EFFECT OF THERMO-DIFFUSION ON CONVECTIVE HEAT AND MASS TRANSFER FLOW IN A NION-UNIFORMILY HEATED VERICAL CHANNEL WITH CHEMICAL REACTION AND HEAT SOURCES

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ABSTRACT

We discuss the effect of thermo-diffusion on mixed convective heat and mass transfer flow of a viscous incompressible fluid n a vertical channel bounded by non-uniformly heated vertical walls.. The viscous dissipation is taken in to account in the energy equation. Assuming the slope of the boundary temperature to be small. we solve the governing momentum, energy and diffusion equations by a perturbation technique. The velocity ,the temperature ,the concentration ,and the rate of heat transfer have been analysed for different variations of the governing parameters. The dissipative effects on the flow, heat and mass transfer are clearly brought out.

Keywords : *Non-Uniform Temperature, Chemical Reaction, Heat Sources, Soret Effect, and Dissipation*

1.INTRODUCTION

This problem of combined buoyancy driven thermal and mass diffusion has been studied in parallel plate geometries by a few authors in the recent times, notably Gebhart [21], Lai[33,33a], Chen and Mout Sogloy [14], Poulikakos [44], Trevisan and Bejan [57], Mehta and Nanda Kumar[40] and Angiras et al[1].

In the risk assessment of nuclear power plants, the possibility and the consequences of a melt down of the reactor core are usually considered. During the course of such an accident molten fuel and coolant may interact. Violent thermal reaction can dispose the molten fuel into fine particles. These small particles quickly solidify in the coolant and settle on internal structures of the reactor pressure vessel forming a saturated porous bed. The question arises, under what conditions the nuclear decay heat can be removed from the particle bed to the ambient coolant by natural convection. Thus the problem of natural convection in saturated porous layers. This analysis of heat transfer in a viscous heat generating fluid also important in engineering processes pertain to flow in which a fluid supports an exothermal chemical or nuclear reaction or problems concerned with dissociating fluids[34,36].The Volumetric heat generation has been assumed to be constant [2,6,7,7a,8,15,42,43] or a function of space variable [12,14,25, 26,29,37]. For example a hypothetical core-disruptive accident in a liquid metal fast breeder reactor (LMFBR) could result in the setting of fragmented fuel debris as horizontal surfaces below the core. The porous debris could be saturated sodium coolant and heat generation will result from the radioactive decay of the fuel particulate(20) The heat losses from the geothermal system in some cases can be treated as if the heat comes from the heat generating sources [27]. Keeping this in view, porous medium with internal heat source have been discussed by several authors [11,20,27,28,43].

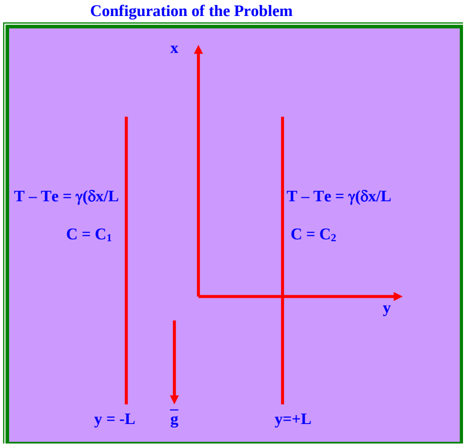
In the above mentioned investigations the bounding walls are maintained at constant temperature. However, there are a few physical situations which warrant the boundary temperature to be maintained non-uniform. It is evident that in forced or free convection flow in a channel (pipe) a secondary flow can be created either by corrugating the boundaries or by maintaining non-uniform wall temperature Such a secondary flow may be of interest in a few technological process. For example in drawing optical glass fibres of extremely low loss and wide bandwidth, the process of modified chemical vapour deposition (MCVD) [32,50] has been suggested in recent times. Performs from which these fibres are drawn are made by passing a gaseous mixture into a fused – silica tube which is heated locally by an oxy-hydrogen flame. Particulates of Sio_2- Geo_2 composition are formed from the mixture and collect on the interior of the tube. Subsequently these are fined to form a victreous deposit as the flame traversed along the tube. The deposition is carried out in the radial direction as the flame traversed along the tube. The deposition is carried out in the radial direction through the secondary flow created due to non-uniform. Ravindranath et al[48b]have studied the combined effect of convective heat and mass transfer on hydromagnetic electrically conducting viscous incompressible fluid through a porous medium in a vertical channel bounded by flat walls which are maintained at non-uniform temperatures.

All the above mentioned studies are based on the hypothesis that the effect of dissipation is neglected. This is possible in case of ordinary fluid flow like air and water under gravitational force. But this effect is expected to be relevant for fluids with high values of the dynamic viscous flows. Moreover Gehart[22], Gebhart and Mollen dorf[23] have shown that that viscous dissipation heat in the natural convective flow is important when the flow field is of extreme size or at extremely low temperature or in high gravitational filed. On the other hand Barletta[3] has pointed out that relevant effect of viscous dissipation on the temperature profiles and on the Nusselt numbers may occur in the fully developed forced convection in tubes. In view of this several authors notably, Soudalgekar and Pop[53]Raptis et al [48], Ramana Murthy and Soundelgekar et al [51], Barletta [3,4], Sreevani[55]. El-hakeing[18], Bulent Yesilata[10]. Rossidi schio[48a] and Israel et al [30] have studied the effect of viscous dissipation on the convective flows past on infinite vertical plates and through vertical channels and Ducts. The effect of viscous dissipation on natural convection has been studied for some different cases including the natural convection from horizontal cylinder. The natural convection from horizontal cylinder embedded in a porous media has been studied by Fand and Brucker[19]. They reported that the viscous dissipation may not be neglected in all cases of natural convection from horizontal cylinders and further, that the inclusion of a viscous dissipation term in porous medium may lead to more accurate correlation equations. The effect of viscous dissipation has been studied by Nakayama and Pop(38) for steady free convection boundary layer over a non-isothermal bodies of arbitrary shape embedded in porous media. They used integral method to show that the viscous dissipation results in lowering the level of the heat transfer rate from the body. Costa[16] has analysed a natural convection in enclosures with viscous dissipation. Recently Jambal et al have discussed the effects of viscous dissipation and fluid axial heat conduction heat transfer for non-Newtonian fluids in ducts with uniform wall temperature. Prasad[44a] has discussed the effect of dissipation on the mixed convective heat and mass transfer flow of a viscous fluid through a porous medium in a vertical channel bounded by flat walls. Vijayabhaskar Reddy[58]has analysed the combined influence of radiation and thermo-diffusion on convective heat And mass transfer flow of a viscous fluid through a porous medium in vertical channel whose walls are maintained at non-uniform temperatures .

In this paper we, discuss the effect of radiation on mixed convective heat and mass transfer flow of a viscous incompressible fluid n a vertical channel bounded by flat walls. A non-uniform temperature is imposed on the walls and the concentration on these walls is taken to be constant. The viscous dissipation is taken in to account in the energy equation. Assuming the slope of the boundary temperature to be small. We solve the governing momentum, energy and diffusion equations by a perturbation technique. The velocity ,the temperature ,the concentration ,the shear stress and the rate of heat transfer have been analysed for different variations of the governing parameters. The dissipative effects on the flow, heat and mass transfer are clearly brought out.

2. FORMULATION OF THE PROBLEM

Configuration of the Problem



We analyse the steady motion of viscous, incompressible fluid in a vertical channel bounded by flat walls which are maintained at a non-uniform wall temperature in the presence of a constant heat source and the concentration on these walls are taken to be constant. The Boussinesq approximation is used so that the density variation will be considered only in the buoyancy force. The viscous is taken into account in the energy equation. Also the kinematic viscosity ν,the thermal conducting k are treated as constants. We choose a rectangular Cartesian system 0(x ,y) with x-axis in the vertical direction and y-axis normal to the walls. The walls of the channel are at y = ± L. The equations governing the steady flow , heat and mass transfer are

Equation of continuity:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Equation of linear momentum:

$$\rho_e\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\rho g \tag{2}$$

$$\rho_e\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \tag{3}$$

Equation of Energy:

$$\rho_e C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\lambda\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+Q+\mu\left(\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial v}{\partial x}\right)^2\right) \tag{4}$$

Equation of Diffusion:

$$\left(u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}\right)=D_1\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right)-k_1'C+k_{11}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{5}$$

Equation of State:

$$\rho-\rho_e=-\beta\rho_e(T-T_e)-\beta^{\bullet}\rho_e(C-C_e) \tag{6}$$

where ρ_e is the density of the fluid in the equilibrium state, Te ,Ce are the temperature and Concentration in the equilibrium state,(u,v) are the velocity components along O(x ,y) directions, p is the pressure, T ,C are the temperature and Concentration in the flow region,ρis the density of the fluid,μ is the constant coefficient of viscosity , Cp is the specific

heat at constant pressure,λis the coefficient of thermal conductivity , β is the coefficient of thermal expansion, $\beta^{\bullet}$ is the coefficient of expansion with mass fraction ,D_1 is the molecular diffusivity ,Q is the strength of the constant internal heat source , q_r is the radiative heat flux and k11 is the cross diffusivity and $k._1$ is chemical reaction coefficient.
In the equilibrium state

$$0 = -\frac{\partial p_e}{\partial x} - \rho_e g \tag{7}$$

Where $p = p_e + p_D, p_D$ being the hydrodynamic pressure.
The flow is maintained by a constant volume flux for which a characteristic velocity is defined as

$$q = \frac{1}{L}\int_{-L}^{L} u \, d\, y. \tag{8}$$

The boundary conditions for the velocity and temperature fields are

u = 0 , v = 0 on y = ±L
$T - T_e = \gamma(\delta x / L)$ on y = ±L
C = C_1 on y = -L
C = C_2 on y = +L (9)

γ is chosen to be twice differentiable function,δ is a small parameter characterizing the slope of the temperature variation on the boundary.
In view of the continuity equation we define the stream function ψ as

$$u = -\psi_y, \; v = \psi_x \tag{9}$$

the equation governing the flow in terms of ψ are

$$\left[\frac{\partial\psi}{\partial x}\frac{\partial(\nabla^2\psi)}{\partial y} - \frac{\partial\psi}{\partial y}\frac{\partial(\nabla^2\psi)}{\partial x}\right] = \nu\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}\right)^2 - \beta g\frac{\partial T}{\partial y} - \beta^{\bullet} g\frac{\partial C}{\partial y} \tag{10}$$

$$\rho_e C_p\left(-\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x} + \frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}\right) = \lambda\nabla^2\theta + Q + \mu\left(\left(\frac{\partial^2\psi}{\partial y^2}\right)^2 + \left(\frac{\partial^2\psi}{\partial x^2}\right)^2\right) \tag{11}$$

$$\left(-\frac{\partial\psi}{\partial y}\frac{\partial C}{\partial x} + \frac{\partial\psi}{\partial x}\frac{\partial C}{\partial y}\right) = D_1\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) + k_{11}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) - k_1' C \tag{12}$$

Introducing the non-dimensional variables in (2 .10)- (2.12) as

$$(x', \; y') = (x, y)/L, \; (u', v') = (u, v)/q, \theta = \frac{T - T_e}{\Delta T_e}, C^{\bullet} = \frac{C - C_1}{C_2 - C_1}$$

$$p' = \frac{p_D}{\rho_e q^2} \quad ,.\gamma' = \frac{\gamma}{\Delta T_e} \tag{13}$$

(under the equilibrium state $\Delta T_e = T_e(L) - T_e(-L) = \frac{QL^2}{\lambda}$)

the governing equations in the non-dimensional form (after dropping the dashes) are

$$R\frac{\partial(\psi, \nabla^2\psi)}{\partial(x, y)} = \nabla^4\psi - \frac{G}{R}(\theta_y + NC_y) \tag{14}$$

and the energy and diffusion equations in the non-dimensional form are

$$PR(-\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}+\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y})=\nabla^2\theta+\alpha+(\frac{P_1R^2E_c}{G})((\frac{\partial^2\psi}{\partial y^2})^2+(\frac{\partial^2\psi}{\partial x^2})^2) \quad (15)$$

$$RSc(-\frac{\partial\psi}{\partial y}\frac{\partial C}{\partial x}+\frac{\partial\psi}{\partial x}\frac{\partial C}{\partial y})=(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2})+\frac{ScS_o}{N}(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2})-k_1C \quad (16)$$

where

$R=\frac{qL}{\nu}$ (Reynolds number) $G=\frac{\beta g\Delta T_e L^3}{\nu^2}$ (Grashof number)

$P=\frac{\mu c_p}{k_1}$ (Prandtl number), $E_c=\frac{\beta g L^3}{C_p}$ (Eckert number)

$N=\frac{\beta^\bullet\Delta C}{\beta\Delta T}$ (Buoyancy Number) $Sc=\frac{\nu}{D_1}$ (Schmidt number)

$\alpha=\frac{QL^2}{\Delta T k_1 C_p}$ (Heat source parameter) $S_0=\frac{k_{11}\beta^\bullet}{\beta\nu}$ (Soret parameter)

$k_1=\frac{k_1^{'}L^2}{D_1}$ (Chemical reaction parameter)

The corresponding boundary conditions are

$$\psi(+1)-\psi(-1)=-1$$

$$\frac{\partial\psi}{\partial x}=0,\ \frac{\partial\psi}{\partial y}=0 \quad at\ y=\pm1 \quad (17a)$$

$$\theta(x,y)=\gamma(\delta x) \quad on \quad y=\pm1 \quad (17b)$$

C= 0 on y = -1

C = 1 on y = 1 (17c)

$$\frac{\partial\theta}{\partial y}=0,\frac{\partial C}{\partial y}=0 \quad at \quad y=0 \quad (18)$$

The value of ψ on the boundary assumes the constant volumetric flow in consistent with the hyphothesis(8) .Also the wall temperature varies in the axial direction in accordance with the prescribed arbitrary function γ(x) .

3. ANALYSIS OF THE FLOW

The main aim of the analysis is to discuss the perturbations created over a combined free and forced convection flow due to non-uniform slowly varying temperature imposed on the boundaries. We introduce the transformation

$$\bar{x}=\delta x$$

With this transformation the equations(14) - (16) reduce to

$$R\delta\frac{\partial(\psi,F^2\psi)}{\partial(x,y)}=F^4\psi-\frac{G}{R}(\theta_y+NC_y) \quad (19)$$

and the energy &diffusion equations in the non-dimensional form are

$$PR\delta(-\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}+\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y})=F^2\theta+\alpha+(\frac{P_1R^2E_c}{G})((\frac{\partial^2\psi}{\partial y^2})^2+\delta^2(\frac{\partial^2\psi}{\partial x^2})^2) \qquad (20)$$

$$\delta RSc(-\frac{\partial\psi}{\partial y}\frac{\partial C}{\partial x}+\frac{\partial\psi}{\partial x}\frac{\partial C}{\partial y})=F^2C+\frac{ScSo}{N}F^2\theta-k_1C \qquad (21)$$

where $F^2=\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$

for small values of the slope δ,the flow develops slowly with axial gradient of order δ and hence we take

$$\frac{\partial}{\partial\bar{x}}\approx O(1)$$

We follow the perturbation scheme and analyse through first order as a regular perturbation problem at finite values of R,G,P ,Sc and D^{-1}

Introducing the asymptotic expansions

$$\psi(x,y)=\psi_0(x,y)+\delta\psi_1(x,y)+\delta^2\psi_2(x,y)+\ldots\ldots$$

$$\theta(x,y)=\theta_0(x,y)+\delta\theta_1(x,y)+\delta^2\theta_2(x,y)+$$

$$C(x,y)=C_0(x,y)+\delta C_1(x,y)+\delta^2C_2(x,y)+\ldots. \qquad (22)$$

On substituting (22) in (19) – (21) and separating the like powers of δ the equations and respective conditions to the zeroth order are

$$\psi_{0,yyyyy}=\frac{G}{R}(\theta_{0,y}+NC_{0,y}) \qquad (23)$$

$$\theta_{0,yy}=-\alpha-\frac{PR^2Ec}{G}\psi_{0,yy}^2 \qquad (24)$$

$$C_{0,yy}-k\,C_0=-\frac{ScSo}{N}\theta_{0,yy} \qquad (25)$$

with $\psi_0(+1)-\Psi(-1)=-1$,

$\psi_{0,y}=0$, $\psi_{0,x}=0$ at $y=\pm1$ (26)

$\theta o(\pm1)=\gamma(x)$ at $y=\pm1$

$C_0(-1)=0$ $C_0(+1)=1$ (27)

and to the first order are

$$\psi_{1,yyyyy}=-\frac{G}{R}(\theta_{1,y}+NC_{1,y})+R(\psi_{0,y}\psi_{0,xyy}-\psi_{0,x}\psi_{0,yyy}) \qquad (28)$$

$$\theta_{1,yy}=PR(\psi_{0,x}\theta_{0,y}-\psi_{0,y}\theta_{0,x})-\frac{PEc}{G}(R^2\psi_{1,yy}^2) \qquad (29)$$

$$C_{1,yy}-k\,C_1=RSc(\psi_{0,y}C_{0,x}-\psi_{0,x}C_{0,y})-\frac{ScSo}{N}\theta_{1,yy} \qquad (30)$$

$\psi_{1(+1)}-\psi_{1(-1)}=0$

$\psi_{1,y}=0$, $\psi_{1,x}=0$ at $y=\pm1$ (31)

$\theta_1(\pm1)=0$ at $y=\pm1$ (32)

$C_1(-1)=0$, $C_1(+1)=0$ (33)

Assuming Ec<<1 to be small we take the asymptotic expansions as

$$\psi_0(x,y)=\psi_{00}(x,y)+Ec\,\psi_{01}(x,y)+\ldots\ldots\ldots..$$

$$\psi_1(x,y)=\psi_{10}(x,y)+Ec\,\psi_{11}(x,y)+\ \ldots\ldots\ldots...$$

$$\theta_0(x,y)=\theta_{00}(x,y)+Ec\,\theta_{01}(x,y)+\ldots\ldots\ldots\ldots..$$

$$\theta_1(x,y)=\theta_{10}(x,y)+Ec\,\theta_{11}(x,y)+\ldots\ldots\ldots\ldots..$$

$$C_0(x,y) = C_{00}(x,y) + EcC_{01}(x,y) + ...$$
$$C_1(x,y) = C_{10}(x,y) + EcC_{11}(x,y) + ... \tag{34}$$

Substituting the expansions(34) in equations (23)-(33) and separating the like powers of Ec we get the following equations

$$\theta_{00,yy} = -\alpha, \quad , \quad \theta_{00}(\pm 1) = \gamma(\bar{x}) \tag{35}$$
$$C_{00,yy} = 0 \quad , \quad C_{00}(-1) = 0, C_{00}(+1) = 1 \tag{36}$$
$$\psi_{00,yyyy} = -\frac{G}{R}(\theta_{00,y} + NC_{00,y}) \ , \tag{37}$$
$$\psi_{00}(+1) - \psi_{00}(-1) = 1, \psi_{00,y} = 0, \psi_{00,x} = 0 \ \ at \ \ y = \pm 1$$
$$\theta_{01,yy} = -\frac{PR^2}{G}\psi^2{}_{00,yy} \quad , \quad \theta_{01}(\pm 1) = 0 \tag{38}$$
$$C_{01,yy} - k\,C_{00} = -\frac{ScSo}{N}\theta_{01,yy} \quad , \quad C_{01}(\pm 1) = 0 \tag{39}$$
$$\left.\begin{aligned} &\psi_{01,yyyy} = -\frac{G}{R}(\theta_{01,y} + NC_{01,y}) \ , \\ &\psi_{01}(+1) - \psi_{01}(-1) = 0, \\ &\psi_{01,y} = 0, \psi_{01,x} = 0 \ \ at \ \ y = \pm 1 \end{aligned}\right| \tag{40}$$
$$\theta_{10,yy} = RP_1(\psi_{00,y}\theta_{00,x} - \psi_{00,x}\theta_{00,y}) \qquad \theta_{10}(\pm 1) = 0 \tag{41}$$
$$C_{10,yy} = RP(\psi_{00,y}C_{00,x} - \psi_{00,x}C_{00,y}) - \frac{ScSo}{N}\theta_{10,yy} \quad C_{10}(\pm 1) = 0 \tag{42}$$
$$\psi_{10,yyyy} = -\frac{G}{R}(\theta_{10,y} + NC_{10,y}) + R(\psi_{00,y}\psi_{00,xyy} - \psi_{00,x}\psi_{00,yyy}) \ , \tag{43}$$
$$\psi_{10}(+1) - \psi_{10}(-1) = 0, \psi_{10,y} = 0, \psi_{10,x} = 0 \ \ at \ \ y = \pm 1$$
$$\theta_{11,yy} = RP_1(\psi_{00,y}\theta_{1,x} - \psi_{1,x}\theta_{00,y}) \quad , \quad \theta_1(\pm 1) = 0 \tag{44}$$
$$C_{11,yy} = RSc(\psi_{00,y}C_{00,x} - \psi_{1,x}C_{00,y}) - \frac{ScSo}{N}\theta_{11,yy} \quad , \quad C_{11}(\pm 1) = 0 \tag{45}$$
$$\psi_{11,yyyy} = -\frac{G}{R}(\theta_{11,y} + NC_{11,y}) + R(\psi_{00,y}\psi_{11,xyy} -$$
$$-\psi_{00,x}\psi_{01,yyy} + \psi_{01,y}\psi_{00,xyy} - \psi_{01.x}\psi_{00,yyy}) \ , \tag{46}$$
$$\psi_{11}(+1) - \psi_{11}(-1) = 0, \psi_{11,y} = 0, \psi_{11,x} = 0 \ \ at \ \ y = \pm 1$$

4. SOLUTION OF THE PROBLEM

Solving the equations (3.14)- (3.22) subject to the relevant boundary conditions we obtain

$$\theta_{00} = 0.5\alpha(1 - y^2) + \gamma(\bar{x})$$
$$C_{00} = 0.5\left(\frac{Ch(\beta_1 y)}{Ch(\beta_1)} + \frac{Sh(\beta_1 y)}{Sh(\beta_1)}\right) + a_3\left(\frac{Ch(\beta_1 y)}{Ch(\beta_1)} - 1\right)$$

$$\psi_{00} = a_6 + a_7 y + a_8 y^2 + a_9 y^3 + \phi_1(y)$$

$$\phi_1(y) = -\frac{G}{R} y - a_4 Sh(\beta_1 y) - a_5 Ch(\beta_1 y)$$

$$\theta_{01} = a_{23} y^2 + a_{25} y^3 + a_{25} y^4 + a_{26} y^5 + a_{27} y^6 + a_{28} y^8 + a_{29} Ch(2\beta_1 y) + a_{30} Sh(2\beta_1 y) +$$

$$+ (a_{31} + a_{34} y + a_{36} y^3) Sh(\beta_1 y) + (a_{32} + a_{33} y + a_{35} y^3) Ch(\beta_1 y)$$

$$C_{01} = a_{53} Ch(\beta_1 y) + a_{54} Sh(\beta_1 y) + \phi_2(y)$$

$$\phi_2(y) = a_{37} + a_{38} y + a_{39} y^2 + a_{40} y^4 + a_{41} y^6 + a_{42} Ch(2\beta_1 y) +$$

$$+ a_{43} Sh(2\beta_1 y) + (a_{44} + a_{46} y + a_{49} y^2 + a_{51} y^4) Ch(\beta_1 y) +$$

$$+ (a_{45} + a_{47} y + a_{48} y^2 + a_{50} y^3 + a_{52} y^4) Sh(\beta_1 y) + a_{46}$$

$$\psi_{01} = a_{74} + a_{75} y + a_{76} y^2 + a_{77} y^3 + \phi_3(y)$$

$$\phi_3(y) = a_{78} y^4 + a_{79} y^5 + a_{80} y^6 + a_{81} y^7 + a_{82} y^8 + a_{83} y^9 + a_{84} y^{11} + a_{85} Sh(2\beta_1 y) +$$

$$+ a_{86} Ch(2\beta_1 y) + (a_{87} + a_{91} y^2 + a_{93} y^3 + a_{96} y^4) Ch(\beta_1 y) + (a_{88} + a_{90} y + a_{92} y^2$$

$$+ a_{94} y^3 + a_{95} y^4) Sh(\beta_1 y)$$

$$\theta_{10} = b_{12} y^2 + b_{13} y^3 + b_{14} y^4 + b_{15} y^5 + b_{16} y^6 + (b_{17} + b_{20} y) Ch(\beta_1 y) +$$

$$+ (b_{18} + b_{19} y) Sh(\beta_1 y) + b_{20} y + b_{22}$$

$$C_{10} = b_{69} Ch(\beta_1 y) + b_{70} Sh(\beta_1 y) + \phi_4(y)$$

$$\phi_4(y) = b_{53} + b_{54} y + b_{55} y^2 + b_{56} y^3 + b_{57} y^4 + b_{58} Ch(2\beta_1 y) + + b_{59} Sh(2\phi_1 y) + (b_{60} y +$$

$$+ b_{62} y^2 + b_{64} y^3 + b_{67} y^4 + b_{68} y^5) Sh(\beta_1 y) + (b_{61} y + b_{63} y^2 + b_{65} y^3 + b_{66} y^4) Ch(\beta_1 y)$$

$$\psi_{10} = d_{20} + d_{21} y + d_{22} y^2 + d_{23} y^3 + \phi_5(y)$$

$$\phi_5(y) = d_2 y^3 + d_3 y^5 + d_4 y^6 + d_5 y^8 + d_6 y^9 + d_7 y^{10} + (d_8 + d_{10} y + d_{13} y^2 + d_{14} y^3$$

$$+ d_{16} y^4) Ch(\beta_1 y) + (d_9 + d_{11} y + d_{12} y^2 + d_{15} y^3 + d_{17} y^4) Sh(\beta_1 y) +$$

$$+ d_{18} Sh(2\beta_1 y) + d_{19} Ch(2\beta_1 y)$$

where $a_1, a_2, ...a_{75}, b_1, b_2, ...b_{53}\ , d_1, ..., d_{23}$ are constants

5. NUSSELT NUMBER and SHERWOOD NUMBER

The local rate of heat transfer coefficient(Nusselt number Nu) on the walls has been calculated using the formula

$$Nu = \frac{1}{\theta_m - \theta_w} \left(\frac{\partial \theta}{\partial y}\right)_{y=\pm 1} \qquad \theta_m = 0.5 \int_{-1}^{1} \theta \, dy$$

and the corresponding expressions are

$$(Nu)_{y=+1} = \frac{(-0.5\alpha_1 + Ec\,d_{25} + \delta\,d_{27}}{(\theta_m - \gamma(x))} \qquad (Nu)_{y=-1} = \frac{(-0.5\alpha_1 + Ec\,d_{26} + \delta\,d_{27})}{(\theta_m - \gamma(x))},$$

$$\theta_m = d_{29} + Ec\,d_{30} + \delta\,d_{31}$$

The local rate of mass transfer coefficient(Sherwood Number Sh) on the walls has been calculated using the formula

$$Sh = \frac{1}{C_m - C_w}\left(\frac{\partial C}{\partial y}\right)_{y=\pm 1} \qquad \text{where} \qquad C_m = 0.5\int_{-1}^{1} C\,dy$$

and the corresponding expressions are

$$(Sh)_{y=+1} = \frac{(d_{32} + Ec\,d_{34} + \delta\,d_{36})}{(C_m - 1)} \qquad (Sh)_{y=-1} = \frac{(d_{33} + Ec\,d_{35} + \delta\,d_{39})}{(C_m)},$$

where $C_m = d_{38} + Ec\,d_{39} + \delta\,d_{40}$ where $d_1.d_2, \ldots\ldots\ldots\ldots, d_{40}$ are constants..

6. DISCUSSION OF THE NUMERICAL RESULTS

In this analysis we discuss the effect of thermo diffusion and dissipation on convective heat and mass transfer flow of a viscous chemically reacting fluid in a non-uniformly heated vertical channel in the presence of heat sources. The analysis has been carried out by employing a perturbation technique with the slope δ of the boundary temperature as a perturbation parameter.

The axial velocity (u) is shown in figures 1 – 5 for different values of Sc, S_0, γ, α_1, Ec. The variation of u with Schmidt number Sc is shown in fig (1). Lesser the molecular diffusivity smaller |u| and for further lowering of the molecular diffusivity larger |u| in the entire flow region. Fig (2) represents u with Soret parameter S_0 . It is found that |u| enhances with increase in $S_0 > 0$ and depreciates in $|S_0|$ (<0). The variation of u with chemical reaction parameter γ is shown in fig (3). Fig (4) represents u with amplitude α_1 of the boundary temperature. It is found that higher the amplitude α_1, smaller |u| in the flow region. The effect of dissipation on u is shown in fig (5). It is found that higher the dissipative heat larger |u| in the flow region.

The secondary velocity (v) which is due to the non-uniform boundary temperature is exhibited in figures 6 - 10 for different parametric values. Fig (6) represents v with Sc. It is found that |v| experiences an enhancement with increase in Sc in the entire flow region. Also |v| enhances in the left half and reduces in the right half of the channel with increase in the Soret parameter $|S_0|$ (Fig 7). From fig (8) we notice that the magnitude of v enhances with increase in the chemical reaction parameter γ. An increase in the amplitude α1 of the boundary temperature depreciates |v| in the left half and enhances in the right half of the channel (fig 9). An increase in Ec leads to an enhancement in |v| thus higher the dissipative heat larger |v| in the flow region (fig 10).

The non-dimensional temperature (θ) is shown in figures 11 – 16 for different parametric values. We follow the convention that the non – dimensional temperature is positive or negative according as the actual temperature is greater / lesser than T_E. Fig (11) represents θ with Sc. It is found that lesser the molecular diffusivity larger the actual temperature in the flow region. The actual temperature reduces with increase in Soret parameter $S_0 > 0$ and enhances with $|S_0|$ (<0) (fig 12). An increase in the chemical reaction parameter γ leads to an enhancement in the actual temperature (fig 13). The variation of θ with heat source parameter α shows that the actual temperature enhances with increase in α>0 and reduces with | α | (α < 0) (fig 14). With respect to the amplitude α1 of the boundary temperature we notice that higher the amplitude $\alpha 1$ larger the actual temperature (fig.15). An increase in Ec results in a depreciation in the actual temperature (fig 16). Thus higher the dissipative heat smaller the actual temperature in the entire flow region.

The non – dimensional concentration (C) is shown in figures 17 – 21 for different parametric values. We follow the convention that the non – dimensional concentration is positive or negative according as the actual concentration is greater / lesser than C_1. With respect to Sc we find that lesser the molecular diffusivity larger the actual concentration except in the vicinity of $y = \pm 1$ and for further lowering of the diffusivity smaller the actual concentration and for still lowering of the diffusivity larger in the region -0.8 , -0.4 and smaller in the remaining region (fig 17). An increase in $S_0 > 0$ reduces the actual concentration in the left half and enhances in the right half , while for $|S_0|$ we notice an enhancement in C in the region -0.8, -0.6 and depreciation in the remaining region (fig 18). From fig (19) we find that the actual concentration reduces in the flow region except in a narrow region adjacent to y = -1 with increase in the chemical reaction parameter γ. With respect to α_1 , we find that the actual concentration depreciates in the left half and enhances in the right half with increase in the amplitude α_1 of the boundary temperature (fig 20). With respect to Ec we observe an enhancement in the actual concentration in the flow region. Thus higher the dissipative heat larger the actual concentration in the entire flow region (fig 21).

The rate off heat transfer (Nusselt Number) at $y = \pm 1$ is shown in tables 1 – 6 for different values of G, R, Sc, S_0, α, $\alpha_{1,}$ γ, N, Ec and x. It is found that the rate of heat transfer at y = +1 depreciates with increase in $R \leq 70$ and for higher $R \geq 140$ the nusselt number reduces in the heating case and enhances in the cooling case. While at y = -1 it depreciates with increase in R. |Nu| enhances with increase in |G| at both the walls. The variation of Nu with Sc shows that lesser the molecular diffusivity larger |Nu| at y = +1. While at y = -1, it depreciates with $Sc \leq 0.6$ and enhances with Sc = 1.3 and again depreciates with higher Sc=2.01. An increase in Soret parameter $S_0 > 0$ depreciates |Nu| at y = +1 and enhances at y=-1 while for $|S_0|$ (<0) it enhances at y = +1 and depreciates at y = -1. An increase in the chemical reaction parameter γ results in an enhancement in |Nu| at both the walls. (Tables 1 & 4). From tables 2 & 5 we notice that the rate of heat transfer enhances with increase in $\alpha>0$ and depreciates with $|\alpha|$ ($\alpha < 0$) at $y = \pm 1$. An increase in amplitude α_1 of the boundary temperature results in an enhancement in |Nu| at both the walls. The variation of Nu with buoyancy ratio N shows that when the molecular buoyancy force dominates over the thermal buoyancy force the rate of heat transfer enhances at y = +1 and depreciates at y = -1 irrespective of the directions of the buoyancy forces. With respect to Ec we find that higher the dissipative heat larger |Nu| and for further higher dissipative heat smaller |Nu| at y = -1, while at y = +1 |Nu| depreciates with the higher dissipative heat. Moving along the axial direction of the channel walls the rate of heat transfer enhances at y = +1 and depreciates at y=-1 for small and higher values of x and for intermediate value $x = \pi$ it depreciates at y = +1 and enhances at y = -1. (Tables 3 & 6).

The rate of mass transfer (Sherwood number) at $y = \pm 1$ is shown in tables 7 – 12 for different parametric values. It is found that the rate of mass transfer depreciates in the heating case and enhances in the cooling case at both the walls. An increase in Reynolds number R results in a depreciation in |Sh| at $y = \pm 1$. With respect to Sc we find that lesser the molecular diffusivity larger |Sh| at $y = \pm$ 1fixing the other parameters. An increase in Soret parameter $S_0 > 0$ depreciates |Sh| at y = +1 and enhances at y = -1 and for $|S_0|$ (<0) it enhances at y = +1 and depreciates at y = -1. The variation of chemical reaction parameter γ shows that the rate of mass transfer enhances in the degenerating chemical reaction case at both the walls (Tables 7 & 10). From tables 8 & 11 we find that the rate of mass transfer enhances at y = +1 and depreciates at y = -1 with increase in the strength of the heat source and in the case of heat sink, |Sh| depreciates at y = +1 and enhances at y = -1. An increase in the amplitude α_1 of the boundary temperature enhances |Sh| at y = +1 and reduces at y = -1. The variation of Sh with buoyancy ratio N shows that when the molecular buoyancy force dominates over the thermal buoyancy force the rate of mass transfer enhances at y = +1 and depreciates at y = -1 when the buoyancy forces act in the same direction and for the forces acting in opposite direction |Sh| depreciates at y = +1 and enhances at y = -1. An increase in Ec, reduces |Sh| at y = -1 and enhances at y = +1. Thus higher the dissipative heat larger the rate of mass transfer at y = +1 and smaller at y = -1. Moving along the axial direction of the channel walls |Sh| at y= +1 enhances at x = $\pi/2$ and 2π and depreciates at x = π. While at y = -1 |Sh| depreciates with x $\leq \pi/2$ and enhances with higher x $\geq \pi$ (Tables 9 & 12).

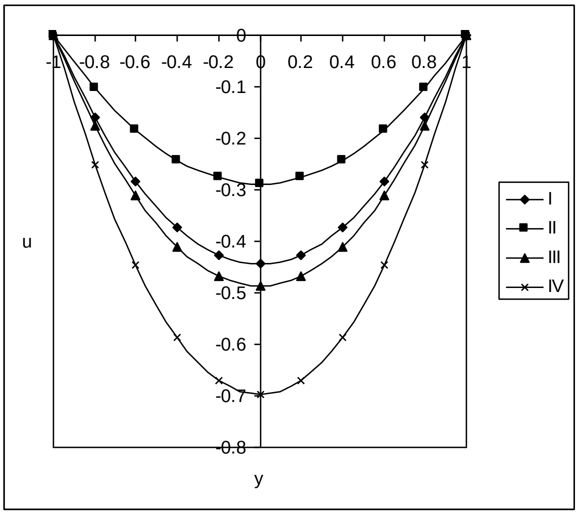


Fig. 1 : Variation of u with Sc

	I	II	III	IV
Sc	0.24	0.6	1.3	2.01

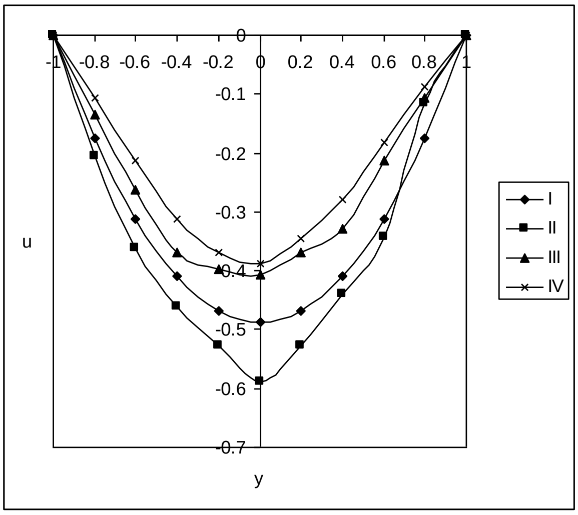


Fig. 2 : Variation of u with S_0

	I	II	III	IV
S_0	0.5	1	-0.5	-1

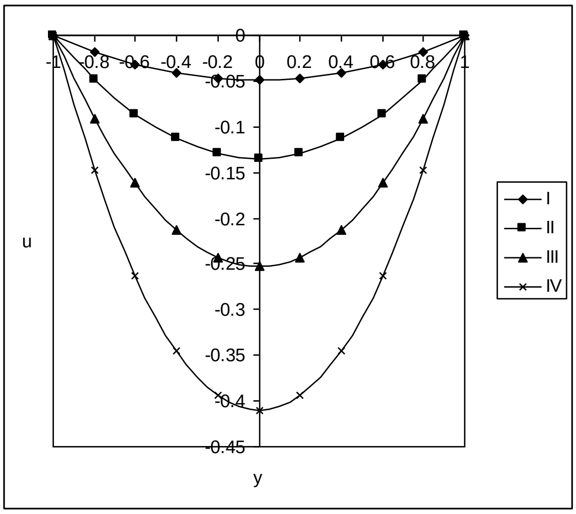


Fig. 3 : Variation of u with γ

	I	II	III	IV
γ	0.5	1.5	2.5	3.5

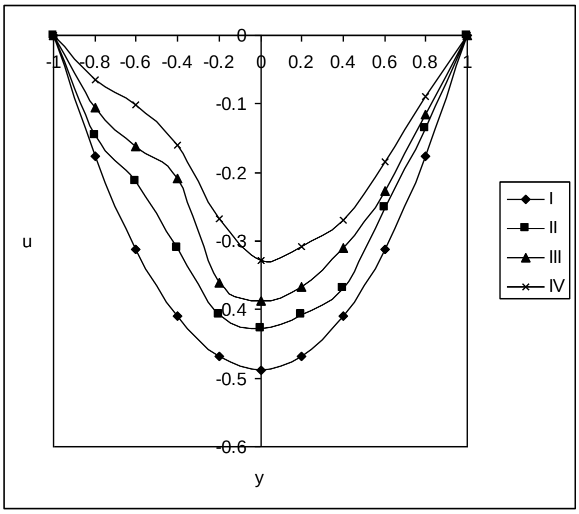


Fig. 4 : Variation of u with α_1

	I	II	III	IV
α_1	0.3	0.5	0.7	0.9

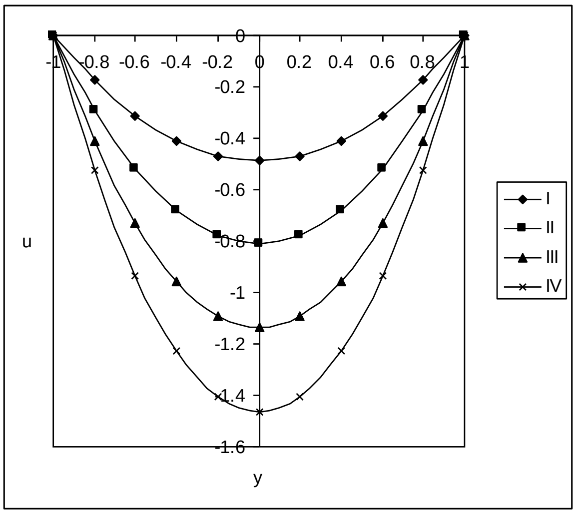


Fig. 5 : Variation of u with

	I	II	III	IV
Ec	0.03	0.05	0.07	0.09

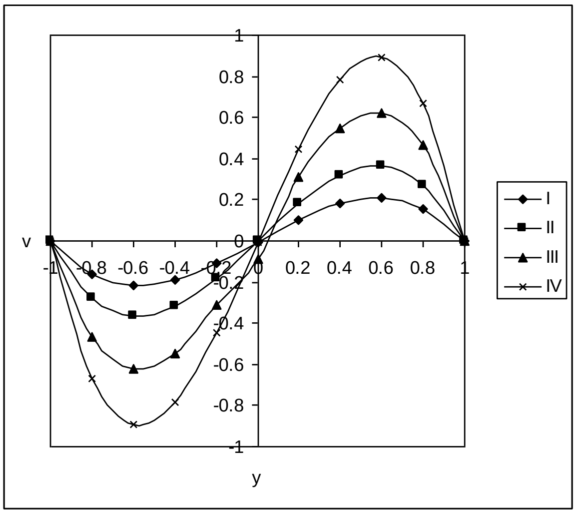


Fig. 6 : Variation of v with Sc

	I	II	III	IV
Sc	0.24	0.6	1.3	2.01

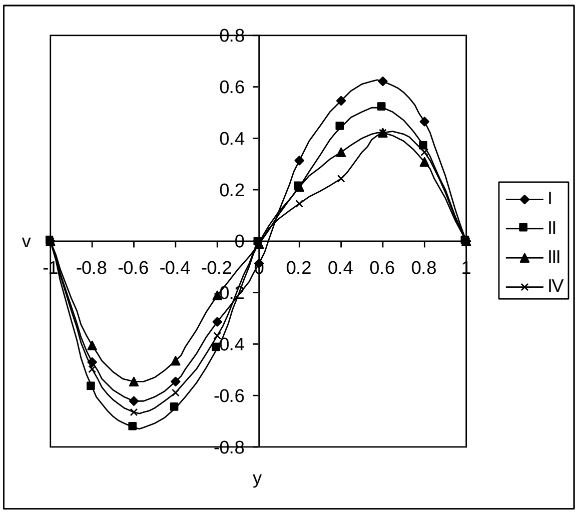


Fig. 7 : Variation of v with S_0

	I	II	III	IV
S_0	0.5	1	-0.5	-1

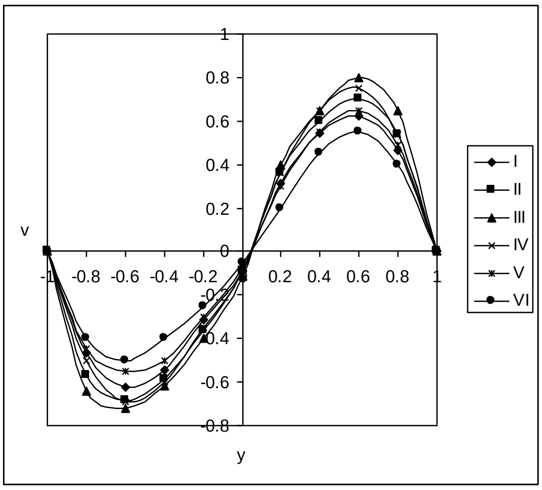


Fig. 8 : Variation of v with α

	I	II	III	IV	V	VI
α	2	4	6	-2	-4	-6

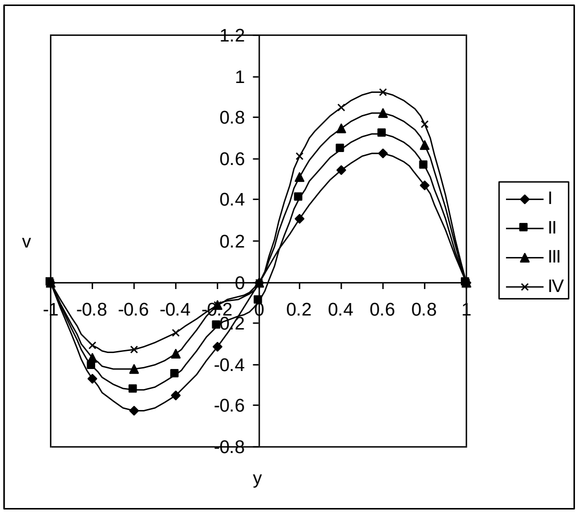


Fig. 9 : Variation of v with α_1

	I	II	III	IV
α_1	0.3	0.5	0.7	0.9

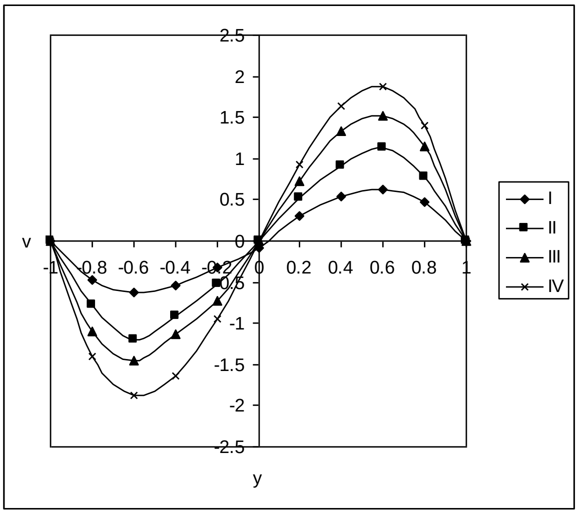


Fig. 10 : Variation of v with

	I	II	III	IV
Ec	0.03	0.05	0.07	0.09

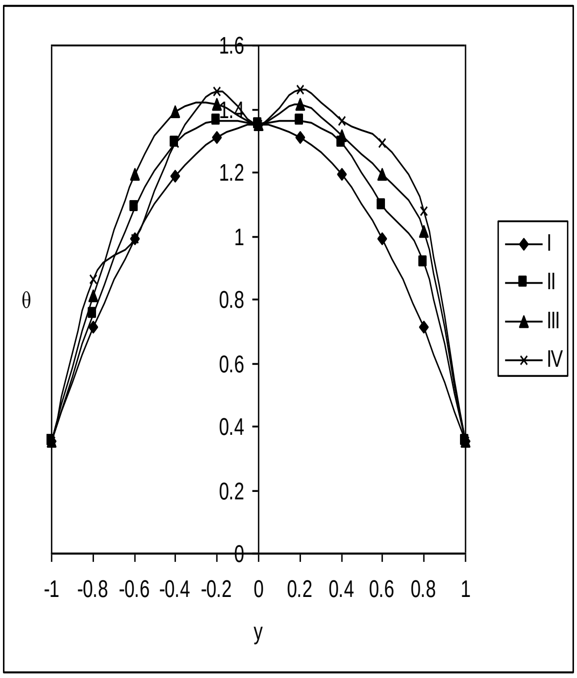


Fig. 11: Variation of θ with Sc

	I	II	III	IV
Sc	0.24	0.6	1.3	2.01

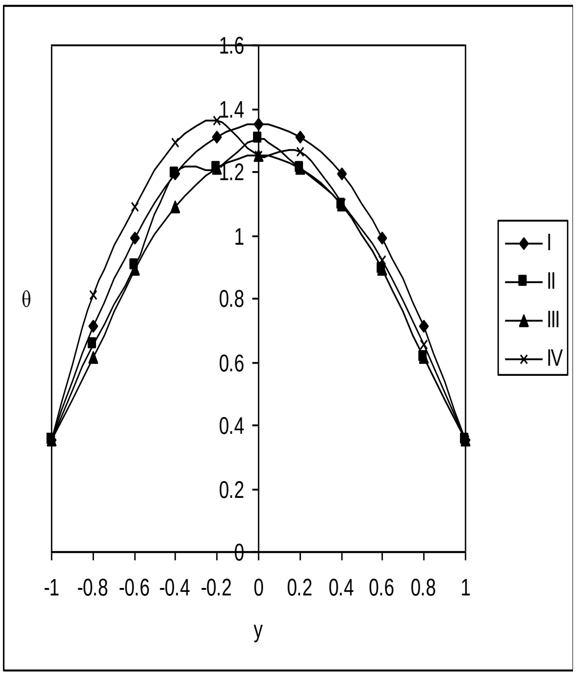


Fig. 12 : Variation of θ with S_0

	I	II	III	IV
S_0	0.5	1	-0.5	-1

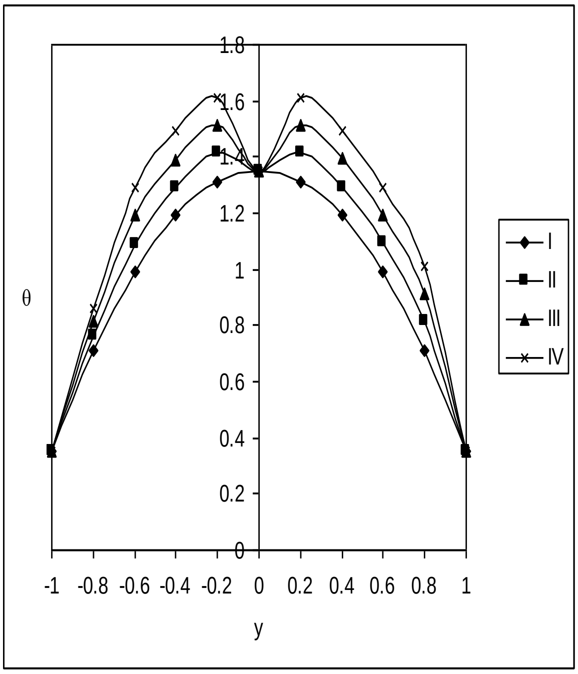


Fig. 13 : Variation of θ with γ

	I	II	III	IV
γ	0.5	1.5	2.5	3.5

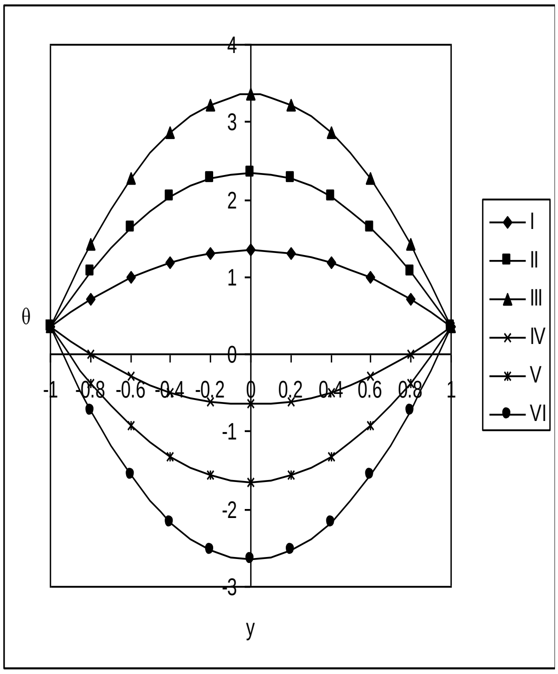


Fig. 14 : Variation of θ with α

	I	II	III	IV	V	VII
α	2	4	6	-2	-4	-6

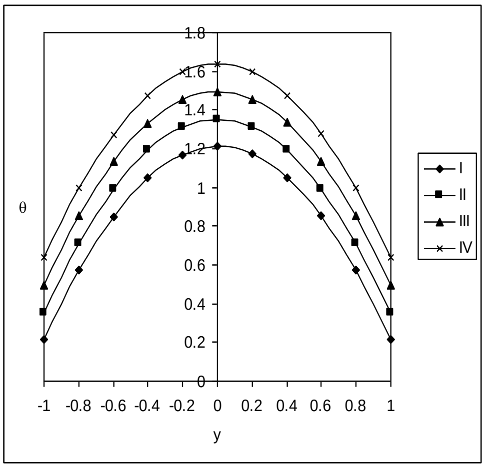


Fig. 15 : Variation of θ with α_1

	I	II	III	IV
α_1	0.3	0.5	0.7	0.9

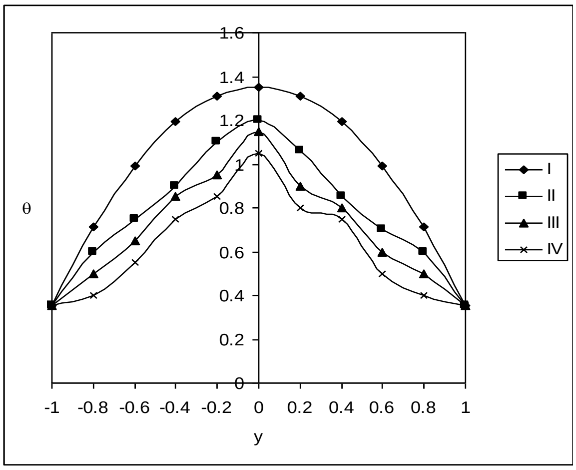


Fig. 16 : Variation of θ with Ec

	I	II	III	IV
Ec	0.03	0.05	0.07	0.09

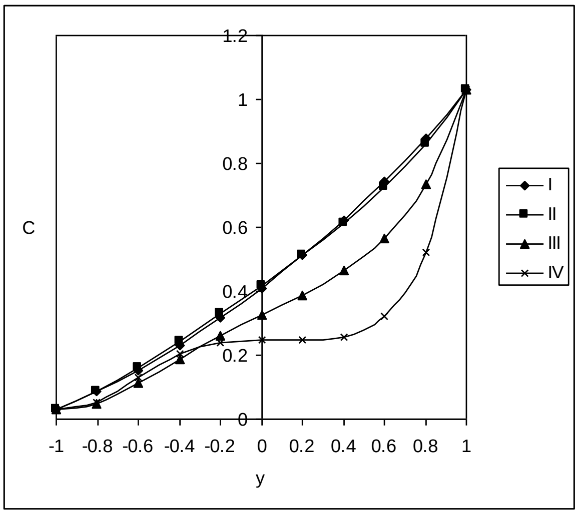


Fig. 17 : Variation of θ with Sc

	I	II	III	IV
Sc	0.24	0.6	1.3	2.01

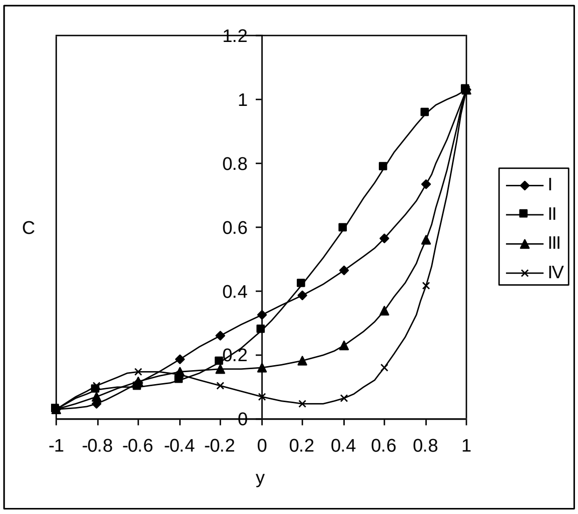


Fig. 18 : Variation of θ with S_0

	I	II	III	IV
S_0	0.5	1	-0.5	-1

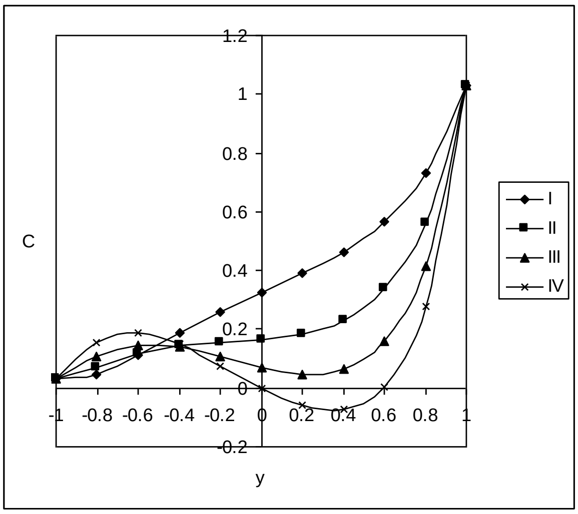


Fig. 19 : Variation of θ with γ

	I	II	III	IV
γ	0.5	1.5	2.5	3.5

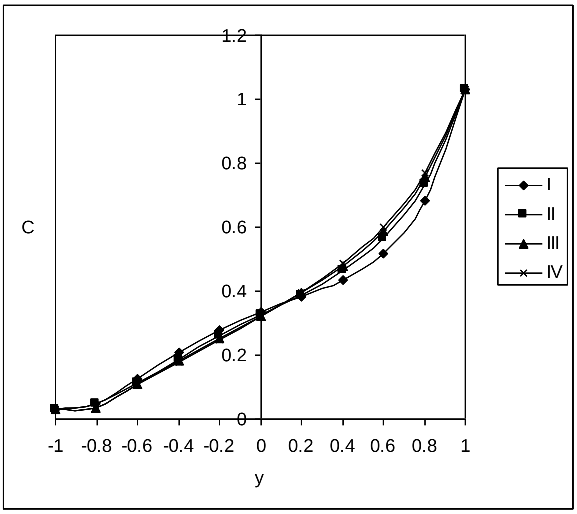


Fig. 20 : Variation of θ with α_1

	I	II	III	IV
α_1	0.3	0.5	0.7	0.9

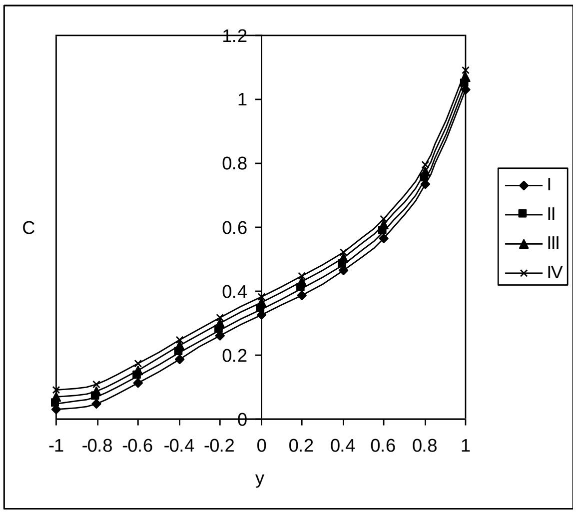


Fig. 21 : Variation of θ with Ec

	I	II	III	IV
Ec	0.03	0.05	0.07	0.09

Nusselt number (Nu) at y = +1 (Table 1)

G	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
10^3	0.2456	0.2386	0.2375	0.2289	0.2349	0.2546	0.2333	0.2337	0.2338	0.3502	0.3629	0.3656
3×10^3	0.2465	0.2395	0.2384	0.2311	0.2358	0.2589	0.2334	0.2339	0.2340	0.3501	0.3629	0.3666
-10^3	0.2346	0.2365	0.2360	0.2265	0.2326	0.2536	0.2338	0.2334	0.2336	0.3503	0.3630	0.3650
-3×10^3	0.2375	0.2370	0.2365	0.2272	0.2346	0.2549	0.2339	0.2336	0.2340	0.3506	0.3636	0.3659
R	**35**	**70**	**140**	**35**	**35**	**35**	**35**	**35**	**35**	**35**	**35**	**35**
Sc	**1.3**	**1.3**	**1.3**	**0.24**	**0.6**	**2.01**	**1.3**	**1.3**	**1.3**	**1.3**	**1.3**	**1.3**
S_0	**0.5**	**0.5**	**0.5**	**0.5**	**0.5**	**0.5**	**1**	**-0.5**	**-1**	**0.5**	**0.5**	**0.5**
γ	**0.5**	**0.5**	**0.5**	**0.5**	**0.5**	**0.5**	**0.5**	**0.5**	**0.5**	**1.5**	**2.5**	**3.5**

Nusselt number (Nu) at y = +1 (Table 2)

G	I	II	III	IV	V	VI	VII	VIII	IX
10^3	0.2456	0.3175	0.3609	-3.8881	1.1386	0.7957	0.2073	0.2670	0.3118
3×10^3	0.2465	0.3185	0.3612	-3.8967	1.1389	0.7967	0.2076	0.2689	0.3127
-10^3	0.2346	0.3179	0.3612	-3.9347	1.1373	0.7955	0.2077	0.2672	0.3120
-3×10^3	0.2375	0.3189	0.3613	-3.9466	1.1380	0.7960	0.2078	0.2673	0.3121
α	**2**	**4**	**6**	**-2**	**-4**	**-6**	**2**	**2**	**2**
α_1	**0.5**	**0.5**	**0.5**	**0.5**	**0.5**	**0.5**	**0.3**	**0.7**	**0.9**

Nusselt number (Nu) at y = +1 (Table 3)

G	I	II	III	IV	V	VI	VII	VIII	IX	X
10^3	0.2456	0.2334	0.2334	0.2330	0.1843	0.1522	0.1297	0.2546	0.2456	0.2646
3×10^3	0.2465	0.2334	0.2333	0.2326	0.1842	0.1522	0.1296	0.2564	0.2462	0.2679
-10^3	0.2346	0.2337	0.2337	0.2334	0.1845	0.1524	0.1298	0.2521	0.2412	0.2612
-3×10^3	0.2375	0.2338	0.2337	0.2334	0.1846	0.1525	0.1299	0.2534	0.2421	0.2624
N	**1**	**2**	**-0.5**	**-0.8**	**1**	**1**	**1**	**1**	**1**	**1**
Ec	**0.03**	**0.03**	**0.03**	**0.03**	**0.05**	**0.07**	**0.09**	**0.03**	**0.03**	**0.03**
x	**$\pi/4$**	**$\pi/4$**	**$\pi/4$**	**$\pi/4$**	**$\pi/4$**	**$\pi/4$**	**$\pi/4$**	**$\pi/2$**	**π**	**2π**

Nusselt number (Nu) at y = -1 (Table 4)

G	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
10^3	0.00005	0.00003	0.00001	0.00003	0.00001	-0.0003	0.42374	0.42245	0.42202	1.05557	1.17947	1.20103
3×10^3	0.00007	0.00004	0.00002	0.00006	0.00003	-0.0005	0.42415	0.42223	0.42158	1.05564	1.17952	1.20106
-10^3	-0.00006	-0.00003	-0.00001	0.00003	0.00001	0.0007	0.42206	0.42336	0.42380	1.05527	1.17929	1.20089
-3×10^3	-0.00010	-0.00005	-0.00002	0.00006	0.00003	0.0013	0.42163	0.42360	0.42424	1.05519	1.17924	1.20085
R	**35**	**70**	**140**	**35**	**35**	**35**	**35**	**35**	**35**	**35**	**35**	**35**
Sc	**1.3**	**1.3**	**1.3**	**0.24**	**0.6**	**2.01**	**1.3**	**1.3**	**1.3**	**1.3**	**1.3**	**1.3**
S_0	**0.5**	**0.5**	**0.5**	**0.5**	**0.5**	**0.5**	**1**	**-0.5**	**-1**	**0.5**	**0.5**	**0.5**
γ	**0.5**	**0.5**	**0.5**	**0.5**	**0.5**	**0.5**	**0.5**	**0.5**	**0.5**	**1.5**	**2.5**	**3.5**

Nusselt number (Nu) at y = -1 (Table 5)

G	I	II	III	IV	V	VI	VII	VIII	IX
10^3	0.00005	0.45874	0.47191	0.61257	0.55098	0.53312	0.34551	0.54674	0.77202
3×10^3	0.00007	0.45899	0.47217	0.61302	0.55134	0.53345	0.34578	0.54692	0.77220
-10^3	-0.00006	0.45777	0.47088	0.61074	0.54953	0.53176	0.34439	0.54602	0.77135
-3×10^3	-0.00010	0.45753	0.47062	0.61028	0.54917	0.53143	0.34410	0.54585	0.77119
α	2	4	6	-2	-4	-6	2	2	2
α_1	0.5	0.5	0.5	0.5	0.5	0.5	0.3	0.7	0.9

Nusselt number (Nu) at y = -1 (Table 6)

G	I	II	III	IV	V	VI	VII	VIII	IX	X
10^3	0.00005	0.42331	0.42335	0.42336	0.28609	0.21606	0.17357	-0.00001	0.00007	0.00003
3×10^3	0.00007	0.42352	0.42357	0.42359	0.28622	0.21615	0.17364	-0.00002	0.00008	0.00001
-10^3	-0.00006	0.42254	0.42246	0.42249	0.28559	0.21569	0.17328	-0.00001	-0.00013	-0.00012
-3×10^3	-0.00010	0.42238	0.42223	0.42229	0.28546	0.21560	0.17321	-0.00002	-0.00021	-0.00021
N	1	2	-0.5	-0.8	1	1	1	1	1	1
Ec	0.03	0.03	0.03	0.03	0.05	0.07	0.09	0.03	0.03	0.03
x	$\pi/4$	$\pi/4$	$\pi/4$	$\pi/4$	$\pi/4$	$\pi/4$	$\pi/4$	$\pi/2$	π	2π

Sherwood number (Sh) at y = +1 (Table 7)

G	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
10^3	-0.01313	-0.008241	-0.005777	-0.0008555	-0.003401	-0.03097	-0.01260	-0.004293	-0.007747	-0.3351	-1.169	-2.122
3×10^3	-0.01312	-0.008243	-0.005781	-0.0008410	-0.003426	-0.03078	-0.01243	-0.004296	-0.007581	-0.3347	-1.169	-2.123
-10^3	-0.01320	-0.008231	-0.005761	-0.0009092	-0.003298	-0.03173	-0.01333	-0.004280	-0.008380	-0.3366	-1.169	-2.119
-3×10^3	-0.01322	-0.008228	-0.005757	-0.0009215	-0.003272	-0.03192	-0.01352	-0.004277	-0.008530	-0.3370	-1.170	-2.119
R	35	70	140	35	35	35	35	35	35	35	35	35
Sc	1.3	1.3	1.3	0.24	0.6	2.01	1.3	1.3	1.3	1.3	1.3	1.3
S_0	0.5	0.5	0.5	0.5	0.5	0.5	1	-0.5	-1	0.5	0.5	0.5
γ	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	1.5	2.5	3.5

Sherwood number (Sh) at y = +1 (Table 8)

G	I	II	III	IV	V	VI	VII	VIII	IX
10^3	-0.01313	-0.01604	-0.01750	-0.01168	-0.01022	-0.008765	-0.01320	-0.01574	-0.01672
3×10^3	-0.01312	-0.01601	-0.01746	-0.01167	-0.01020	-0.008764	-0.01315	-0.01573	-0.01671
-10^3	-0.01320	-0.01616	-0.01764	-0.01173	-0.01025	-0.008760	-0.01339	-0.01579	-0.01675
-3×10^3	-0.01322	-0.01619	-0.01767	-0.01174	-0.01028	-0.008762	-0.01344	-0.01581	-0.01676
α	2	4	6	-2	-4	-6	2	2	2
α_1	0.5	0.5	0.5	0.5	0.5	0.5	0.3	0.7	0.9

Sherwood number (Sh) at y = +1 (Table 9)

G	I	II	III	IV	V	VI	VII	VIII	IX	X
10^3	-0.01313	-0.01716	-0.008248	-0.006091	-0.01459	-0.01469	-0.01519	-0.01417	-0.01344	-0.01601
3×10^3	-0.01312	-0.01719	-0.008336	-0.006051	-0.01457	-0.01467	-0.01527	-0.01418	-0.01392	-0.019608
-10^3	-0.01320	-0.01706	-0.007888	-0.005888	-0.01468	-0.01470	-0.01538	-0.01412	-0.006080	-0.01236
-3×10^3	-0.01322	-0.01703	-0.007796	-0.005855	-0.01471	-0.01476	-0.01531	-0.01411	-0.005243	-0.01324
N	1	2	-0.5	-0.8	1	1	1	1	1	1
Ec	0.03	0.03	0.03	0.03	0.05	0.07	0.09	0.03	0.03	0.03
x	$\pi/4$	$\pi/4$	$\pi/4$	$\pi/4$	$\pi/4$	$\pi/4$	$\pi/4$	$\pi/2$	π	2π

Sherwood number (Sh) at y = -1 (Table 10)

G	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
10^3	0.006465	0.004887	0.004096	-0.0004739	-0.0005029	0.02121	0.006714	0.01393	0.01145	0.1783	0.6766	1.269
3×10^3	0.006297	0.004806	0.004056	-0.0004498	-0.0003431	0.02048	0.006233	0.01416	0.01169	0.1770	0.6741	1.266
-10^3	0.007154	0.005214	0.004255	-0.0005737	-0.0001163	0.02417	0.008723	0.01305	0.01053	0.1836	0.6867	1.281
-3×10^3	0.007330	0.005297	0.004295	-0.0005996	-0.0001333	0.02493	0.009248	0.01284	0.01032	0.1849	0.6892	1.284
R	35	70	140	35	35	35	35	35	35	35	35	35
Sc	1.3	1.3	1.3	0.24	0.6	2.01	1.3	1.3	1.3	1.3	1.3	1.3
S_0	0.5	0.5	0.5	0.5	0.5	0.5	1	-0.5	-1	0.5	0.5	0.5
γ	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	1.5	2.5	3.5

Sherwood number (Sh) at y = -1 (Table 11)

G	I	II	III	IV	V	VI	VII	VIII	IX
10^3	0.006465	0.005995	0.005761	0.006700	0.006935	0.007171	0.006809	0.005611	0.005040
3×10^3	0.006297	0.005830	0.005598	0.006530	0.006763	0.006997	0.006498	0.005506	0.004968
-10^3	0.007154	0.006669	0.006427	0.007397	0.007639	0.007882	0.008101	0.006039	0.005332
-3×10^3	0.007330	0.006842	0.006598	0.007575	0.007819	0.008064	0.008437	0.006148	0.005406
α	2	4	6	-2	-4	-6	2	2	2
α_1	0.5	0.5	0.5	0.5	0.5	0.5	0.3	0.7	0.9

Sherwood number (Sh) at y = -1 (Table 12)

G	I	II	III	IV	V	VI	VII	VIII	IX	X
10^3	0.006465	0.004688	0.01078	0.01204	0.006230	0.006130	0.006030	0.005884	0.1052	0.2178
3×10^3	0.006297	0.004591	0.01068	0.01187	0.006063	0.005964	0.005864	0.005895	0.8709	1.5417
-10^3	0.007154	0.005083	0.01119	0.01274	0.006912	0.006812	0.006812	0.005839	-0.01132	0.01813
-3×10^3	0.007330	0.005185	0.01130	0.01292	0.007086	0.006986	0.006986	0.005828	-0.01695	0.02417
N	**1**	**2**	**-0.5**	**-0.8**	**1**	**1**	**1**	**1**	**1**	**1**
Ec	**0.03**	**0.03**	**0.03**	**0.03**	**0.05**	**0.07**	**0.09**	**0.03**	**0.03**	**0.03**
x	$\pi/4$	$\pi/4$	$\pi/4$	$\pi/4$	$\pi/4$	$\pi/4$	$\pi/4$	$\pi/2$	π	2π

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