$(g\ \alpha)^*$- CLOSED SETS IN TOPOLOGICAL SPACES

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ABSTRACT

In this paper we introduce a new class of sets namely, $(g\alpha)^*$ - closed sets in topological spaces, which settled in between the class of closed sets and the class of $g\alpha$-closed sets. Applying these sets, we introduce five new classes of spaces namely, $T_{g\alpha}^*$ - spaces, $T_{g\alpha}$ - spaces, $^*T_{g\alpha}$ - spaces, $T_{g\alpha}^{\alpha}$ - spaces and $T_{(g\alpha)^*}^{\alpha}$ - spaces. Further we introduce $(g\alpha)^*$ - continuous maps and $(g\alpha)^*$- irresolute maps.

Key words: $(g\alpha)^*$ - closed sets, $T_{g\alpha}^*$ - spaces, $T_{g\alpha}$ - spaces, $^*T_{g\alpha}$ - spaces, $T_{g\alpha}^{\alpha}$ - spaces, $T_{(g\alpha)^*}^{\alpha}$ - spaces, $(g\alpha)^*$ - continuous maps and $(g\alpha)^*$- irresolute maps.

1. INTRODUCTION

Levine [9] introduced the class of g - closed sets in 1970. Maki.et.al [11] defined αg - closed sets in 1994. .P Arya and T. Nour [3] defined gs - closed sets in 1990. J. Dontchev [7], Y. Gnanambal [8], N. Palaniappan and K.C.Rao [17] introduced gsp - closed sets, gpr - closed sets and rg - closed sets respectively. H. Maki, J. Umehara and T. Noiri [13], H. Maki, R. Devi and K. Balachandran [12], M.K.R.S. Veerakumar [19] introduced gp - closed sets, $g\alpha$ - closed sets and g^* - closed sets respectively.

Devi.et.al [4] introduced $_{\alpha}T_b$- spaces, Devi.et.al [5] introduced T_b- spaces. Applying $(g\alpha)^*$ - closed sets, we introduce five new class of spaces namely $T_{g\alpha}^*$ - spaces, $T_{g\alpha}$ - spaces, $^*T_{g\alpha}$ - spaces, $T_{g\alpha}^{\alpha}$ - spaces, $T_{(g\alpha)^*}^{\alpha}$ - spaces. Further we introduce $(g\alpha)^*$ - continuous maps and $(g\alpha)^*$- irresolute maps.

2. PRELIMINARIES

Throughout this paper (X,τ), (Y,σ) and (Z,η) represent non - empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X,τ), cl(A) and int(A) denote the closure and the interior of A respectively.

Definition 2.1 A subset A of a topological space X is said to be,

1) a pre closed set [14] if cl [int(A)] $\subseteq$ A and a pre open set if A $\subseteq$ (int [cl(A)])
2) a semi closed set [10] if (int [cl(A)]) $\subseteq$ A and a semi open set if A $\subseteq$ cl [int(A)]
3) a g- closed set [9] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.
4) a gs- closed set [3] if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.
5) a gp- closed set [13] if pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.
6) a regular closed set [10] if A = (cl [int (A)]) and a regular open set if A = (int [cl(A)])
7) a α- closed set [15] if cl [int[cl(A)]] $\subseteq$ A and a α-open set[16] if A $\subseteq$ (int[cl(int(A))])
8) a semi pre closed set [1] if (int[cl(int(A))]) $\subseteq$ A and a semi pre open set if A $\subseteq$ (cl[int[cl(A)]])
9) a gα - closed set [16] if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is α- open in X.
10) a αg- closed set [11] if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.
11) a rg- closed set [17] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is regular open in X.
12) a gpr- closed set [8] if pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.
13) a g^*- closed set [19] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is g - open in X.
14) a gsp- closed set [7] if spcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.
15) a g^{**}- closed set [18] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is g^*- open in X.

Definition 2.2 A space (X, τ) is called

1) an ${}_{\alpha}T_b$ - space [4] if every αg - closed set in it is closed.
2) a T_b - space [5] if every gs - closed set in it is closed.

Definition 2.3 A function f **:** $(X,\tau) \rightarrow (Y,\sigma)$ is called,

1) a gsp - continuous [7] if $f^{-1}(V)$ is gsp - Closed in (X, τ) for every closed set V of (Y, σ).
2) a gp - continuous [2] if $f^{-1}(V)$ is a gp - Closed set of (X, τ) for every closed set V of

(Y,σ).

3) a gs - continuous [6] if $f^{-1}(V)$ is a gs - Closed set of (X, τ) for every closed set V of (Y, σ).
4) an gα - continuous [12] if $f^{-1}(V)$ is a gα - Closed set of (X, τ) for every closed set V of (Y, σ).
5) an αg - continuous [8] if $f^{-1}(V)$ is a αg - Closed set of (X, τ) for every closed set V of (Y, σ).

3. BASIC PROPERTIES OF $(g\alpha)^*$ - CLOSED SETS

We introduce the following definition

Definition 3.1 A subset A of a topological space (X,τ) is called $(g\alpha)^*$- closed if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is gα - open in X.

The class of $(g\alpha)^*$ - closed subsets of (X,τ) is denoted by $G_\alpha^*\, C$(X,τ)

Proposition 3.2 Every closed set is $(g\alpha)^*$ - Closed.

Proof: Let A be closed. Then Cl(A) = A Let $A \subseteq U$ and U be gα - open. $\alpha Cl(A) \subseteq Cl(A) = A \subseteq U$. $\therefore$ A is $(g\ \alpha)^*$ - closed.

A $(g\ \alpha)^*$- closed set need not be closed.

Example3.3 Let X = {a,b,c}, τ= { ϕ, X, {a}} Let A = {b} is $(g\alpha)^*$ - closed but not closed. So the class of (gα)*- closed sets properly contains the class of closed sets.

Proposition 3.4 Every $(g\alpha)^*$ - closed set is gα - closed. Converse is not true.

Example 3.5 Let X = {a,b,c} , τ = { ϕ,X,{a},{b,c}} A= {b} is gα - closed but not $(g\alpha)^*$-closed. $\therefore$The class of $(g\alpha)^*$- closed sets is properly contained in the class of gα-closed sets.

Proposition 3.6 Every α - closed set is $(g\alpha)^*$- closed.

Proof: Let A be α - closed. Then αCl(A) = A. Whenever $A \subseteq U$ and U is gα- open, $\alpha Cl(A) \subseteq U$. $\therefore$ A is $(g\alpha)^*$- closed.

The converse of the above proposition need not be true in general as it can be seen from the following example.

Example 3.7 Let X = {a, b, c}, τ = {ϕ, X, {a,b}} $\therefore$ A = {a,c} is $(g\alpha)^*$- closed but not α-closed.

Proposition 3.8 Every $(g\alpha)^*$ closed set is gs- closed.

Proof follows from the definitions.

Example 3.9 Let X= {a,b,c}, τ = {ϕ, X,{a},{a,c}} A = {a,b} is gs - closed but not $(g\alpha)^*$ - closed. $\therefore$The class of $(g\alpha)^*$- closed sets is properly contained in the class of gs - closed sets.

Proposition 3.10 Every $(g\alpha)^*$ closed set is αg – closed.

Proof: Let A be $(g\alpha)^*$ - closed. Then $\alpha cl(A) \subseteq U$, whenever $A \subseteq U$ and U is gα - open.

Let us prove that $\alpha cl(A) \subseteq U$, when ever $A \subseteq U$ and U is open. Let $A \subseteq U$ and U be open.

Then $A \subseteq U$ and U is gα-open. $\alpha Cl(A) \subseteq U$, since A is $(g\alpha)^*$ - closed. $\therefore$A is αg- closed.

Example 3.11 In example 3.9, A = {a,b} is αg - closed but not $(g\alpha)^*$ - closed.

$\therefore$The class of αg - closed sets properly contains the class of $(g\alpha)^*$ - closed sets.

Proposition 3.12 Every $(g\alpha)^*$closed set is gsp -closed.

The following example shows that the converse of the above proposition need not be true in general.

Example 3.13 In example 3.9, A = {a,b} is gsp - closed but not $(g\alpha)^*$ - closed.

Proposition 3.14 Every $(g\alpha)^*$closed set is gp –closed.

Proof: Let A be $(g\alpha)^*$closed. Then $\alpha Cl(A) \subseteq U$, whenever $A \subseteq U$ and U is gα - open.

Let us prove that A is gp - closed. Let us prove that $pcl(A) \subseteq U$, whenever $A \subseteq U$ and U is open. Let $A \subseteq U$ and U be open. Then $A \subseteq U$ and U is gα - open. Then $\alpha cl(A) \subseteq U$. $pcl(A) \subseteq \alpha cl(A) \subseteq U$. $\therefore pcl(A) \subseteq U$, whenever $A \subseteq U$ and U is open.$\therefore$A is gp- closed.

Example 3.15 Let X= {a,b,c}, τ = {ϕ,X,{a},{a,c}} A = {a,b} is gp-closed but not $(g\alpha)^*$-closed.

$\therefore$ The class of $(g\alpha)^*$- closed sets is properly contained in the class of gp - closed sets.

Example 3.16 A gpr - closed set need not be $(g\alpha)^*$ - closed. In example 3.9, A = {a,b} is gpr - closed but not $(g\alpha)^*$- closed.

Example 3.17 A rg - closed set need not be $(g\alpha)^*$ - closed.

In example 3.9, rg - closed sets are all the subsets of X. A={a,b} is rg -closed but not $(g\alpha)^*$- closed. $\therefore$ A is rg - closed but $(g\alpha)^*$- closed.

Remark 3.18 $(g\alpha)^*$ - closedness is independent of g - closedness.

In example 3.9, A = {a,b} is g -closed but not $(g\alpha)^*$- closed. A = {c} is $(g\alpha)^*$- closed but not g - closed. $\therefore (g\alpha)^*$ - closedness is independent of g - closedness.

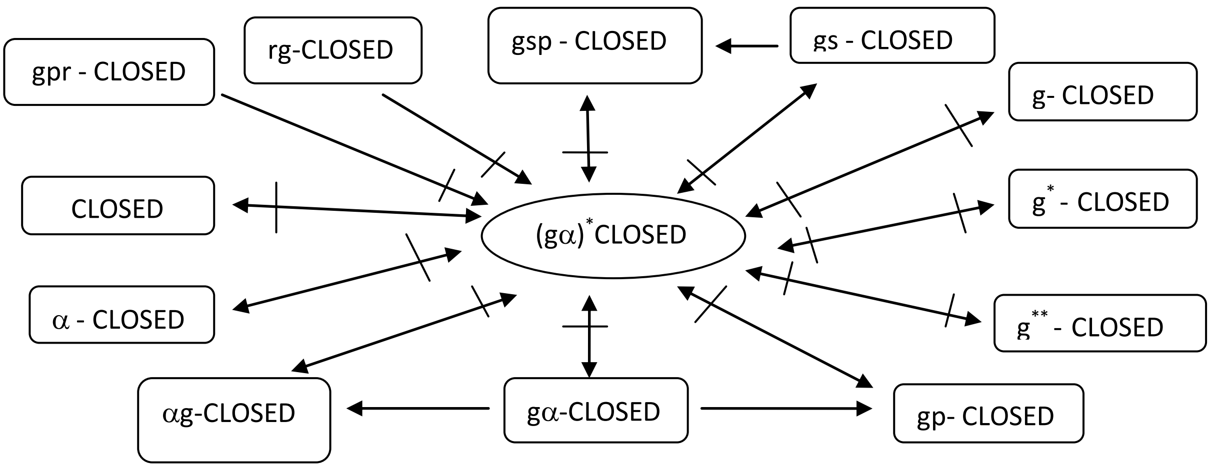
Remark 3.19 $(g\alpha)^*$ - closedness is independent of g^*- closedness.

In example 3.9, A = {a,b} is g^* - closed but not $(g\alpha)^*$- closed and A = {c} is $(g\alpha)^*$- closed but not g^* - closed. $\therefore (g\alpha)^*$ - closedness is independent of g^* -closedness.

Remark 3.20 $(g\alpha)^*$ - closedness is independent of g^{**}- closedness.

In example 3.9, A = {a,b} is g^{**} - closed but not $(g\alpha)^*$- closed and B= {c} is $(g\alpha)^*$- closed but not g^{**} - closed. $\therefore (g\alpha)^*$ - closedness is independent of g^{**} -closedness.

The above results can be represented in the following figure.



4. APPLICATION OF (gα)* -CLOSED SETS

As application of $(g\alpha)^*$- closed sets, five new spaces namely $T^*_{g\alpha}$-spaces, $T_{g\alpha}$ -spaces, ${}^*T_{g\alpha}$ -spaces, $T^{\alpha}_{g\alpha}$ - spaces, $T^{\alpha}_{(g\alpha)^*}$ -spaces are introduced.

Definition 4.1 A space (X,τ) is called,

1. a $T^*_{g\alpha}$ - space if every $(g\alpha)^*$- closed set is closed.
2. a $T_{g\alpha}$ - space if every $g\alpha$ - closed set is closed.
3. a ${}^*T_{g\alpha}$ - space if every $g\alpha$- closed set is $(g\alpha)^*$- closed.
4. a $T^{\alpha}_{g\alpha}$ - space if every $g\alpha$-closed set is α- closed.
5. a $T^{\alpha}_{(g\alpha)^*}$ - space if every $(g\alpha)^*$- closed set is α- closed.

Proposition 4.2 Every $T_{g\alpha}$ - space is a $T^*_{g\alpha}$ - space.

Proof: Let (X,τ) be a $T_{g\alpha}$ - space. Let A be a $(g\alpha)^*$-closed set. Then A is $g\alpha$ - closed. $\therefore$ A is closed, since the space is $T_{g\alpha}$ - space. $\therefore$ Every $(g\alpha)^*$- closed set is closed. $\therefore (X,\tau)$ is a $T^*_{g\alpha}$ - space.

Converse is not true.

Example 4.3 In example 3.5, A = {b} is $g\alpha$ - closed but not closed. $\therefore (X,\tau)$ is not a $T_{g\alpha}$ - space. $(g\alpha)^*$- closed sets are ϕ, X, {a},{b,c} and all these sets are closed. $\therefore (X,\tau)$ is a $T^*_{g\alpha}$ - space. $\therefore$ A $T^*_{g\alpha}$ - space need not be a $T_{g\alpha}$ - space.

Proposition 4.4 A space (X,τ) which is both $T^*_{g\alpha}$ and ${}^*T_{g\alpha}$ is a $T_{g\alpha}$ - space.

Example 4.5 In example 3.9, A = {c} is $g\alpha$ - closed but not closed. $\therefore (X,\tau)$ is not a $T_{g\alpha}$ - space. B = {c} is $(g\alpha)^*$- closed but not closed. $\therefore (X,\tau)$ is not a $T^*_{g\alpha}$ - space.

Every $g\alpha$ - closed set is $(g\alpha)^*$- closed and hence (X,τ) is a ${}^*T_{g\alpha}$- space. $\therefore$ A ${}^*T_{g\alpha}$ - space need not be either $T_{g\alpha}$ - space or $T^*_{g\alpha}$ - space.

Proposition 4.6 Every $T^{\alpha}_{g\alpha}$ - space is a $T^{\alpha}_{(g\alpha)^*}$ - space.

Proof: Let A be a $(g\alpha)^*$ - closed set. $\therefore$ A is $g\alpha$ - closed. Then A is α - closed, since (X,τ) is a $T^{\alpha}_{g\alpha}$ - space. $\therefore$ Every $(g\alpha)^*$ - closed set is α - closed. $\therefore$ (X,τ) is a $T^{\alpha}_{(g\alpha)^*}$ - space.

Converse of the above proposition is not true.

Example 4.7 Let X = {a,b,c} , τ = { ϕ,X,{a},{b,c}}. $(g\alpha)^*$- closed sets are ϕ, X, {a},{b,c} and all these sets are α - closed. $\therefore$(X,τ) is a $T^{\alpha}_{(g\alpha)^*}$ - space. A = {b} is gα - closed but not α - closed. $\therefore$(X,τ) is not a $T^{\alpha}_{g\alpha}$ - space. Hence a $T^{\alpha}_{(g\alpha)^*}$ - space need not be a $T^{\alpha}_{g\alpha}$ - space.

Proposition 4.8 Every $T^{*}_{g\alpha}$- space is a $T^{\alpha}_{(g\alpha)^*}$- space. Converse is not true.

Example 4.9 In example 3.9, A = {c} is $(g\alpha)^*$ - closed but not closed. $\therefore$(X,τ) is not a $T^{*}_{g\alpha}$ - space. Every $(g\alpha)^*$ - closed set in it is closed. $\therefore$ It is a $T^{\alpha}_{(g\alpha)^*}$ -space.

$\therefore$ A $T^{\alpha}_{(g\alpha)^*}$ - space need not be a $T^{*}_{g\alpha}$ - space.

Proposition 4.10 Every $T_{g\alpha}$ - space is a $T^{\alpha}_{g\alpha}$ - space.

Proof: Let A be a gα - closed set. A is closed since (X,τ) is a $T_{g\alpha}$ - space. Then A is α-closed. Hence every gα - closed set is α- closed. $\therefore$ The space is a $T^{\alpha}_{g\alpha}$ - space. Converse is not true.

Example 4.11 In example 3.9, A = {c} is gα - closed but not closed. $\therefore$(X,τ) is not a $T_{g\alpha}$ - space. Every gα - closed set in this space is α - closed and hence it is a $T^{\alpha}_{g\alpha}$ - space. $\therefore$A $T^{\alpha}_{g\alpha}$ - space need not be a $T_{g\alpha}$- space.

Proposition 4.12 Every ${}_{\alpha}T_b$ - space is a $T^{*}_{g\alpha}$ - space. Converse is not true.

Example 4.13 In example 3.9, A = {a,b} is αg - closed but not closed. $\therefore$ The space (X,τ) is not a ${}_{\alpha}T_b$ - space. Every gα - closed set in this space is $(g\alpha)^*$ - closed. $\therefore$ (X,τ) is a $T^{*}_{g\alpha}$ - space. $\therefore$A $T^{*}_{g\alpha}$ - space need not be a ${}_{\alpha}T_b$ - space.

Proposition 4.14 Every ${}_{\alpha}T_b$ - space is a $T_{g\alpha}$ - space.

Proof: Let A be a gα - closed set. Then A is αg- closed. $\therefore$A is closed since it is a ${}_{\alpha}T_b$ - space. Every gα - closed set is closed. $\therefore$(X,τ) is a $T_{g\alpha}$ - space. $\therefore$Every ${}_{\alpha}T_b$ - space is a $T_{g\alpha}$ - space.

Proposition 4.15 Every ${}_{\alpha}T_b$ - space is a $T^{\alpha}_{(g\alpha)^*}$ - space.

Example 4.16 Every $T^{\alpha}_{(g\alpha)^*}$ - space need not be a $_{\alpha}T_b$ - space. In example 3.11, A = {a,b} is αg - closed but not closed. ∴The space is not a $_{\alpha}T_b$ - space, but every $(g\alpha)^*$- closed set in (X,τ) is α - closed. ∴It is a $T^{\alpha}_{(g\alpha)^*}$ - space but not a $_{\alpha}T_b$ - space.

Proposition 4.17 Every $_{\alpha}T_b$ - space is a $T^{\alpha}_{g\alpha}$ - space.

Example 4.18 In example 3.9 and 3.11, A = {a,b} is αg - closed but not closed. ∴The space is not a $_{\alpha}T_b$ - space. Every gα - closed set is α - closed. ∴The space is a $T^{\alpha}_{g\alpha}$ - space. ∴ Every $T^{\alpha}_{g\alpha}$ - space need not be a $_{\alpha}T_b$ - space.

Proposition 4.19 Every T_b - space is a $T^{*}_{g\alpha}$ - space.

Proof: Let A be a $(g\alpha)^*$ - closed set. A is gs-closed since it is $(g\alpha)^*$ - closed. A is closed since the space is a T_b - space. ∴Every $(g\alpha)^*$ - closed set is closed. ∴ The space is a $T^{*}_{g\alpha}$ - space.

Example 4.20 In example 3.5, every $(g\alpha)^*$ - closed set is closed. ∴(X,τ) is a $T^{*}_{g\alpha}$ - space. A = {b} is gs - closed but not a closed set. ∴(X,τ) is not a T_b-space. ∴A $T^{*}_{g\alpha}$ - space need not be a T_b - space.

Proposition 4.21 Every T_b - space is a $T_{g\alpha}$ - space. Converse is not true.

Example 4.22 Let X = {a,b,c}, τ = {ϕ, X, {a}, {b}, {a,b}}, A = {b} is gs - closed but not closed. ∴The space is not a T_b - space. Every gα - closed set is closed in this space. ∴ The space is a $T_{g\alpha}$ - space. ∴ Every $T_{g\alpha}$ - space need not be a T_b - space.

Proposition 4.23 Every T_b - space is a $T^{\alpha}_{g\alpha}$ - space.

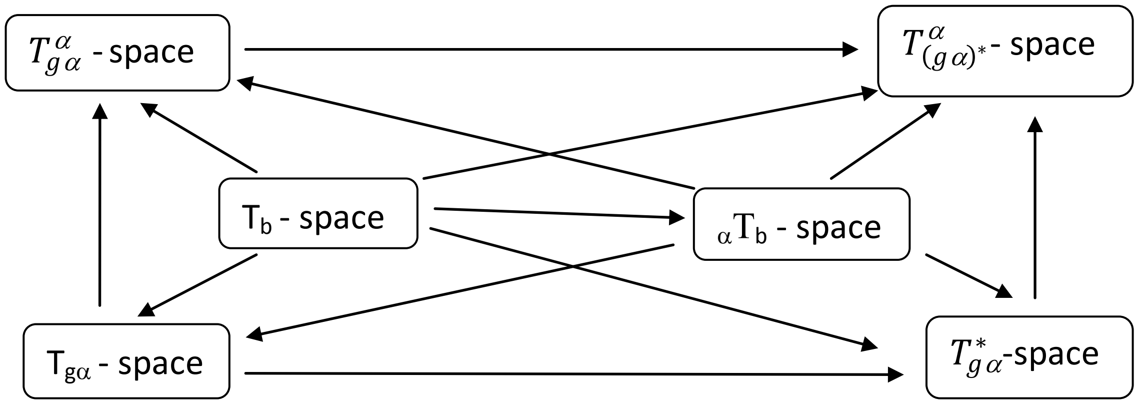
Example 4.24 Every $T^{\alpha}_{g\alpha}$ - space need not be a T_b - space. In example 3.9, A = {c} is gs - closed but not closed. ∴The space is not a T_b - space. But every gα - closed set in this space is α - closed and hence it is a $T^{\alpha}_{g\alpha}$ - space. ∴ A $T^{\alpha}_{g\alpha}$ - space need not be a T_b – space**.**

Proposition 4.25 Every T_b - space is a $T^{\alpha}_{(g\alpha)^*}$- space.

Proof: Let A be a $(g\alpha)^*$ - closed set. Then A is gs- closed. A is closed since it is a T_b - space. Then A is α - closed. ∴ Every $(g\alpha)^*$ closed set is α - closed. Hence (X,τ) is a $T^{\alpha}_{(g\alpha)^*}$ -space. ∴Every T_b - space is a $T^{\alpha}_{(g\alpha)^*}$- space.

Example 4.26 In example 3.9, A = {a,b} is gs - closed but not closed. ∴The space is not a T_b - space. But every $(g\alpha)^*$ - closed set in it is α - closed and hence it is a $T^{\alpha}_{(g\alpha)^*}$ - space. ∴ Every $T^{\alpha}_{(g\alpha)^*}$ - space need not be a T_b -space.

The following diagram summarizes the above discussion.



5. (gα)* - CONTINUOUS AND (gα)* - IRRESOLUTE MAPS

We introduce the following definitions.

Definition 5.1 A function f : (X,τ) → (Y,σ) is called,

1. a $(g\alpha)^*$- continuous map if f^{-1}(V) is a $(g\alpha)^*$- closed set of (X,τ) for every closed set V of (Y,σ).
2. a $(g\alpha)^*$- irresolute if f^{-1}(V) is a $(g\alpha)^*$- closed set of (X,τ) for every $(g\alpha)^*$- closed set of (Y,σ).

Theorem 5.2 Every continuous map is $(g\alpha)^*$- continuous.

Proof: Let V be a closed subset of (Y,σ). Then f^{-1}(V) is closed, since f is continuous. Now f^{-1}(V) is a $(g\alpha)^*$- closed, since every closed set is $(g\alpha)^*$- closed. ∴ f is $(g\alpha)^*$- continuous.

Converse of the above theorem is not true.

Example 5.3 Let X = Y = {a, b, c}, τ = { ϕ, X, {a}, {a,c} }, and σ = { ϕ, X, {a,b}}.

ϕ, Y, and {c} are the closed subsets of Y. Define f : (X,τ) → (Y,σ) by, f (a) = b, f(b) = a, and f (c) = c, f^{-1}({c}) = {c} is $(g\alpha)^*$- Closed set in (X,τ), but it is not closed in (X,τ). ∴f is $(g\alpha)^*$- continuous, but it is not continuous. ∴A $(g\alpha)^*$- continuous map need not be continuous.

Theorem 5.4 Every $(g\alpha)^*$- continuous map is gsp – continuous, gp – continuous, gs – continuous, $g\alpha$ - continuous and αg - continuous.

Proof: Let f : $(X,\tau) \rightarrow (Y,\sigma)$ be a $(g\alpha)^*$- continuous map. Let V be a closed set of (Y,σ).

Then $f^{-1}(V)$ is a $(g\alpha)^*$- Closed since f is $(g\alpha)^*$- continuous. By proposition 3.13 $f^{-1}(V)$ is a gsp- Closed and hence f is gsp - continuous. By proposition 3.15 $f^{-1}(V)$ is a gp- Closed and hence f is gp - continuous. By proposition 3.4 $f^{-1}(V)$ is a $g\alpha$- Closed and hence f is $g\alpha$ - continuous. By proposition 3.8 $f^{-1}(V)$ is a gs- Closed and hence f is gs - continuous. By proposition 3.10 $f^{-1}(V)$ is a αg- Closed and hence f is αg - continuous.

Converse of the above theorem is not true.

Example 5.5 X = Y = {a,b,c} $\tau = \{\phi, X,\{a\},\{a,c\}\}$, $\sigma = \{\phi, Y,\{a\},\{b\},\{a,b\}\}$. Define f : $(X,\tau) \rightarrow (Y,\sigma)$ by, f (a) = a, f (b) = c, f (c) = b, $f^{-1}(\{b,c\}) = \{b,c\}$, $f^{-1}(\{b\}) = \{c\}$ and $f^{-1}(\{a,c\}) = \{a,b\}$ are gsp - closed in (X,τ). $\therefore$ $f^{-1}(V)$ are gsp - closed in (X,τ) for all closed sets V in Y. $\therefore$ f is gsp - continuous, but since $f^{-1}(\{a,c\}) = \{a,b\}$ is not closed in (X,τ), f is not continuous. $\therefore$ A gsp - continuous map need not be continuous.

Example 5.6 X = Y = {a,b,c} $\tau = \{\phi, X, \{c\},\{b,c\}\}$, $\sigma = \{\phi,Y,\{a\},\{a,b\}\}$ Define f : $(X,\tau) \rightarrow (Y,\sigma)$ by f (a) = a, f (b) = a, f (c) = b. ϕ, Y, {b,c} and {c} are the closed subsets of Y. $f^{-1}(\{b,c\}) = \{a,b\}$ and $f^{-1}(\{c\}) = \{b\}$ are $(g\alpha)^*$- closed in (X,τ). $\therefore$ f is $(g\alpha)^*$- continuous. $f^{-1}(\{c\}) = \{b\}$ is not closed in X. $\therefore$ f is not continuous. $\therefore$ A $(g\alpha)^*$ - continuous map need not be continuous.

Example 5.7 X = Y = {a,b,c} $\tau = \{\phi, X,\{c\},\{b,c\}\}$, $\sigma = \{\phi, Y,\{a\},\{a,b\}\}$ Define f : $(X,\tau) \rightarrow (Y,\sigma)$ by f (a) = c, f (b) = a, f (c) = b, $f^{-1}(\{b,c\}) = \{a,c\}$ is not $(g\alpha)^*$- closed in (X,τ). $\therefore$ f is not $(g\alpha)^*$- continuous. $f^{-1}(\{b,c\}) = \{a,c\}$ is gp - closed in (X,τ). $f^{-1}(\{c\}) =\{b\}$ is gp - closed in (X,τ). $\therefore$ f is gp - continuous.

$\therefore$ A gp - continuous map need not be $(g\alpha)^*$- continuous.

Example 5.8 X = Y = {a,b,c} $\tau = \{\phi, X,\{a\},\{b,c\}\}$, $\sigma = \{\phi, Y, \{a,b\}, \{c\}\}$ Define f : $(X,\tau) \rightarrow (Y,\sigma)$ by f(a)= b, f(b)=c, f(c)=a, $f^{-1}(\{a,b\}) = \{a,c\}$ and $f^{-1}(\{c\}) = \{b\}$ are $g\alpha$- closed in (X,τ). $\therefore$ f is $g\alpha$ - continuous. $f^{-1}(\{a,b\}) = \{a,c\}$ is not $(g\alpha)^*$- closed. $\therefore$ f is not $(g\alpha)^*$- continuous. $\therefore$ A $g\alpha$ - continuous map need not be $(g\alpha)^*$- continuous.

Example 5.9 X = Y = {a,b,c} τ = {ϕ, X,{a},{a,c}}, σ = {ϕ, Y,{a,b}} Define f : (X,τ) $\rightarrow$ (Y,σ) by f (a) = c, f (b) = c, f (c) = a In example 3.9, f^{-1}({c}) = {a,b} is gs - closed in (X,τ) $\therefore$ f is gs - continuous. f^{-1}({c}) = {a,b} is not $(g\alpha)^*$- closed. $\therefore$ f is not $(g\alpha)^*$- continuous.

$\therefore$ A gs - continuous map need not be $(g\alpha)^*$- continuous.

Theorem 5.10 Every $(g\alpha)^*$- irresolute function is $(g\alpha)^*$- continuous.

Proof: Let f : (X,τ) $\rightarrow$ (Y,σ) be a $(g\alpha)^*$- irresolute function. We have to prove that f is $(g\alpha)^*$- continuous. Let V be a closed set in Y. Then V is $(g\alpha)^*$- Closed in Y. Then f^{-1}(V) is $(g\alpha)^*$- closed in X, since f is $(g\alpha)^*$- irresolute. $\therefore$ f is $(g\alpha)^*$- continuous. Converse is not true.

Example 5.11 Let X = Y = {a,b,c}, τ = {ϕ, X,{a},{b},{a,b}{a,c}}, σ = {ϕ, Y, {a,b}} Define h : (X,τ) $\rightarrow$ (Y,σ) by h(a) = b, h(b) = c, h(c) = a. {b,c} is $(g\alpha)^*$- closed in Y but h^{-1}({b,c}) = ({a,b}) is not $(g\alpha)^*$- closed in X. $\therefore$ h is not $(g\alpha)^*$ - irresolute.

ϕ, Y, {c} are the closed subsets of Y. h^{-1}({c}) = ({b}) is $(g\alpha)^*$- closed in X. $\therefore$f is $(g\alpha)^*$- continuous. $\therefore$ A $(g\alpha)^*$- continuous map need not be $(g\alpha)^*$- irresolute function.

Theorem 5.12 Every continuous function f : (X,τ) $\rightarrow$ (Y,σ) is $(g\alpha)^*$- irresolute, if the space (Y,σ) is a $T^*_{g\alpha}$ - space.

Proof: Let f : (X,τ) $\rightarrow$ (Y,σ) be continuous. Let V be a $(g\alpha)^*$- closed set in Y. Since Y is a $T^*_{g\alpha}$ - space, V is closed. Since f is continuous, f^{-1}(V) is closed in X. Since every closed set is $(g\alpha)^*$- closed, f^{-1}(V) is $(g\alpha)^*$- closed. $\therefore$ f is $(g\alpha)^*$- irresolute.

Theorem 5.13 Let f :(X,τ) $\rightarrow$ (Y,σ) and g : (Y,σ) $\rightarrow$ (Z,η) be any two functions. Then,

1) g$\circ$f is $(g\alpha)^*$- continuous if g is continuous and f is $(g\alpha)^*$- continuous.
2) g$\circ$f is $(g\alpha)^*$- irresolute if both f and g are $(g\alpha)^*$- irresolute.
3) g$\circ$f is $(g\alpha)^*$- continuous if g is $(g\alpha)^*$- continuous and f is $(g\alpha)^*$- irresolute.

Proof: 1) Let V be a closed set in Z. Since g is continuous g^{-1}(V) is closed in Y.

$f^{-1}(g^{-1}(V)) = (g\circ f)^{-1}(V)$ is $(g\alpha)^*$- closed in X, since f is $(g\alpha)^*$- continuous.$\therefore$ g$\circ$f is $(g\alpha)^*$- continuous.

2) Let V be a $(g\alpha)^*$- closed set in Z. Then $g^{-1}(V)$ is $(g\alpha)^*$- closed set in Y, since g is $(g\alpha)^*$- irresolute. $f^{-1}(g^{-1}(V)) = (g\circ f)^{-1}(V)$ is $(g\alpha)^*$- closed, since f is $(g\alpha)^*$- irresolute. $\therefore$ $g\circ f$ is $(g\alpha)^*$- irresolute.

3) Let V be a closed set in Z. Then $g^{-1}(V)$ is $(g\alpha)^*$- closed set in Y. $\therefore$ $f^{-1}(g^{-1}(V)) = (g\circ f)^{-1}(V)$ is $(g\alpha)^*$- closed in X, since f is $(g\alpha)^*$- irresolute. $\therefore$ $g\circ f$ is $(g\alpha)^*$- continuous.

Theorem 5.14 Let $f : (X,\tau) \to (Y,\sigma)$ be a $(g\alpha)^*$- continuous map. If (X,τ) is a $T^*_{g\alpha}$ - space then f is continuous.

Proof: Let V be a closed set in Y. Then $f^{-1}(V)$ is $(g\alpha)^*$- closed in X, since f is $(g\alpha)^*$- continuous map. Since the space is $T^*_{g\alpha}$ - space, $f^{-1}(V)$ is closed. $\therefore$ f is continuous

Theorem 5.15 Let $f : (X,\tau) \to (Y,\sigma)$ be a $(g\alpha)^*$- continuous map. If (X,τ) is a $T^{\alpha}_{(g\alpha)^*}$- space then f is α - continuous.

Proof: Let V be a closed set in Y. We have to prove that $f^{-1}(V)$ is α- closed in X. Now $f^{-1}(V)$ is $(g\alpha)^*$- closed, since f is $(g\alpha)^*$- continuous map. Since the space is $T^{\alpha}_{(g\alpha)^*}$ - space, every $(g\alpha)^*$- closed set is α- closed. $\therefore$ $f^{-1}(V)$ is α - closed and hence f is α - continuous.

Theorem 5.16 Let $f : (X,\tau) \to (Y,\sigma)$ be a $g\alpha$- continuous map. If (X,τ) is $T^{\alpha}_{g\alpha}$- space, then f is α -continuous.

Proof: Let V be a closed set in Y. Then $f^{-1}(V)$ is $g\alpha$- closed in X. Since the space is $T^{\alpha}_{g\alpha}$ - space, every $g\alpha$ - closed set is α - closed. $\therefore$ $f^{-1}(V)$ is α - closed and hence f is α - continuous.

Theorem 5.17 Let $f : (X,\tau) \to (Y,\sigma)$ be a $g\alpha$- continuous map. If (X,τ) is ${}^*T_{g\alpha}$ - space, then f is $(g\alpha)^*$ -continuous.

Proof: Let V be a closed set in Y. Then $f^{-1}(V)$ is $g\alpha$- closed in X, since f is $g\alpha$ - continuous. Since X is ${}^*T_{g\alpha}$ - space, every $g\alpha$ - closed set is $(g\alpha)^*$ - closed set. $\therefore$ $f^{-1}(V)$ is $(g\alpha)^*$- closed and hence f is $(g\alpha)^*$ - continuous.

Theorem 5.18 Let $f : (X,\tau) \to (Y,\sigma)$ be a $g\alpha$- continuous map. If (X,τ) is a $T_{g\alpha}$– space, then f is $(g\alpha)^*$ -continuous.

Proof: Let V be a closed set in Y. $\therefore f^{-1}$(V) is gα - closed in X. Since X is $T_{g\alpha}$ - space, f^{-1}(V) is closed. $\therefore f^{-1}$(V) is $(g\alpha)^*$ - closed. Hence f is $(g\alpha)^*$ - continuous.

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