αg**-closed sets in topological spaces

Pauline Mary Helen M
Associate Professor in Mathematics, Nirmala College, Coimbatore, India
Mythili. S
PG Student, Nirmala College, Coimbatore, India

ABSTRACT

In this paper we introduce a new class of sets namely *αg**-closed sets* which is settled properly in between the class of α-closed and the class of *g**-closed sets*. The notion of the αg**-continuous maps and αg**- irresolute maps are introduced and certain results regarding the above said maps are found. $T_\alpha{**}-space$ and $^*T_\alpha{*}-space$ are introduced and studied.

Keywords: *αg**-closed set, αg**-continuous map, αg**-irresolute maps,* $T_\alpha{**}-spaces$; $^*T_\alpha{}^*-spaces$

1.INTRODUCTION

Levine **[8]** introduced the class of generalized closed sets, a super class of closed sets in 1970. Andrijevic**[1]** defined semi- pre-open sets in 1986. Dontchev**[6]** introduced on generalizing semi- pre-open sets in 1995. Balachandran**[3]**, Sundaram and Maki introduced on generalized continuous maps in topological spaces in 1991. Arya**[2]** and Nour defined Characterizations of s-normal spaces in 1990. Pauline Mary Helen **[13]**, PonnuthaiSelvarani and Veronica Vijayan introduced g**-closed sets in topological spaces in 2012. Veerakumar**[14]** defined g*-closed sets in 1996.

Levine **[9]**, Njasted**[12]** introduces semi- open sets, pre-open sets, α-closed sets. The complement of a semi-open (resp. pre-open, α-open, semi- pre-open) set in 1963. Maki **[11]**, Devi and Balachandran defined associated topologies of generalized α-closed sets and α-generalized closed sets in 1994. Devi **[4]**, Maki and Balachandran introduced Semi-generalized closed maps and generalized closed maps in 1993. Gnanbambal **[7]** defined on generalized pre regular closed sets in topological spaces in 1997. Devi **[5]**, Maki and Balachandran introduced Semi-generalized homeomorphisms and generalized semi-homeomorphism in topological spaces in 1995. We proved that g**- closedness is independent from αg**-closedness. Applying αg**-closed sets, two new spaces namely, $T\alpha^{**}$-spaces and ${}_*T_\alpha{}^*$-spaces are introduced. Maragathavalli**[10]** and Sheik Jhon introduced on sαg**-closed sets in topological spaces in 2005.

2.Preliminaries

Throughout this paper (X,τ), (Y,σ) and (Z,η) represent non- empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X,τ), cl(A) and int(A) denote the closure and the interior of A respectively.

Definition 2.1: A subset A of a topological space (X,τ) is called

1) a semi-open set **[9]** if $A \subseteq cl(int(A))$ and semi-closed set if $int(cl(A)) \subseteq A$.
2) a semi-pre-open set **[1]** if $A \subseteq cl(int(cl(A)))$ and semi-pre closed set **[1]** if $int(cl(int(A))) \subseteq A$.
3) an α-open set if $A \subseteq int(cl(int(cl(A)))$ and an α-closed set **[12]** if $cl(int(cl(A))) \subseteq A$.

Definition 2.2: A subset A of a topological space (X,τ) is called

1) a generalized closed set (briefly g-closed) **[8]** if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ).
2) a generalized semi-closed set (briefly gs-closed) **[2]** if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ).
3) a α generalized closed set (briefly αg-closed) **[11]**if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ).

4) a generalized * closed set (briefly g*-closed) **[14]** if cl(A)$\subseteq$U whenever A$\subseteq$U and U is g-open in (X, τ).

5) a generalized ** closed set (briefly g**-closed) **[13]** if cl(A)$\subseteq$U whenever A$\subseteq$U and U is g*-open in (X, τ).

6) a generalized semi-pre closed set (briefly gsp-closed) **[9]** if spcl(A)$\subseteq$U whenever A$\subseteq$ U and U is open in (X, τ).

7) a semi α generalized * closed set (briefly sα g*-closed)**[10]** if α cl(A)$\subseteq$U whenever A$\subseteq$U and U is g*-open in (X, τ).

Definition 2.3: A function *f: (X,τ) →(Y,σ)* is called

1) a continuous if the inverse image of every closed set in (Y,σ) is closed in (X,τ).

2) an $\alpha g-continuous$ **[7]** if the inverse image of every closed set in (Y,σ) is $\alpha g-closed$ in (X,τ).

3) ags-continuous **[5]** if the inverse image of every closed set in (Y,σ) is gs-closed in (X,τ).

4) agsp-continuous **[6]** if the inverse image of every closed set in (Y,σ) is gsp-closed in (X,τ).

5) a g*-continuous **[14]** if the inverse image of every closed set in (Y,σ) is g*-closed in (X,τ).

6) a sαg*-continuous **[14]** if the inverse image of every closed set in (Y,σ) is sαg*-closed in (X,τ).

7) a $g*-irresolute$ **[14]** if the inverse image of every g*-closed set in (Y,σ) is g*-closed in (X,τ)

Definition 2.4: A topological space (X,τ) is said to be

1) a $T_{1/2}* - space$ **[14]** if every g*-closed set in (X,τ) is closed in (X,τ).

2) an $_{\alpha}T_c - space$ **[14]** if every $\alpha g - closed$ set in (X,τ) is g*-closed in (X,τ).

3) a $T_b - space$ **[4]** if every gs-closed set in (X,τ) is closed in (X,τ).

4) a $_{\alpha}T_b - space$ **[11]** if every $\alpha g - closed$ set in (X,τ) is closed in (X,τ).

5) a $_{\alpha}T_b - space$ **[3]** if every $\alpha g - closed$ set in (X,τ) is closed in (X,τ).

3.Basic properties of αg**-closed sets

We now introduce the following definition.

Definition 3.1: A subset *A* of (X,τ) is said to be a α*g**-closed set* if $\alpha cl(A)\subseteq U$ whenever $A\subseteq U$ and *U* is *g**-open* in X.

The class of *αg**-closed* subset of (X,τ) is denoted by αg**C (X,τ).

Proposition 3.2: Every *closed set* is *αg**-closed*.

Proof follows from the definition.

The converse of the above proposition need not be true in general as seen in the following example.

Example 3.3: Let X = {a,b,c} and τ = {Φ,X,,{b},{b,c}}. Let A={c}, then A is αg**-closed but not closed.

So, the class of αg**-closed setg is properly contained in the class of closed sets.

Proposition 3.4: Every *g*-closed* set is *αg**-closed*.

Proof follows from the definition.

The converse of the above proposition need true and in general it can be seen from the following example.

Example 3.5: Let X= {a,b,c}, τ = {Φ,X,{a},{a,b}}. Let A = {b} is αg**-closed but not g*-closed.

Proposition 3.6: Every α*g**-closed* set *is gs-closed*.

Proof follows from the definition.

The converse of the above proposition need true and in general it can be seen from the following example.

Example 3.7: Let X={a,b,c}, τ = {Φ,X,{a},{b,c}}. let A={c} & {a,c} is gs-closed but not αg**-closed.

Proposition 3.8: Every αg**-closed set is αg-closed.

Proof follows from the definition.

The converse of the above proposition need true and in general it can be seen from the following example.

Example 3.9: Let X={a,b,c}, τ = {Φ,X,{a},{b,c}}. let A={a,c} is αg-closed but not αg**-closed.

Proposition 3.10: Every α*g**-closed* set is gsp-closed.

Proof follows from the definition.

The converse of the above proposition need true and in general it can be seen from the following example.

Example 3.11: Let X={a,b,c}, τ={Φ,X,{a}}. Let A={a,c} then A is gsp-closed but not αg**-closed.

Proposition 3.12: Every α*g**-closed* set is sαg*-closed.

Proof follows from the definition.

The converse of the above proposition need true and in general it can be seen from the following example.

Example 3.13: Let X={a,b,c}, τ = {Φ,X,{a},{b,c}}. let A={b} is sαg*-closed but not αg**-closed.

Theorem 3.14: For each $x \in X$ either {x} is g**- closed (or) $\{x\}^c$ is $\alpha g^{**}-closed$ in X.

Proof: If {x} is not g**-closed then the only g**-open set containing $\{x\}^c$ is X.

$\therefore \alpha cl\{x\}^c \subseteq X$ and hence $\{x\}^c$ is $\alpha g^{**}-closed$.

Theorem 3.15: A is a $\alpha g^{**}-closed$ set of (X,τ) if $\alpha cl(A)\setminus A$ does not contains any non-empty g**-closed set.

Proof: Let F be a g**-closed set of (X,τ) such that $F \subseteq \alpha cl(A)\setminus A$. Then $A \subseteq X \setminus F$ since A is $\alpha g^{**}-closed$ and $X \setminus F$ is g**-open, $\alpha cl(A) \subseteq X \setminus F$. This implies $F \subseteq X \setminus \alpha cl(A)$ so

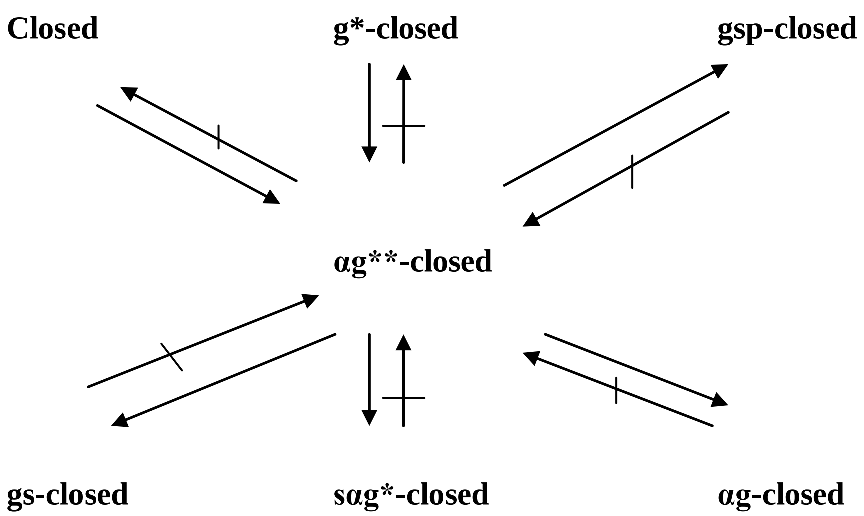
$$F \subseteq (X \setminus \alpha cl(A)) \cap (\alpha cl(A) \setminus AS) \subseteq (X \setminus \alpha cl(A)) \cap \alpha cl(A) = \phi \Rightarrow F = \phi$$

Remark 3.16: $g^{**}-closedness$ and $\alpha g^{**}-closedness$ are independent.

In example (3.5) A={b} is $\alpha g^{**}-closed$ but not g**-closed.

In example (3.10) A= {c} is $g^{**}-closed$ but not $\alpha g^{**}-closed$.

The above results can be represented in the following figure.



Where A → B(resp. A ↮ B) represents A implies B (resp. A & B are independent)

4.αg**-CONTINUOUS MAPS AND αg**-IRRESOLUTE MAPS.

We now introduce the following definitions.

Definition 4.1: A map $f:(X,\tau)\to(Y,\sigma)$ from a topological space (X,τ) to a topological space (Y,σ) is called $\alpha g**-continuous$ if the inverse image of every closed set in (Y,σ) is $\alpha g**-closed$ in (X,τ).

Theorem 4.2: Every *continuous map* is *αg**-continuous*.

Proof: Let $f:(X,\tau)\to(Y,\sigma)$ be *continuous*. Let F be closed set in (Y,σ) then f^{-1}(F) is closed in (X,τ). Since every closed set is *αg**-closed*, f^{-1}(F) is α*g**-closed* in(X,τ).

$\therefore f$ is α*g**-continuous* in (X,τ).

The converse of the above theorem need not be true and in general it can be seen from the following example.

Example 4.3: Let X=Y = {a,b,c}, ,τ = {Φ,X,{a,b}},σ = {Φ,Y,{a}} $f:(X,\tau)\to(Y,\sigma)$ is defined as the identity map. The inverse image of all the closed sets in (Y,σ) are αg**-closed in (X,τ). But f^{--1}*({b,c})= {b,c}* is not closed in (X,τ). Therefore *f* is *αg**-continuous* but not *continuous.*

Theorem 4.4: Every *g*- continuous* map is α*g**-continuous*.

Proof: Let $f:(X,\tau)\to(Y,\sigma)$ be g*-continuous. Let F be closed set in (Y,σ) then f^{-1}(F) is g*-closed in (X,τ). Since every g*-closed set is α*g**-closed*, f^{-1}(F) is α*g**-closed* in(X,τ).

$\therefore f$ is α*g**-continuous* in (X,τ).

The converse of the above theorem need not be true and in general it can be seen from the following example.

Example: 4.5: Let X = Y= {a,b,c} and τ = {X,Φ,{a},{a,b}},σ = {Y,Φ,{b,c},{a}}. $f:(X,\tau)\to(Y,\sigma)$ is defined as f(a)=b; f(b)=a; f(c)=c . The inverse image of all the closed sets in (Y,σ) are αg**-closed in (X,τ). But $f^{--1}(\{a\})= \{b\}$ is not g*-closed in (X,τ). Therefore *f is αg**-continuous* but not *g*-continuous.*

Theorem 4.6: Every α*g**- continuous* map is *gs-continuous*.

Proof: Let $f : (X,\tau) \rightarrow (Y,\sigma)$ be sg**-continuous. Let F be closed set in (Y,σ) then f^{-1}(F) is αg**-closed in (X,τ). Since every αg**-closed set is gs-*closed*, f^{-1}(F) is *αg**-closed* in (X,τ). $\therefore f$is *gs-continuous* in (X,τ).

The converse of the above theorem need not be true and in general it can be seen from the following example.

Example: 4.7: Let X = Y= {a,b,c} and τ = {X,Φ,{a},{b,c}},σ = {Y,Φ,{a},{a,b}}. $f : (X,\tau) \rightarrow (Y,\sigma)$ is defined as f(a)=b; f(b)=c; f(c)=a . The inverse image of all the closed sets in (Y,σ) are gs-closed in (X,τ). But $f^{-1}(\{c\}) = \{b\}$ is not αg*-closed in (X,τ). Therefore *f is gs-continuous* but not αg**-continuous.

Theorem: 4.8 Every αg**- continuous map is αg-continuous.

Proof: Let $f : (X,\tau) \rightarrow (Y,\sigma)$ be αg**-continuous. Let F be closed set in (Y,σ) then f^{-1}(F) is αg**-closed in (X,τ). Since every αg**-*closed* set is αg-closed, f^{-1}(F) is *αg-closed* in (X,τ). $\therefore f$ is *αg-continuous* in (X,τ).

The converse of the above theorem need not be true and in general it can be seen from the following example.

Example: 4.9: Let X = Y= {a,b,c} and τ = {X,Φ,{a},{b,c}},σ = {Y,Φ,{a}}. $f : (X,\tau) \rightarrow (Y,\sigma)$ is defined as f(a)=c; f(b)=a; f(c)=b . The inverse image of all the closed sets in (Y,σ) are αg-closed in (X,τ). But $f^{-1}(\{b,c\}) = \{a,c\}$ is not αg*-closed in (X,τ). Therefore *f is αg-continuous* but not αg**-continuous.

Theorem: 4.10: Every αg**- continuous map is gsp-continuous.

Proof: Let $f : (X,\tau) \rightarrow (Y,\sigma)$ be αg**-continuous. Let F be closed set in (Y,σ) then f^{-1}(F) is αg**-closed in (X,τ). Since every α*g**-closed* set is gsp-closed, f^{-1}(F) is *gsp-closed* in (X,τ). $\therefore f$ is *gsp-continuous* in (X,τ).

The converse of the above theorem need not be true and in general it can be seen from the following example.

Example: 4.11: Let X = Y= {a,b,c} and τ = {X,Φ,{a}},σ = {Y,Φ,{a},{b,c}}.

$f : (X,\tau) \rightarrow (Y,\sigma)$ is defined as the identity map. The inverse image of all the closed sets in (Y,σ) are gsp-closed in (X,τ). But f^{-1}({b,c}) = {a,c} is not αg*-closed in (X,τ). Therefore *f is gsp-continuous* but not αg**-continuous.

Theorem 4.12: Every αg**- continuous map is sαg*-continuous.

Proof: Let $f : (X,\tau) \rightarrow (Y,\sigma)$ be αg**-continuous. Let F be closed set in (Y,σ) then f^{-1}(F) is αg**-closed in (X,τ). Since every *αg**-closed* set is sαg*-closed, f^{-1}(F) is *sαg*-closed* in (X,τ). $\therefore f$ is *sαg*-continuous* in (X,τ).

The converse of the above theorem need not be true and in general it can be seen from the following example.

Example 4.13: Let X = Y= {a,b,c} and τ = {X,Φ,{a},{b,c}},σ = {Y,Φ,{a}}.

$f : (X,\tau) \rightarrow (Y,\sigma)$ is defined as f(a)=c; f(b)=a; f(c)=b. The inverse image of all the closed sets in (Y,σ) are sαg*-closed in (X,τ). But f^{-1}({b,c}) = {a,c} is not αg*-closed in (X,τ). Therefore *f is sαg*-continuous* but not αg**-continuous.

We now introduce the following definition.

Definition4.14: A function $f:(X,\tau)\rightarrow(Y,\sigma)$ is said to be $\alpha g^{**}-irresolute$ if the inverse image of every $\alpha g^{**}-closed$ set in (Y,σ) is $\alpha g^{**}-closed$ in (X,τ)

Theorem 4.15: Let $f:(X,\tau)\rightarrow(Y,\sigma)$ and $g:(Y,\sigma)\rightarrow(Z,\eta)$ be any two functions then,

(i) $g \circ f : (X, \tau) \to (Z, \eta)$ is $\alpha g^{**} - continuous$ if f is $\alpha g^{**} - irresolute$ and g is $\alpha g^{**} - continuous$.

(ii) $g \circ f : (X, \tau) \to (Z, \eta)$ is $\alpha g^{**} - irresolute$ if f and gare $\alpha g^{**} - irresolute$.

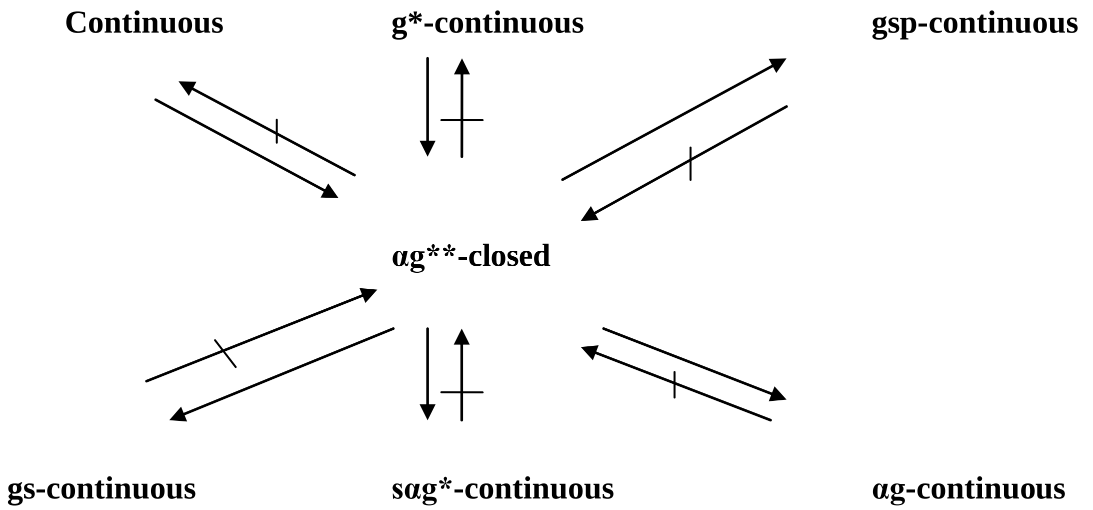
(iii) $g \circ f : (X, \tau) \to (Z, \eta)$ is $\alpha g^{**} - continuous$ if f is $\alpha g^{**} - irresolute$ and g is continuous.

Proof follows from the definitions.

Theorem 4.16: Every g*-irresolute map is $\alpha g^{**} - continuous$

Proof follows from the definitions.

The above results can be represented in the following figure.



Where A $\rightarrow$ B(resp. A $\nleftrightarrow$ B) represents A implies B (resp. A & B are independent)

5. APPLICATIONS OF sg-CLOSED SET.**

As applications of αg**-closed sets , new spaces namely, T_α^{**}-*space* and ${}^{*}T_\alpha^{*} - space$ are introduced.

We introduce the following definitions.

Definition 5.1: A space (X, τ) is said to be $T_\alpha^{**} - space$, if every αg**-closed set in (X, τ) is closed in (X, τ).

Definition 5.2:A space (X,τ) is said to be ${}^{*}T_{\alpha}{}^{*}-space$, if every αg**-closed set in (X,τ) is g*-closed in (X,τ).

Theorem5.3: Every $T_{\alpha}{}^{**}-space$ is $T_{1/2}{}^{*}-space$ but not conversely.

Proof follows from the definitions.

The converse of the above theorem need not be true in general as seen in the following example.

Example 5.4: In example (3.11), where X = {a,b,c} $\tau=\{\phi,X,\{a\}\}$ every g*-closed set is closed.. $\therefore$ (X,τ) is a $T_{1/2}{}^{*}-space$. In this space {b} is αg**-closed but not closed in (X,τ). Therefore every αg**-closed set is not closed in (X,τ) $\therefore$ (X,τ) is not a $T_{\alpha}{}^{**}-space$.

Theorem5.5: Every $T_{1/2}{}^{*}-space$ and ${}_{\alpha}T_{c}-space$ is $T_{\alpha}{}^{**}-space$.

Proof follows from the definitions.

Theorem 5.6: Every $T_{b}-space$ is $T_{\alpha}{}^{**}-space$.

Proof follows from the definitions.

Theorem 5.7: Every ${}_{\alpha}T_{b}-space$ is $T_{\alpha}{}^{**}-space$.

Proof follows from the definitions.

Theorem 5.8: Let $f:(X,\tau)\to(Y,\sigma)$ be a $\alpha g**-continuous$ map. If (X,τ) is $T_{\alpha}{}^{**}-space$ then f is continuous.

Proof: Let $f:(X,\tau)\to(Y,\sigma)$ be $\alpha g**-continuous$.Let F be closed in (Y,σ) then since f is $\alpha g**-continuous$ $f^{-1}(F)$ is $\alpha g**-closed$ in (X,τ) Also since (X,τ) is a $T_{\alpha}{}^{**}-space$, $f^{-1}(F)$ is closed in (X,τ)

The inverse image $f^{-1}(F)$ is closed in (X,τ) $\therefore f$ is continuous.

Theorem 5.9: Every ${}_{\alpha}T_{c}-space$ is ${}^{*}T_{\alpha}{}^{*}-space$.

Proof: Let (X,τ) be $_{\alpha}T_c$ $-space$ Let A be αg**-closed in (X,τ)

Then by proposition (3.8), A is αg-closed set in (X,τ) and since (X,τ) is $_{\alpha}T_c$ $-space$, A is g*-closed in (X,τ) Therefore every αg**-closed set in (X,τ) is g*-closed in (X,τ) $\therefore$ (X,τ) is ${}^{*}T_{\alpha}{}^{*}-space$.

Theorem 5.10: Every T_c $-space$ is ${}^{*}T_{\alpha}{}^{*}-space$.

Proof follows from the definition.

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