

“Strongly g , g^* , g^{**} Closed Sets In Bitopological Space”

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Abstract

In this paper we introduce strongly g , g^* , g^{**} closed sets in bitopological spaces. Properties of these sets are investigated and we introduce six new spaces namely (i, j) - $T_{\frac{1}{2}s}$ space, (i, j) - $T_{\frac{1}{2}s}^*$ space, (i, j) - $T_{\frac{1}{2}s}^{**}$ space, (i, j) - $^*T_{\frac{1}{2}s}$ space, (i, j) - $^{**}T_{\frac{1}{2}s}$ space, (i, j) - $^*T_{\frac{1}{2}s}^*$ space.

Key words: (i, j) - strongly g closed sets, (i, j) - strongly g^* closed sets and (i, j) - strongly g^{**} closed sets.

1.INTRODUCTION

A triple (X, τ_1, τ_2) where X is a non-empty set and τ_1, τ_2 are topologies in X is called a bitopological space and Kelley[5] initiated the study of such spaces. In 1985, Fukutake[2] introduced the concepts of g -closed sets in bitopological spaces. M.K.R.S. Veerakumar[11] introduced and studied the concepts of g^* -closed sets and g^* -continuity in topological spaces. Sheik John. M and sundaram. P [10] introduced and studied the concepts of g^* -closed sets in bitopological spaces in 2002. The purpose of this paper is to introduce the concept of strongly g , g^* , g^{**} closed sets in bitopological spaces. Six new spaces namely, (i, j) - $T_{\frac{1}{2}s}$ space, (i, j) - $T_{\frac{1}{2}s}^*$ space, (i, j) - $T_{\frac{1}{2}s}^{**}$ space, (i, j) - $^*T_{\frac{1}{2}s}$ space, (i, j) - $^{**}T_{\frac{1}{2}s}$ & (i, j) - $^*T_{\frac{1}{2}s}^*$ space in bitopological spaces are introduced and some of their properties are investigated.

2.PRELIMINARIES

Throughout this paper (X, τ) represents non-empty topological space on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ) , $cl(A)$ and $int(A)$ denote the closure and the interior of A respectively.

Definition2.1: A subset A of a topological space (X, τ) is said to be

1. a semi open set [6] if $A \subseteq cl(int(A))$ and semi closed set if $int(cl(A)) \subseteq A$.
2. a regular open set [6] if $A = int(cl(A))$.
3. a generalized closed set (briefly g -closed) [7] if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is open in (X, τ) .
4. a generalized star closed set (briefly g^* -closed set) [12] if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is g open in (X, τ) .
5. a generalized star star closed set (briefly g^{**} -closed) [8] if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is g^* open in (X, τ) .

6. a strongly generalized closed set (briefly sg-closed) [9] if $cl(int(A)) \subseteq U$, whenever $A \subseteq U$ and U is open in (X, τ) .
7. a strongly generalized star closed set (briefly sg*-closed) [13] if $cl(int(A)) \subseteq U$, whenever $A \subseteq U$ and U is g open in (X, τ) .
8. a strongly generalized star star closed set (briefly sg**-closed) [9] if $cl(int(A)) \subseteq U$, whenever $A \subseteq U$ and U is g open in (X, τ) .

Definition 2.2: A subset A of a bitopological space (X, τ_1, τ_2) is called

1. an (i, j) -g closed [2] if $\tau_j - cl(A) \subseteq U$, whenever $A \subseteq U$ and U is open in τ_i .
2. an (i, j) -g* closed [10] if $\tau_j - cl(A) \subseteq U$, whenever $A \subseteq U$ and U is g open in τ_i .
3. an (i, j) -g**closed [8] if $\tau_j - cl(A) \subseteq U$, whenever $A \subseteq U$ and U is g* open in τ_i .
4. an (i, j) -rg closed [1] if $\tau_j - cl(A) \subseteq U$, whenever $A \subseteq U$ and U is regular open in τ_i .

Definition 2.3: A bitopological space (X, τ_1, τ_2) is called

1. an (i, j) - $T_{1/2}$ space [2] if every (i, j) -g closed set is τ_j -closed.
2. an (i, j) - $T_{1/2}^*$ space [10] if every (i, j) -g* closed set is τ_j -closed.
3. an (i, j) - $T_{1/2}^{**}$ space [8] if every (i, j) -g** closed set is τ_j -closed.

3. (i, j) -strongly g, g*, g** closed sets

In this section we introduce the concept of (i, j) -strongly g, g*, g** closed sets in bitopological spaces.

Definition 3.1: A subset A of a bitopological space (X, τ_1, τ_2) is called (i, j) -strongly g closed if $\tau_j - cl(int(A)) \subseteq U$, whenever $A \subseteq U$ and U is open in τ_i .

Definition 3.2: A subset A of a bitopological space (X, τ_1, τ_2) is called (i, j) -strongly g* closed if $\tau_j - cl(int(A)) \subseteq U$, whenever $A \subseteq U$ and U is g open in τ_i .

Definition 3.3: A subset A of a bitopological space (X, τ_1, τ_2) is called (i, j) -strongly g** closed if $\tau_j - cl(int(A)) \subseteq U$, whenever $A \subseteq U$ and U is g* open in τ_i .

Remark 3.4:

- (i) By setting $\tau_1 = \tau_2$ in definition (4.1), a (i, j) -strongly g closed set is a strongly g closed set.
- (ii) By setting $\tau_1 = \tau_2$ in definition (4.2), a (i, j) -strongly g* closed set is a strongly g* closed set.
- (iii) By setting $\tau_1 = \tau_2$ in definition (4.3), a (i, j) -strongly g** closed set is a strongly g** closed set.

Theorem 3.5: Every τ_j -closed set is (i, j) -strongly g closed.

Proof: let A be τ_j -closed. Since A is τ_j -closed, $\tau_j - cl(A) = A$. let $A \subseteq U$ and U be τ_i -open. $\tau_j - cl(int(A)) \subseteq \tau_j - cl(A) = A \subseteq U$. Therefore A is (i, j) -strongly g closed.

The converse of the above theorem is not true as seen in the following example.

Example 3.6: Let $X = \{a, b, c\}$, $\tau_1 = \{\Phi, \{c\}, \{a, c\}, X\}$ and $\tau_2 = \{\Phi, \{a\}, X\}$. Then the set $A = \{b\}$ is (i, j) -strongly g closed but not τ_j -closed in (X, τ_1, τ_2) .

Theorem 3.7: Every τ_j -closed set is (i, j) -strongly g*closed.

proof follows from the definition.

The converse of the above theorem is not true as seen in the following example.

Example 3.8: In example (3.6), $A = \{a, b\}$ is (i, j) -strongly g* closed but not τ_j -closed.

Theorem 3.9: Every τ_j -closed set is (i, j)-strongly g^{**} closed.

The converse of the above theorem is not true as seen in the following example.

Example 3.10: in example (3.6), $A = \{b\}$ is (i, j)-strongly g^{**} closed but not τ_j -closed.

Theorem 3.11: Every (i, j)-g closed set is (i, j)-strongly g closed.

Proof: Let A be (i, j)-g closed. Let $A \subseteq U$ and U be τ_i -open. Since A is (i, j)-g closed,

$\tau_j - cl(A) \subseteq U \therefore \tau_j - cl(int(A)) \subseteq \tau_j - cl(A) \subseteq U$. Hence A is (i, j)-strongly g closed.

The converse of the above theorem is not true as seen in the following example.

Example 3.12: Let $X = \{a, b, c\}$, $\tau_1 = \{\Phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $\tau_2 = \{\Phi, \{a\}, \{a, b\}, X\}$. Then the set $A = \{b\}$ is (i, j)-strongly g closed but not (i, j)-g closed.

Theorem 3.13: Every (i, j)- g^* closed set is (i, j)-strongly g^* closed.

The converse of the above theorem is not true as seen in the following example.

Example 3.14: In example (3.6), $A = \{c\}$ is (i, j)-strongly g^* closed but not a (i, j)- g^* closed set.

Theorem 3.15: Every (i, j)- g^{**} closed set is (i, j)-strongly g^{**} closed.

proof follows from the definition.

The converse of the above theorem is not true as seen in the following example.

Example 3.16: let $X = \{a, b, c\}$, $\tau_1 = \{\Phi, \{b\}, X\}$ and $\tau_2 = \{\Phi, \{c\}, X\}$. Then $A = \{b\}$ is (i, j)-strongly g^{**} closed but not (i, j)- g^{**} closed.

Theorem 3.17: Every (i, j)-strongly g^* closed set is (i, j)-strongly g closed.

Proof: Let A be (i, j)-strongly g^* closed. Let $A \subseteq U$ and U be τ_i -open. Then $A \subseteq U$ and U is τ_i -g open. Since A is (i, j)-strongly g^* closed, $\tau_j - cl(int(A)) \subseteq U$. Therefore A is (i, j)-strongly g closed.

The converse of the above theorem is not true as seen in the following example.

Example 3.18: let $X = \{a, b, c\}$, $\tau_1 = \{\Phi, \{a\}, X\}$ and $\tau_2 = \{\Phi, \{a\}, \{a, b\}, X\}$. Then $A = \{a, c\}$ is (i, j)-strongly g closed but not (i, j)-strongly g^* closed.

Theorem 3.19: Every (i, j)-strongly g^{**} closed set is (i, j)-strongly g closed.

proof follows from the definition.

Theorem 3.20: Every (i, j)-strongly g^* closed set is (i, j)-strongly g^{**} closed.

Proof: Let A be (i, j)-strongly g^* closed set. Let $A \subseteq U$ and U is τ_i - g^* open. Then $A \subseteq U$ and U is τ_i -g open. Since A is (i, j)-strongly g^* closed, $\tau_j - cl(int(A)) \subseteq U$. Therefore A is (i, j)-strongly g^{**} closed.

The converse of the above theorem is not true as seen in the following example.

Example 3.21: Let $X = \{a, b, c\}$, $\tau_1 = \{\Phi, \{c\}, X\}$ and $\tau_2 = \{\Phi, \{a\}, X\}$. Then $A = \{a\}$ is (i, j)-strongly g^{**} closed but not (i, j)-strongly g^* closed set.

Theorem 3.22: If A is both τ_i -open and (i, j)-strongly g^* closed in (X, τ_1, τ_2) , then it is (i, j)-regular closed in (X, τ_1, τ_2) .

Proof: Let A be both τ_i -open and (i, j)-strongly g^* closed. Since A is open, $\tau_j - int(A) = A$. A is τ_i -open \Rightarrow A is τ_i -g open. (i.e.,) $A \subseteq \tau_j - cl(int(A))$, where A is τ_i -g open. Since A is (i, j)-strongly g^* closed, $\tau_j - cl(int(A)) \subseteq A$ ----- (1).

$A \subseteq \tau_j - cl(A) = \tau_j - cl(int(A)) \therefore A \subseteq \tau_j - cl(int(A))$ ----- (2). By (1) & (2)

$A = \tau_j - cl(int(A))$ and hence A is regular closed in (X, τ_1, τ_2) .

Theorem 3.23: If a subset A of a bitopological space (X, τ_1, τ_2) is both (i, j)-strongly g^* closed and τ_j -semi open, then it is (i, j)- g^* closed.

Proof : Let A be both (i, j) -strongly g^* closed and τ_j -semi open. Let $A \subseteq U$ and U be τ_i - g open. Since A is (i, j) -strongly g^* closed, $\tau_j - cl(int(A)) \subseteq U$. Since A is τ_j -semi open, $A \subseteq \tau_j - cl(int(A))$.

Then, $\tau_j - cl(A) \subseteq \tau_j - cl(cl(int(A))) = \tau_j - cl(int(A)) \subseteq U$. Therefore A is (i, j) - g^* closed.

Theorem 3.24: If A is both (i, j) -strongly g closed and τ_j -open, then A is (i, j) -rg closed.

Proof: Let A be (i, j) -strongly g closed. Let $A \subseteq U$ and U be τ_i -regular open $\Rightarrow A \subseteq U$ and U is τ_i -open. Since A is (i, j) -strongly g closed, $\tau_j - cl(int(A)) \subseteq U$. $\tau_j - cl(A) = \tau_j cl(int(A)) \subseteq U$. Therefore A is (i, j) -rg closed.

Theorem 3.25: Every (i, j) - g^* closed set is (i, j) -strongly g closed.

Proof: Let A be (i, j) - g^* closed. Let $A \subseteq U$ and U is τ_i -open. Then $A \subseteq U$ and U is τ_i - g open. Since A is (i, j) - g^* closed, $\tau_j - cl(A) \subseteq U$. $\tau_j - cl(int(A)) \subseteq \tau_j - cl(A) \subseteq U$. Therefore A is (i, j) -strongly g closed.

The converse of the above theorem is not true as seen in the following example.

Example 3.26: Let $X = \{a, b, c\}$, $\tau_1 = \{\Phi, \{a\}, X\}$ and $\tau_2 = \{\Phi, \{c\}, X\}$. Then $A = \{b\}$ is (i, j) -strongly g closed but not (i, j) - g^* closed.

Theorem 3.27: Every (i, j) - g^* closed set is (i, j) -strongly g^{**} closed.
proof follows from the definition.

The converse of the above theorem is not true as seen in the following example.

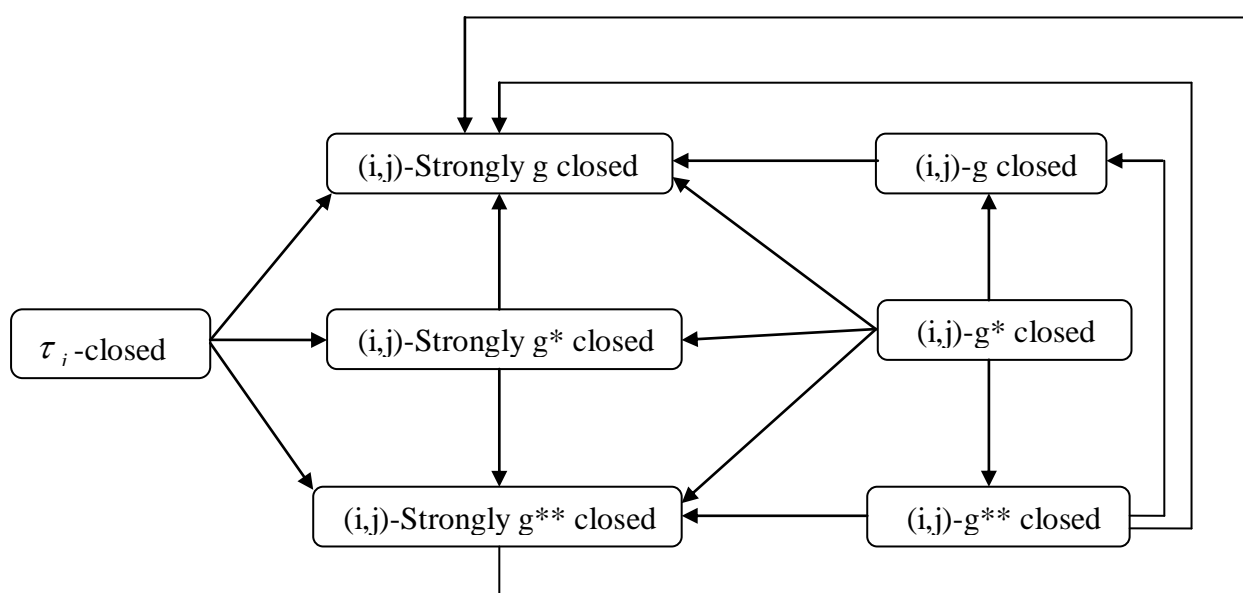
Example 3.28: let $X = \{a, b, c\}$, $\tau_1 = \{\Phi, \{b\}, X\}$ and $\tau_2 = \{\Phi, \{a\}, X\}$. Then $A = \{b\}$ is (i, j) -strongly g^{**} closed but not (i, j) - g^* closed.

Theorem 3.29: Every (i, j) - g^{**} closed set is (i, j) -strongly g closed.
proof follows from the definition.

The converse of the above theorem is not true as seen in the following example.

Example 3.30: Let $X = \{a, b, c\}$, $\tau_1 = \{\Phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $\tau_2 = \{\Phi, \{a\}, X\}$. Then $A = \{b\}$ is (i, j) -strongly g closed but not (i, j) - g^{**} closed.

The above results can be represented in the following figure.



where $A \rightarrow B$ represents A implies B but not conversely.

4. Applications of (i, j)-strongly g, g*, g** closed sets.

In this section we introduce six new spaces namely (i, j) - $T_{\frac{1}{2}s}$ space, (i, j) - $T_{\frac{1}{2}s}^*$ space, (i, j) - $T_{\frac{1}{2}s}^{**}$ space, (i, j) - $^*T_{\frac{1}{2}s}$ space, (i, j) - $^{**}T_{\frac{1}{2}s}$ & (i, j) - $^*T_{\frac{1}{2}s}^*$ space.

Definition 4.1: A bitopological space (X, τ_1, τ_2) is said to be an (i, j)- $T_{\frac{1}{2}s}$ space if every (i, j)-strongly g closed set is τ_j -closed.

Definition 4.2: A bitopological space (X, τ_1, τ_2) is said to be an (i, j)- $T_{\frac{1}{2}s}^*$ space if every (i, j)-strongly g*closed set is τ_j -closed.

Definition 4.3: A bitopological space (X, τ_1, τ_2) is said to be an (i, j)- $T_{\frac{1}{2}s}^{**}$ space if every (i, j)-strongly g**closed set is τ_j -closed.

Definition 4.4: A bitopological space (X, τ_1, τ_2) is said to be an (i, j)- $^*T_{\frac{1}{2}s}$ space if every (i, j)-strongly g closed set is (i, j)-strongly g* closed.

Definition 4.5: A bitopological space (X, τ_1, τ_2) is said to be an (i, j)- $^{**}T_{\frac{1}{2}s}$ space if every (i, j)-strongly g closed set is (i, j)-strongly g** closed.

Definition 4.6: A bitopological space (X, τ_1, τ_2) is said to be an (i, j)- $^*T_{\frac{1}{2}s}^*$ space if every (i, j)-strongly g** closed set is (i, j)-strongly g* closed.

Theorem 4.7: Every (i, j)- $T_{\frac{1}{2}s}$ space is a (i, j)- $T_{\frac{1}{2}}$ space.

Proof: Let (X, τ_1, τ_2) be a (i, j)- $T_{\frac{1}{2}s}$ space. Let A be a (i, j)-g closed set. Then A is (i, j)-strongly g closed. \therefore A is τ_j -closed since (X, τ_1, τ_2) is a (i, j)- $T_{\frac{1}{2}s}$ space. Therefore (X, τ_1, τ_2) is a (i, j)- $T_{\frac{1}{2}}$ space.

Theorem 4.8: Every (i, j)- $T_{\frac{1}{2}s}$ space is a (i, j)- $T_{\frac{1}{2}s}^*$ space.

Proof: Let (X, τ_1, τ_2) be a (i, j)- $T_{\frac{1}{2}s}$ space. Let A be a (i, j)-strongly g*closed set. Then A is (i, j)-strongly g closed & hence A is τ_j closed since (X, τ_1, τ_2) is a (i, j)- $T_{\frac{1}{2}s}$ space. Therefore (X, τ_1, τ_2) is a (i, j)- $T_{\frac{1}{2}s}^*$ space.

Theorem 4.9: Every (i, j)- $T_{\frac{1}{2}s}^{**}$ space is a (i, j)- $T_{\frac{1}{2}s}^*$ space.

proof: Let (X, τ_1, τ_2) be a (i, j)- $T_{\frac{1}{2}s}^{**}$ space. Let A be a (i, j)-strongly g*closed set. Then A is (i, j)-strongly g** closed. \therefore A is τ_j -closed since (X, τ_1, τ_2) is a (i, j)- $T_{\frac{1}{2}s}^{**}$ space. Therefore every (i, j)-strongly g*closed set is τ_j -closed & hence (X, τ_1, τ_2) is a (i, j)- $T_{\frac{1}{2}s}^*$ space.

Theorem 4.10: Every (i, j)- $T_{\frac{1}{2}s}$ space is a (i, j)- $T_{\frac{1}{2}s}^{**}$ space.

Proof: Let (X, τ_1, τ_2) be a (i, j)- $T_{\frac{1}{2}s}$ space. Let A be a (i, j)-strongly g**closed set. Then A is (i, j)-strongly g closed & hence A is τ_j -closed since (X, τ_1, τ_2) is a (i, j)- $T_{\frac{1}{2}s}$ space. \therefore Every (i, j)-strongly g**closed set is τ_j -closed. Therefore (X, τ_1, τ_2) is a (i, j)- $T_{\frac{1}{2}s}^{**}$ space.

Theorem 4.11: Every (i, j)- $T_{\frac{1}{2}s}$ space is (i, j)- $T_{\frac{1}{2}}^*$ space.

Proof: Let (X, τ_1, τ_2) be a (i, j) - $T_{\frac{1}{2}s}$ space. Let A be a (i, j) - g^* closed set. Then A is a (i, j) -strongly g closed. Then A is τ_j -closed since (X, τ_1, τ_2) is a (i, j) - $T_{\frac{1}{2}s}$ space. Therefore (X, τ_1, τ_2) is a (i, j) - $T_{\frac{1}{2}}^*$ space.

The converse of the above theorem is not true as seen in the following example.

Example 4.12: Let $x = \{a, b, c\}$, $\tau_1 = \{\Phi, \{a\}, X\}$ and $\tau_2 = \{\Phi, \{a\}, \{b, c\}, X\}$. Then $A = \{b\}$ is (i, j) -strongly g closed but not τ_j -closed. $\therefore (X, \tau_1, \tau_2)$ is not a (i, j) - $T_{\frac{1}{2}s}$ space. Here all the (i, j) - g^* closed sets are τ_j -closed. Therefore (X, τ_1, τ_2) is a $T_{\frac{1}{2}}^*$ space & hence every (i, j) - $T_{\frac{1}{2}}^*$ space need not be a (i, j) - $T_{\frac{1}{2}s}$ space.

Theorem 4.13: Every (i, j) - $T_{\frac{1}{2}s}^*$ space is a (i, j) - $T_{\frac{1}{2}}^*$ space.

The converse of the above theorem is not true as seen in the following example.

Example 4.14: In example (4.12), $A = \{b\}$ is (i, j) -strongly g^* closed but not τ_j -closed. \therefore

(X, τ_1, τ_2) is not a (i, j) - $T_{\frac{1}{2}s}^*$ space. All the (i, j) - g^* closed sets are τ_j -closed and hence (X, τ_1, τ_2) is (i, j) - $T_{\frac{1}{2}}^*$ space. Therefore (i, j) - $T_{\frac{1}{2}}^*$ space need not be a (i, j) - $T_{\frac{1}{2}s}^*$ space.

Theorem 4.15: Every (i, j) - $T_{\frac{1}{2}s}^{**}$ space is (i, j) - $T_{\frac{1}{2}}^*$ space.

proof follows from the definition.

The converse of the above is not true as seen in the example.

Example 4.16: In example (4.12), $A = \{b\}$ is (i, j) -strongly g^{**} closed but not τ_j -closed. Therefore (X, τ_1, τ_2) is not a (i, j) - $T_{\frac{1}{2}s}^{**}$ space. (i, j) - g^* closed sets are $\Phi, \{b, c\}, X$ and all these sets are τ_j -closed and hence (X, τ_1, τ_2) is (i, j) - $T_{\frac{1}{2}}^*$ space. Therefore every (i, j) - $T_{\frac{1}{2}}^*$ space need not be a (i, j) - $T_{\frac{1}{2}s}^*$ space.

Theorem 4.17: Every (i, j) - $^*T_{\frac{1}{2}s}$ space is (i, j) - $^{**}T_{\frac{1}{2}s}$ space.

Proof: Let (X, τ_1, τ_2) be (i, j) - $^*T_{\frac{1}{2}s}$ space. Since (X, τ_1, τ_2) is a (i, j) - $^*T_{\frac{1}{2}s}$ space, every (i, j) -strongly g closed set is (i, j) -strongly g^* closed. But every (i, j) -strongly g^* closed set is (i, j) -strongly g^{**} closed. Therefore every (i, j) -strongly g closed set is (i, j) -strongly g^{**} closed. $\therefore (X, \tau_1, \tau_2)$ is a (i, j) - $^{**}T_{\frac{1}{2}s}$ space.

The converse of the above theorem is not true as seen in the following example.

Example 4.18: Let $X = \{a, b, c\}$, $\tau_1 = \{\Phi, \{a\}, X\}$ and $\tau_2 = \{\Phi, \{a\}, \{a, b\}, X\}$. Then $A = \{b\}$ is (i, j) -strongly g closed but not τ_j -closed. (i.e.,) (i, j) -strongly g closed set is not a (i, j) -strongly g^* closed. Therefore (X, τ_1, τ_2) is not a (i, j) - $^*T_{\frac{1}{2}s}$ space. But every (i, j) -strongly g closed set is (i, j) -strongly g^{**} closed. Therefore (X, τ_1, τ_2) is a (i, j) - $^{**}T_{\frac{1}{2}s}$ space. Therefore every (i, j) - $^{**}T_{\frac{1}{2}s}$ space need not be (i, j) - $^*T_{\frac{1}{2}s}$ space.

Theorem 4.19: Every (i, j) - $T_{\frac{1}{2}s}$ space is a (i, j) - $^*T_{\frac{1}{2}s}$ space.

proof follows from the definition.

The converse of the above theorem is not true as seen in the following example.

Example 4.20: In example (4.14) we have proved that the (i, j) -strongly g^* closed sets are all the subsets of X and in example (4.12) we have proved that all these sets are (i, j) -strongly g closed. Therefore every (i, j) -strongly g closed sets are (i, j) -strongly g^* closed. $\therefore (X, \tau_1, \tau_2)$ is a (i, j) - $^*T_{\frac{1}{2}s}$

space. In the same example we have proved that (X, τ_1, τ_2) is not a $T_{\frac{1}{2}s}$ space. Therefore every (i, j) - $T_{\frac{1}{2}s}$ space need not be a (i, j) - $T_{\frac{1}{2}s}$ space.

Theorem 4.21: Every (i, j) - $T_{\frac{1}{2}s}^{**}$ space is a (i, j) - $T_{\frac{1}{2}s}^*$ space.

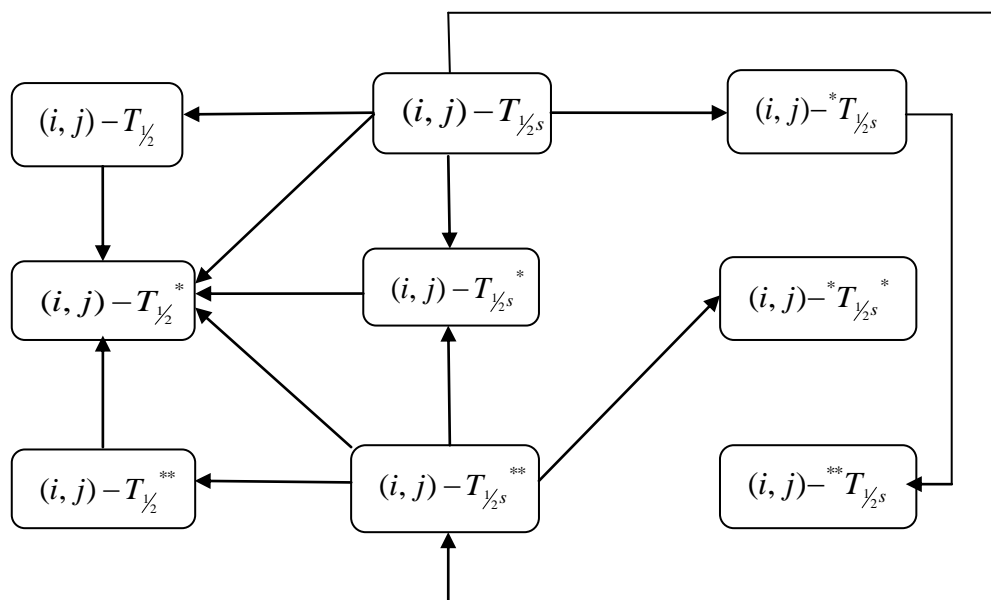
The converse of the above theorem is not true as seen in the following example.

Example 4.22: In example (4.16) we have proved that (X, τ_1, τ_2) is not a (i, j) - $T_{\frac{1}{2}s}^{**}$ space. In the same example we have proved that all the (i, j) -strongly g^{**} -closed sets are (i, j) -strongly g^* -closed. Therefore (X, τ_1, τ_2) is a (i, j) - $T_{\frac{1}{2}s}^*$ space & hence a (i, j) - $T_{\frac{1}{2}s}^*$ space need not be a (i, j) - $T_{\frac{1}{2}s}^{**}$ space.

Theorem 4.23: Every space (X, τ_1, τ_2) which is both a (i, j) - $T_{\frac{1}{2}s}^*$ space and a (i, j) - $T_{\frac{1}{2}s}^{**}$ space is a (i, j) - $T_{\frac{1}{2}s}^{**}$ space.

Proof: Let (X, τ_1, τ_2) be both (i, j) - $T_{\frac{1}{2}s}^*$ space and (i, j) - $T_{\frac{1}{2}s}^{**}$ space. Let A be (i, j) -strongly g^{**} -closed. Then A is (i, j) -strongly g^* -closed since (X, τ_1, τ_2) is a (i, j) - $T_{\frac{1}{2}s}^*$ space and A is τ_j -closed since (X, τ_1, τ_2) is (i, j) - $T_{\frac{1}{2}s}^{**}$ space. $\therefore A$ is τ_j -closed. Therefore (X, τ_1, τ_2) is (i, j) - $T_{\frac{1}{2}s}^{**}$ space.

The above results can be represented in the following figure.



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