+,I), then $bI^+cl(A) \setminus A$ does not contain any non empty closed set .

Proof:

Let F be a closed set such that $F \subseteq bI^+cl(A) \setminus A$. Then(X\F) is open and

$F \subseteq bI^+cl(A) \cap A^c \longrightarrow$ (1)

$\Rightarrow F \subseteq A^c$

$\Rightarrow (X\setminus F) \supset A$. Since A is gbI^+ closed we have $bI^+cl(A) \subseteq (X\setminus F)$.

Hence $F \subseteq X \setminus bI^+cl(A) \longrightarrow$ (2)

From (1) & (2) we have

$F \subseteq (X \backslash bI^{+}cl(A)) \cap bI^{+}cl(A) = \phi$ ie., $F = \phi$.Hence $bI^{+}cl(A) \backslash A$ does not contain any non empty closed set . Hence the proof.

Theorem 3.6:

If A be a gbI^{+} closed set of a SEITS (X, τ^{+}, I) and $A \subseteq B \subseteq bI^{+}cl(A)$ then B is also gbI^{+} closed.

Proof:

Let A be a gbI^{+} closed set) and $A \subseteq B \subseteq bI^{+}cl(A)$.Then $bI^{+}cl(A) \subseteq bI^{+}cl(B) \subseteq bI^{+}cl(A)$ which implies $bI^{+}cl(A) = bI^{+}cl(B)$ let us now consider U to be a open set in (X, τ^{+}, I) containing B. Then $A \subseteq U$ and A is gbI^{+} closed .

$\Rightarrow bI^{+}cl(A) \subseteq U$

$\Rightarrow bI^{+}cl(B) \subseteq U \Rightarrow$ B is gbI^{+} closed.

We now provide a necessary and sufficient condition for a gbI^{+} closed set to be bI^{+} closed.

Theorem 3.7:

A gbI^{+} closed set A is bI^{+} closed if and only if $bI^{+}cl(A) \backslash A$ is closed.

Proof:

Let A be bI^{+} closed , then $A = bI^{+}cl(A)$.

$\Rightarrow$ if $bI^{+}cl(A) \backslash A = \phi$ which is closed.

Conversely , let $bI^{+}cl(A) \backslash A$ is closed. By theorem 3.5 we know that $bI^{+}cl(A) \backslash A$ does not contain any non empty closed set. Therefore $bI^{+}cl(A) \backslash A = \phi \Rightarrow bI^{+}cl(A) = A$. Hence A is bI^{+} closed.

Theorem 3.8 :

If A and B are gbI^{+} closed sets such that $D(A) \subseteq D_{bI}^{+}(A)$ and $D(B) \subseteq D_{bI}^{+}(B)$ .Then $A \cup B$ is gbI^{+} closed.

Proof:

Let U be an open set such that $A \cup B \subseteq U$ .Then since A and B are gbI^{+} closed sets we have $bI^{+}cl(A) \subseteq U$ and $bI^{+}cl(B) \subseteq U$. Since $D(A) \subseteq D_{bI}^{+}(A)$, thus $D(A) = D_{bI}^{+}(A)$ and by lemma 2.19 , $cl(A) = bI^{+}cl(A)$ and similarly $cl(B) = bI^{+}cl(B)$.

Thus $bI^{+}cl(A \cup B) \subseteq cl(A \cup B) = cl(A) \cup cl(B) = bI^{+}cl(A) \cup bI^{+}cl(B) \subseteq U$.

This implies $A \cup B$ is gbI^{+} closed.

Definition 3.9:

Let $B \subseteq A \subseteq X$. The set B is said to be gbI^{+} closed relative to A if $bI^{+}cl_{A}(B) \subseteq U$ whenever $B \subseteq U$ and U is open in A, where $bI^{+}cl_{A}(B) = A \cap bI^{+}cl(B)$

Theorem 3.10:

If $B \subseteq A \subseteq X$ and A is gbI^+ closed and open ,then B is gbI^+ closed relative to A if and only if B is gbI^+ closed in X.

Proof:

Let A be gbI^+ closed and open. Let B be gbI^+ closed relative to A. Since A is gbI^+ closed and open, we have $bI^+cl(A) \subseteq A$.

Therefore $bI^+cl(B) \subseteq bI^+cl(A) \subseteq A$

Therefore $bI^+cl_A(B) \subseteq bI^+cl(B) \cap A = bI^+cl(B)$.

Now let U be open in X and $B \subseteq U$.

Then $U \cap A$ is open in A and $B \subseteq U \cap A$. Since B is gbI^+ closed relative to A we have

$bI^+cl_A(B) \subseteq U \cap A$. Hence $bI^+cl_A(B) \subseteq U \cap A \subseteq U$. Therefore B is gbI^+ closed.

Conversely , let B be gbI^+ closed in X.

Consider U be open in A and $B \subseteq U$. Then $U = V \cap A$ where V is open in (X,τ^+,I).

Now $B \subseteq V$ and B is gbI^+ closed in X. This implies $bI^+cl(B) \cap A \subseteq V \cap A = U$.

ie., $bI^+cl_A(B) \subseteq U$.

Therefore B is gbI^+ closed relative to A. Hence the proof.

Definition 3.11:

A set A is said to be gbI^+ open if and only if $(X \backslash A)$ is gbI^+ closed.

The family of all gbI^+ open subsets of X is denoted by $GBI^+O(X)$.

The largest gbI^+ open set contained in X is called the gbI^+ interior of A and is denoted by $gbI^+(int(A))$.Also A is gbI^+ open if and only if $gbI^+(int(A)=A$.

Theorem 3.12:

$bI^+cl(X \backslash A) = X \backslash bI^+(int(A)$

Proof:

Let $x \in bI^+cl(X \backslash A)$

$\Leftrightarrow$ every bI^+ open set U containing x intersects $(X \backslash A)$

$\Leftrightarrow$ there is no bI^+ open set containing x and contained in A.

$\Leftrightarrow x \in X \backslash bI^+(int(A)$

Theorem 3.13:

A subset A of a SEITS (X , τ^+,I) is gbI^+ open if and only if $F \subseteq bI^+(int(A)$ whenever F is closed and $F \subseteq A$.

Proof:

Let A be gbI^+ open and suppose that F is closed and $F \subseteq A$. Then (X\A) is gbI^+ closed and

$(X\backslash F) \supset (X\backslash A)$.Now (X\F) is open and (X\A) is gbI^+ closed. Therefore $bI^+cl(X \backslash A) \subseteq (X\backslash F)$.

By theorem 3.12, $bI^+cl(X \backslash A) = X\backslash bI^+(int(A)$.

Hence $X\backslash bI^+(int(A) \subseteq (X\backslash F)$.

ie,$F \subseteq bI^+(int(A)$.

Conversely , let $F \subseteq bI^+(int(A)$ whenever F is closed and $F \subseteq A$.

Now to prove A is gbI^+ open is the same as proving (X\A) is gbI^+ closed .Let G be an open set containing (X\A) then F= (X\G) is a closed set such that $F \subseteq A$.

Therefore , $F \subseteq bI^+(int(A)$

ie., $(X\backslash F) \supset (X\backslash bI^+cl(X \backslash A)) = bI^+cl(X \backslash A)$

$bI^+cl(X \backslash A) \subseteq G$.

Therefore (X \A) is gbI^+ closed.ie., A is gbI^+ open . Hence the proof.

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