

Procedures and tables for construction and selection of Bayesian Chain Sampling plan (BChSP-1)

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ABSTRACT

Bayesian chain sampling plans (BChSP-1) allow significant reductions in sample size under conditions of a continuing succession of lots from a stable, trusted supplier. This paper presents procedures and tables for the construction of such plans and for selection of plans by specified properties. Suresh and Latha (2002) have developed Bayesian chain sampling plan (BChSP-1) and selection procedure has been provided for different performance measures. This paper provides a method for the selection of Bayesian chain sampling plan (BChSP-1) on the basis of different combinations of entry parameters. Gamma distribution is considered as prior distribution. Tables for determining the associated AQL and OAOQL are also given.

Key words: Bayesian chain sampling plan (BChSP-1), OAOQL, AQL, Selection procedure.

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INTRODUCTION

Bayesian Acceptance Sampling approach is associated with utilization of prior process history for the selection of distributions viz., Gamma Poisson, Beta Binomial, Beta Geometric to describe the random fluctuations involved in Acceptance Sampling. Calvin (1984) provides procedures and tables for implementing Bayesian Sampling Plan. Dodge (1955) has proposed Chain Sampling Plan in which Chain Sampling Plan allows significant reduction in sample size and the condition for a continuing succession of lots from a stable and trusted supplier. Latha (2002) has studied Bayesian Chain Sampling Plan – 1 involving designing of Bayesian Chain Sampling Plan indexed through AQL and OAOQL. The main thrust of this paper is to account for the possibility of dependence among the items of a sample. Latha and Rajeswari have discussed about the cost and regret function for Bayesian Chain Sampling Plan – 1. Latha and Jeyabharathi have given the procedure for the selection of Bayesian Chain Sampling Plan based on geometric distribution.

A study of construction of Bayesian chain sampling plan (BChSP-1), corresponding to given value of the acceptable quality level (AQL) and the overall average outgoing quality limit (OAOQL) is presented. Comparison is made with conventional sampling plan ChSP-1 for given AQL and AOQL values.

CHAIN SAMPLING PLAN (ChSP-1)

For situation in which testing is destructive or very expensive sampling plans with small sample sizes are usually selected. These small sample size plans often have acceptance number of zero. Plans with zero acceptance numbers are often undesirable, however, their OC curves are convex throughout, which means that the probability of lot acceptance begins to drop very rapidly as the lot fraction defective becomes greater than zero. This is often unfair to the producer, and in situations where rectifying inspection is used requires the consumer to screen a large number of lots which are essentially of acceptable quality.

Dodge (1955) suggested an alternate procedure, known as chain sampling that might be a substitute for ordinary single-sampling plans with zero acceptance numbers in certain circumstances. chain sampling plans make use of the cumulative results of several preceding lots. The conditions for application and the operating procedure for the ChSP-1 plan are given as follows:

CONDITIONS FOR APPLICATION OF ChSP-1

- (i) The cost of destructiveness of testing is such that a relatively small sample size is necessary, although other factors make a larger sample desirable.
- (ii) The product to be inspected comprises a series of successive lots produced by a continuing process.
- (iii) Normally lots are expected to be essentially of the same quality.
- (iv) The consumer has faith in the integrity of the producer.

OPERATING PROCEDURE

The plan is implemented in the following way

- (i) For each lot, select a sample of n units and test each unit for conformance to the specified requirements.
- (ii) Accept the lot if d (the observed number defectives) is zero in the sample of n units, and reject it if $d > 1$.
- (iii) Accept the lot if d is equal to 1 and if no defective is found in the immediately preceding i samples of size n .

AVERAGE PROBABILITY OF ACCEPTANCE (APA) Curves

The probability of acceptance for chain sampling plan of type ChSP-1 based on Poisson model is

$$Pa(p; n, i) = e^{-np} + npe^{-np(1+i)} \quad \text{-----(1)}$$

When p follows gamma prior distribution with density function

$$w(p) = e^{-tp} p^{s-1} \Gamma(s), p > 0, s, t > 0 \quad \text{-----(2)}$$

Where s and t are the parameters and the mean value of distribution, $\mu = s/t$. The average probability of acceptance is given by

$$\bar{P} = \int P(p; n, i) w(p) dp \quad \text{-----(3)}$$

$$\text{Which gives } \bar{P} = \frac{s}{(s+n\mu)^2} + n\mu \frac{s}{(s+n\mu(1+i))^{s+1}} \quad \text{-----}(4)$$

OVERALL AVERAGE OUTGIONG QUALITY LIMIT (OAOQL)

The Overall Average Outgoing Quality (OAOQL) for a Bayesian sampling plan is given by

$$OAOQL = \int pPa(p) dw(p)$$

Where W(p) is the distribution function of the product quality p, OAOQL is maximum value of OAOQ.

The expression in the right hand side of (4) is equated to the required probability of acceptance and then $n\mu$ values are obtained by using Newton's method of approximation. Overall Average Outgoing Quality (OAOQ)for Bayesian sampling plan is defined by

$$OAOQ = \int pPa(p)dw(p) \quad \text{-----}(5)$$

For BChSP-1,

$$nOAOQ = \frac{n\mu^{s+1}}{(s+n\mu)^{s+1}} + \frac{(n\mu)^2(s+1)s^{s+1}}{(s+n\mu(1+i))^{s+2}} \quad \text{-----}(6)$$

Differentiating equation (6) with respect to μ and equating to zero which results with

$$\frac{(1-n\mu)}{(s+n\mu)^{s+2}} + \frac{(s+1)(2n\mu-(n\mu)^2(1+i))}{(s+n\mu(1+i))^{s+3}} = 0 \quad \text{-----}(7)$$

Equation (7) is solved using Newton's method of successive approximation and the values of $n\mu(=n\mu_m)$ can be calculated for different values of s and i .Substituting $n\mu_m$ in equation (6) OAOQL values are obtained.

SELECTION PROCEDURE of BAYESIAN CHAIN SAMPLING PLAN-1 FOR GIVEN OAOQL AND s

Table 1 is constructed for the selection of a Bayesian chain sampling plan-1 (BChSP-1) plan with given s, the parameter of prior distribution and for the required OAOQL. Such table can be extended for any value of s and OAOQL.For example, when OAOQL = 1%,s=1,the plans can be (528,0),(324,1),(282,2) or (266,3) one of which may be chosen according to the requirement of inspection

SELECTION PROCEDURE BASED OF BAYESIAN CHAIN SAMPLING PLAN-1 BASED ON OAOQL AND AQL= μ_1

Table 2 is constructed for the selection of Bayesian chain sampling plan-1 for the given value of OAOQL and AQL. For given values AQL and OAOQL the ratio OAOQL/AQL is obtained. The value closer to this required ratio is searched in the table of Suresh and Latha (2001) (Table 3) and the corresponding (s,i) combination can be obtained. Dividing $n\mu_1$ by μ_1 , the AQL, the sample size n is obtained and hence a combination (s,n,i) for given OAOQL and AQL for the Bayesian chain sampling plan-1 is obtained.

For example, when $OAQL = 0.1$ and $AQL = 0.05$, the table values closer to the ratio $OAQL/AQL = 2$ is obtained as 2.1007 for which $(s,i) = (1,2)$ and 2.1117, for which $(s,i) = (3,0)$. Similarly more combination of (s,i) can be formed as per the inspection requirement.

COMPARISON WITH CONVENTIONAL PLAN

The parameters of Bayesian chain sampling plan-1 given by Suresh and Latha (2002) as in Table 3 can be compared with the parameters of ChSP-1 given by Soundararajan (1978) for given $OAQL$ and AQL . When $AOQL = 0.25$ and $AQL = 0.10$ in ChSP-1, the optimum plan is (168,2). In case of BChSP-1, for the same combination of $OAQL$ and AQL the appropriate plans are (104,4), (135,3), (152,2) and (156,2) for $s = 1, 3, 5, 7$ respectively. This implies that the sample size required for BChSP-1 is less than ChSP-1. It is observed that for small values of s , the optimum sample size is less, and i , the number of proceeding sample is more compared with the classical plan. which is much favourable to the consumer.

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Table 1: Selection Procedure Based on OAOQL

AOQL in percent						
S	i	0.1	0.25	0.5	0.75	1.0
1	0	528	212	106	71	53
	1	324	130	65	43	33
	2	282	113	56	38	28
	3	266	106	53	36	27
3	0	692	277	138	92	69
	1	420	168	84	56	42
	2	359	144	72	48	36
	3	336	135	67	45	34
5	0	742	742	148	99	74
	1	449	449	90	60	45
	2	381	381	76	51	38
	3	355	355	71	47	36
7	0	767	307	153	102	78
	1	463	185	93	62	46
	2	391	156	78	52	39
	3	364	146	73	49	36

Table 2: Selection Procedure Based on AQL and OAOQL

AQL in %	OAOQL in %			
		0.1	0.25	0.5
	S	n , i	n , i	n , i
0.05	1	282,2	-	-
	3	692,0		
	5	742,0		
	7	767,0		
0.075	1	528,0	128,7	69,0
	3		135,6	
	5		-	
	7			
0.10	1	528, 0	104,4	-
	3		135,3	
	5		152,2	
	7		156,2	
0.15	1	-	-	-
	3			
	5			
	7			

Table:3 Certain parametric values of Bayesian Chain Sampling Plans (BChSP-1)

s	i	$n\mu_1$	nOAOQL	OAOQL/ μ_1	s	i	$n\mu_1$	nOAOQL	OAOQL/ μ_1
1	0	0.2880	0.5282	1.8340	5	0	0.3353	0.7424	2.2141
	1	0.1686	0.3241	1.9223		1	0.1953	0.4487	2.2975
	2	0.1340	0.2815	2.1007		2	0.1538	0.3808	2.4759
	3	0.1163	0.2662	2.2889		3	0.1322	0.3553	2.6876
	4	0.1052	0.2593	2.4648		6	0.1017	0.3376	3.3196
	∞	0.0526	0.2500	4.7529		∞	0.0516	0.3349	6.4903
3	0	0.3245	0.6922	2.1331	7	0	0.3405	0.7671	2.2529
	1	0.1892	0.4201	2.2204		1	0.1983	0.4626	2.3932
	2	0.1493	0.3589	2.4039		2	0.1860	0.3912	2.5077
	3	0.1286	0.3362	2.6143		3	0.1339	0.3642	2.7199
	7	0.0940	0.3184	3.3872		∞	0.0515	0.3436	6.6718
	∞	0.0517	0.3164	6.1192					