Non-Linear Study of Heat and Mass Transfer of Two Immiscible Viscous Fluids Induced by Thermal Waves in Vertical Wavy Channel

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Abstract

The non-linear study of heat and mass transfer of two immiscible viscous fluids in vertical wavy channel which is induced by thermal waves is analyzed. The governing equations in the problem are solved by perturbation technique for both hydrodynamic and hydro magnetic cases. The contributions of the Hartmann number M^2, in particular and those of the other parameter G, W, P and α in general, to the flow and heat transfer characteristics are found to be quite significant.

Keywords: Non-linear study, Heat and Mass transfer, Thermal waves, Vertical Wavy Channel.

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Introduction

From a technological point of view, the study of viscous fluid flows bounded by wavy walls is of special interest and has practical application to transpiration cooling of re-entry vehicles and rocket boosters, cross-hatching on ablative surfaces and film vaporization in combustion chambers. In view of these applications, Shankar and Sinha [15] have made a detailed study of the Rayleigh problem for a wavy wall and arrived at certain interesting conclusions, namely, that at low Reynolds numbers the waviness of the wall quickly ceases to be of importance as the liquid is dragged along by the wall, while at large Reynolds numbers the effects of viscosity are confined to a thin layer close to the wall, and the known potential solution emerges in time. Transient force and free convection along a vertical wavy surface in micro polar fluids study has arrived at convincing results done by Cheng [5]. Kathyayani et al [11] had studied the effect of chemical reaction and radiation absorption on unsteady mixed convective heat and mass transfer flow through a porous medium in a vertical wavy channel with oscillatory flux and obtained the significant results. Rajesh Sharma [14] has analyzed the effect of viscous dissipation and heat sources on unsteady boundary layer flow and heat transfer past a stretching surface embedded in a porous medium using element free Galerkin method. Aziz et al [1] has estimated the MHD flow over an inclined radiating plate with the temp-dependent thermal conductivity with variable reactive index and heat generation. Malashetty et al [10] has worked on magneto convection of two immiscible fluids in a vertical enclosures. Prasada rao[12] has investigated free convection in hydro magnetic flow in a vertical wavy channel. Alazmi [2] analyzed the analysis of fluid flow and heat transfer interfacial conditions between a porous medium and a fluid layer.

In 2009, A J Chamka et al [3] has analyzed the natural convection flow in a rotating fluid on a vertical plate embedded in thermal stratified high –porosity medium. Doufer and Soret effects on free convective heat and mass transfer from an arbitrarily inclined plate in a porous medium with constant wall temperature and concentration was studied by Cheng [6]. Nabel [16] had made the numerical study of viscous dissipation effect on free convection heat and mass transfer through a porous medium. Bhuvanavijaya[13] had arrived at a good result on the study made on Double diffusive convection of a rotating fluid over a vertical plate embedded in Darcy –Forchhermer porous medium with non-uniform heat sources. Dulal pal [9] had analyzed the effect of variable viscosity on MHD non-Darcy mixed convective heat transfer over a stretching sheet embedded in a porous medium with non-uniform heat source/sink. Recently Devika et al [8] have arrived at a satisfactory result made on the research made on the effects of chemical reaction made on the effect of chemical reaction on the unsteady convective heat and mass transfer flow in a vertical wavy channel with oscillatory flux and heat sources.

In [4], the following four different configurations of the wavy channels (see Fig. 1) are considered:

- The crest of a wall corresponds to the crest of the other wall of the channel;
- (II) One of the walls considered in (i) has a phase-advance/lag;
- The crest of the wall corresponds to the trough of the other; and
- One of the walls considered in (iii) has a phase-advance/lag.

As the problem is highly nonlinear, it is solved by a perturbation technique wherein the solution is assumed to be made up of two parts: a mean part corresponding to the fully developed mean flow, and a small perturbed part. The mean part, the perturbed part, and the total solution of the problem are evaluated numerically for various values of the free convection parameter G, the frequency parameter ω, the Prandtl number P and the heat source/sink parameter α. The contributions of these parameters are found to be quite significant. Inspired by this hydrodynamics analysis of the unsteady convection problem and types of channels under consideration flow configure. The author strongly feels that its hydromantic extension would be interesting and have useful applications.

The main objective of this paper is to investigate the combined free and forced convection in hydromantic flows in vertical wavy channels with traveling thermal waves. Using the long wave approximation, the governing equation of the problem are solved by the perturbation technique for both hydrodynamic and hydro magnetic cases. At each stage, a comparison is made between the hydrodynamic and hydro magnetic cases. The contributions of the Hertmann number M^2, in particular, and those of the other parameters G, ω, P and α, in general, to the flow and heat transfer characteristics are found to be quite significant.

Formulation of the Problem

We consider the wavy wall in which x-axis is taken vertically upward, and parallel to the direction of buoyancy, and the y-axis is normal to it. The wavy walls are represented by $y = d + a\cos\lambda x$ and $y = -d - a\cos\lambda x$, where the latter can be conveniently represented by $y = -d - a\cos(\lambda x + \theta)$. We study the combined convective heat transfer and fluid flow in an incompressible electrically conducting vicious fluid confined to the vertical wavy channels in four different configurations (as mentioned in the Fig. 1) with the values of $\theta = 0, \pi/2, \pi$, and $3\pi/2$.

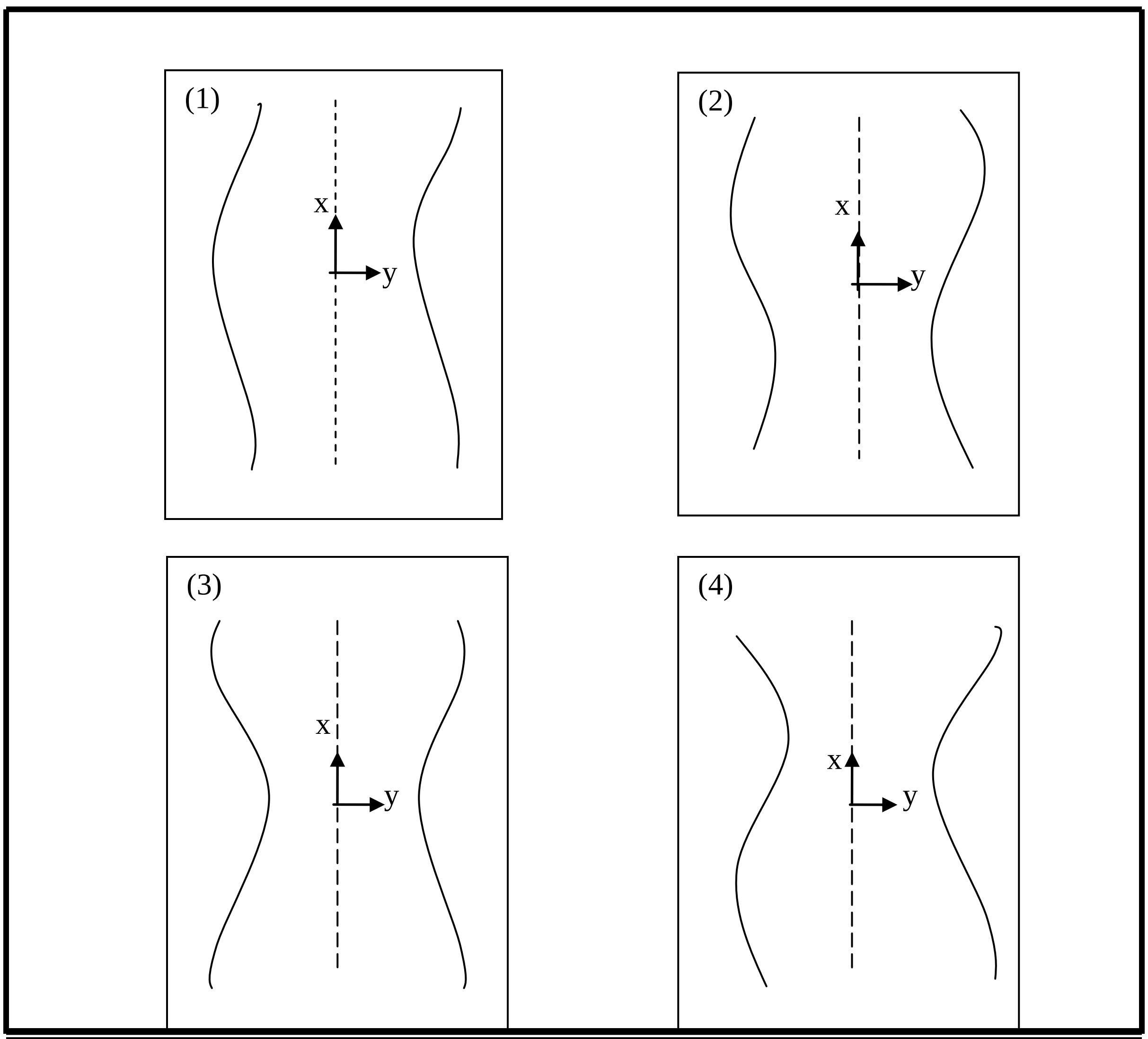

Fig. 1: Flow of Configuration and Types of Channel under Consideration

We make the following assumptions:

- The fluid properties are assumed to be constant and the Boussinesq approximation will be used that the density variation is retained only in the buoyancy term;
- The flow is Laminar and two-dimensional (that is the flow is identical in vertical layers which is a valid assumption);
- The viscous dissipation and the work done by pressure are sufficiently small in comparison with both heat flow by conduction and the wall temperature
- The volumetric heat source/sink term in the energy equation is constant;
- The wave length of the wavy walls which is proportional to $1/\lambda$ is large; the electric field is zero; and
- The induced magnetic field is negligible compared to the applied magnetic field.

With these assumptions, the unsteady flow and heat transfer in a viscous incompressible conducting fluid are governed by the momentum equations, the continuity equation, and the energy equation in the form. With these assumptions, the steady flow and heat transfer in a viscous incompressible conducting fluid are governed by the momentum equations, the continuity equation, and the energy equation in the form,

Region: I

$$\rho^{(1)}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)^{(1)}=-\frac{\partial p^{(1)}}{\partial x}+\mu^{(1)}\nabla^2 u-\rho^{(1)}g \tag{1}$$

$$\rho^{(1)}\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)^{(1)}=-\frac{\partial p^{(1)}}{\partial y}+\mu^{(1)}\nabla^2 v^{(1)} \tag{2}$$

$$\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)^{(1)}=0 \tag{3}$$

$$\rho^{(1)}c_p\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)^{(1)}=\left(k\nabla^2 T\right)^{(1)} \tag{4}$$

Region: II

$$\rho^{(2)}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)^{(2)}=-\frac{\partial p^{(2)}}{\partial x}+\mu^{(2)}\nabla^2 u-\rho^{(2)}g \tag{5}$$

$$\rho^{(2)}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)^{(2)}=-\frac{\partial p^{(2)}}{\partial y}+\mu^{(2)}\nabla^2 v^{(2)} \tag{6}$$

$$\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)^{(2)}=0 \qquad (7)$$

$$\rho^{(2)}c_p\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)^{(2)}=\left(k\nabla^2 T\right)^{(2)} \qquad (8)$$

Where u, v are the velocity components, T is the temperature, P is the pressure, B_0 is the transverse magnetic filed, σ is the coefficient if the electric conductivity, Q is the constant heat addition/absorption, ∇^2 is the two dimensional Lapalcian, and the other symbols have their meanings. The relevant boundary conditions of the problems are

$$u^{(1)}=v^{(1)}=0 \qquad T^{(1)}=T_{(1)}\left(1+\varepsilon\cos\lambda x\right)$$

$$\text{at } \; y=h^{(1)}+a\cos\lambda x=\hat{T}_1^{(1)} \qquad (9)$$

$$u^{(1)}=u^{(2)}=0 \qquad T^{(2)}=T_{(2)}\left(1+\varepsilon\cos\left(\lambda x+\theta\right)\right)$$

$$\text{at } \; y=-h^{(2)}+a\cos\left(\lambda x+\theta\right)=\hat{T}_2^{(2)} \qquad (10)$$

$$u^{(1)}=u^{(2)} \;;\; v^{(1)}=v^{(2)} \;;\; T^{(1)}=T^{(2)} \text{ at } \; y=0$$

The boundary conditions on the temperature field physically indicate that there are traveling thermal waves moving in the negative x-direction.

We next introduce the non dimensional flow and heat transfer variables as

$$\left.\begin{aligned}
&\left(x^*,y^*\right)=\frac{1}{d}\left(x,y\right), \quad t^*=t\upsilon/d^2,\\
&\left(u^*,v^*\right)=\frac{1}{d}\left(u,v\right), \quad p^*=p/\rho\left(\frac{\upsilon}{d}\right)^2,\\
&\text{And} \quad T^*=\left(T-\hat{T}_1\right)/\left(\hat{T}_2-\hat{T}_1\right),
\end{aligned}\right\} \qquad (11)$$

Where $\upsilon=\mu/\rho$ is the kinematic viscosity. In terms of these non-dimensional variables in equation (11), the basic equations (1) - (4) and (5) – (8) the equations can be express in the non-dimensional form dropping the asterisks,

Region: I

$$\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)^{(1)}=\left(-\frac{\partial p}{\partial x}+\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+GT\right)^{(1)} \qquad (12)$$

$$\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)^{(1)}=\left(-\frac{\partial p}{\partial y}+\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)^{(1)} \qquad (13)$$

$$\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)^{(1)}=\frac{1}{p_r}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)^{(1)} \tag{14}$$

Region: II

$$\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)^{(2)}=\left(-\frac{\partial p}{\partial x}+\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+G\beta h^3 m^2 r^2 T\right)^{(2)} \tag{15}$$

$$\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)^{(2)}=\left(-\frac{\partial p}{\partial y}+\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)^{(2)} \tag{16}$$

$$\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)^{(2)}=\frac{km}{p_r}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)^{(2)} \tag{17}$$

Boundary conditions reduces to,

$$u^{(1)}=v^{(1)}=0\,,T^{(1)}=0 \quad \text{at} \quad y=1+\varepsilon\cos\lambda^{(1)}x \tag{18}$$

$$u^{(2)}=v^{(2)}=0\,,T^{(2)}=1 \quad \text{at} \quad y=-1+\left(\frac{\varepsilon}{h}\right)\varepsilon\cos\left(\lambda^{(2)}+x\right) \tag{19}$$

The interfaces conditions are,

$$\left.\begin{aligned}
&u^{(1)}=\frac{1}{rmh}u^2 \quad \text{at} \quad y=0\\
&v^{(1)}=\frac{1}{rmh}v^2 \quad \text{at} \quad y=0\\
&T^{(1)}=T^2 \quad \text{at} \quad y=0\\
&\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^{(1)}=\frac{1}{rm^2h^2}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^{(2)} \quad \text{at} \quad y=0\\
&\left(\frac{\partial T}{\partial y}+\frac{\partial T}{\partial x}\right)^{(1)}=\frac{k}{h}\left(\frac{\partial T}{\partial y}+\frac{\partial T}{\partial x}\right)^{(2)} \quad \text{at} \quad y=0
\end{aligned}\right\} \tag{20}$$

Where $G=d^3 g\beta\left(\hat{T}_2-\hat{T}_1\right)/\upsilon^2$, the Grashof number, $P=\mu C_p/k$, Prandtl number, $\lambda\left(=\lambda^*\right)=\lambda d$, the non dimensional wave number, $\varepsilon=a/d$, the amplitude parameter.

Using small perturbation method,

$$\left.\begin{aligned} u^{(i)}(x,y) &= u_0^{(i)}(y) + \varepsilon u^{(i)}{}_1(x,y) \\ v^{(i)}(x,y) &= \varepsilon v^{(i)}{}_1(x,y) \\ p^{(i)}(x,y) &= p_0^{(i)}(x,y) + \varepsilon p^{(i)}{}_1(x,y) \\ T^{(i)}(x,y) &= T_0^{(i)}(y) + \varepsilon T^{(i)}{}_1(x,y) \\ &\text{where, i = 1, 2} \end{aligned}\right\} \quad (21)$$

Numerical Solution

We next solve the problem by the method of perturbation. We assume that the solution is made up of a mean and a perturbed part, so that the velocity and temperature distribution will take the forms, respectively

$$\left.\begin{aligned} \psi(x,y,z) &= \psi_0(y) + \psi_1(x,y,t) \\ T(x,y,t) &= T_0(y) + T_1(x,y,t) \\ \text{Where} \quad & \\ \psi_1(x,y,t) &= \varepsilon \exp\left[i\left(\cos(\lambda x)\right)\right]\bar{\psi}_1(y) \\ T_1(x,y,t) &= \varepsilon \exp\left[i\left(\cos(\lambda x)\right)\right]\bar{T}_1(y) \end{aligned}\right\} \quad (22)$$

And the perturbed quantities ψ_1 and T_1 are small compared to their mean quantities ψ_0 and T_0 respectively.

Zeroth order equations are,

Region : I

$$\frac{d^2T_0^{(1)}}{dy^2} = 0 \quad (23)$$

$$\frac{d^2u_0^{(1)}}{dy^2} = 0 \quad (24)$$

Region : II

$$\frac{d^2T_0^{(2)}}{dy^2} = 0 \quad (25)$$

$$\frac{d^2u_0^{(2)}}{dy^2} = -G\beta m^2 r^2 h^3 T_0^{(2)} \quad (26)$$

Zeroth order boundary and interface conditions are,

$$u_0^{(1)}(1) = 0,\ u_0^{(1)}(-1) = 0,\quad T_0^{(1)}(1) = 0,\ T_0^{(1)}(-1) = 0 \quad (27)$$

Zeroth order interface conditions are,

$$\left.\begin{aligned} & u_0^{(1)} = \frac{1}{rmh}\left(u_0\right)^{(2)} \\ & \left(\frac{du_0}{dy}\right)^{(1)} = \frac{1}{rm^2h^2}\left(\frac{du_0}{dy}\right)^{(2)} \\ & T_0^{(1)} = T_0^{(2)} \\ & \left(\frac{dT_0}{dy}\right)^{(1)} = \frac{k}{h}\left(\frac{dT_0}{dy}\right)^{(2)} \end{aligned}\right\} \quad (28)$$

First order equations are,

Region: I

$$\left(u_0\frac{\partial u_1}{\partial x} + v_1\frac{\partial u_0}{\partial y}\right)^{(1)} = \left(-\frac{\partial p_1}{\partial x} + \frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial x^2} + GT_1\right)^{(1)} \quad (29)$$

$$\left(u_0\frac{\partial v_1}{\partial x}\right)^{(1)} = \left(-\frac{\partial p_1}{\partial y} + \frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial y^2}\right)^{(1)} \quad (30)$$

$$\left(u_0\frac{\partial T_1}{\partial x} + v_1\frac{\partial T_0}{\partial y}\right)^{(1)} = \frac{1}{P_r}\left(\frac{\partial T_1}{\partial x^2} + \frac{\partial^2 T_1}{\partial y^2}\right)^{(1)} \quad (31)$$

Region: II

$$\left(u_0\frac{\partial u_1}{\partial x} + v_1\frac{\partial u_0}{\partial y}\right)^{(2)} = \left(-\frac{\partial p_1}{\partial x} + \frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial x^2} + G\beta m^2 r^2 h^3 T_1\right)^{(2)} \quad (32)$$

$$\left(u_0\frac{\partial v_1}{\partial x}\right)^{(2)} = \left(-\frac{\partial p_1}{\partial y} + \frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_1}{\partial x^2}\right)^{(2)} \quad (33)$$

$$\left(u_0\frac{\partial T_1}{\partial x} + v_1\frac{\partial T_0}{\partial y}\right)^{(2)} = \frac{km}{P_r}\left(\frac{\partial T_1}{\partial x^2} + \frac{\partial^2 T_1}{\partial y^2}\right)^{(2)} \quad (34)$$

First order boundary conditions for region-I & region-II,

$$\left.\begin{array}{l} u^{(1)}{}_1(1) = -\cos\lambda x \dfrac{\partial u_0{}^{(1)}}{\partial y}, \qquad v^{(1)}{}_1(1) = 0 \\ T^{(1)}{}_1(1) = -\cos\lambda x \dfrac{\partial T_0^{(1)}}{\partial y}(1), \quad u^{(2)}{}_1(-1) = -\dfrac{1}{h}\cos(\lambda x+\theta)\dfrac{\partial u_0{}^{(2)}}{\partial y}(-1) \\ T^{(2)}{}_1(-1) = -\dfrac{1}{h}\cos(\lambda x+\theta)\dfrac{\partial T_0^{(1)}}{\partial y}(-1), \qquad v^{(2)}{}_1(-1) = 0 \end{array}\right\} \quad (35)$$

First order interface conditions are,

$$\left.\begin{array}{ll} u_1^{(1)} = \dfrac{1}{rmh}u_1^{(2)}, \; v_1^{(1)} = \dfrac{1}{rmh}v_1^{(2)}, \; T_1^{(1)} = T_1^{(2)} & \text{at} \quad y=0 \\ \left(\dfrac{\partial u_1}{\partial y}+\dfrac{\partial v_1}{\partial x}\right)^{(1)} = \dfrac{1}{rm^2h^2}\left(\dfrac{\partial u_1}{\partial y}+\dfrac{\partial v_1}{\partial x}\right)^{(2)} & \text{at} \quad y=0 \\ \left(\dfrac{\partial T_1}{\partial y}+\dfrac{\partial T_1}{\partial x}\right)^{(1)} = \dfrac{k}{h}\left(\dfrac{\partial T_1}{\partial y}+\dfrac{\partial T_1}{\partial x}\right)^{(2)} & \text{at} \quad y=0 \\ \left(u_0\dfrac{\partial u_1}{\partial y}+v_1\dfrac{\partial u_0}{\partial x}-\nabla^2 u_1 - GT_1\right)^{(1)} = \left(u_0\dfrac{\partial u_1}{\partial x}+v_1\dfrac{\partial u_0}{\partial y}-\nabla^2 u_1 - G\beta m^2 r^2 h^3 T_1\right)^{(2)} & \text{at } y=0 \\ \left(u_0\dfrac{\partial u_1}{\partial x}-\nabla^2 v_1\right)^{(1)} = \left(u_0\dfrac{\partial v_1}{\partial x}-\nabla^2 v_1\right)^{(2)} & \text{at} \quad y=0 \end{array}\right\} (36)$$

Now introducing stream function ψ defined by

$$u^{(i)} = -\frac{\partial \psi^{(i)}}{\partial y} \quad \text{and} \quad v^{(i)} = \frac{\partial \psi^{(i)}}{\partial x} \qquad \text{where i = 1, 2}$$

Substitute in to equations (29)-(34), and (35, 36), and eliminating the non dimensional pressure p, we get,

Region-I

$$\left(u_0(\psi_1)_{xyy} - u_0''(\psi_1)_x + u_0(\psi_1)_{xxx} - (\psi_1)_{yyyy} - 2(\psi_1)_{xxyy} - (\psi_1)_{xxxx} + G(T_1)_y\right)^{(1)} = 0 \quad (37)$$

$$\left(p_r\left(u_0\frac{\partial T_1}{\partial x}+v_1\frac{\partial T_0}{\partial y}\right) = \left(\frac{\partial^2 T_1}{\partial x^2}+\frac{\partial^2 T_1}{\partial y^2}\right)\right)^{(1)} \qquad (38)$$

Region-II

$$\left(u_0(\psi_1)_{xyy} - u_0''(\psi_1)_x + u_0(\psi_1)_{xxx} - (\psi_1)_{yyyy} - 2(\psi_1)_{xxyy} - (\psi_1)_{xxxx} + G\beta m^2 r^2 h^3 (T_1)_y\right)^{(2)} = 0 \quad (39)$$

$$\left(\left(u_0 \frac{\partial T_1}{\partial x} + v_1 \frac{\partial T_0}{\partial y}\right) = \frac{km}{P_r}\left(\frac{\partial^2 T_1}{\partial x^2} + \frac{\partial^2 T_1}{\partial y^2}\right)\right)^{(2)} \quad (40)$$

Where the subscripts denote partial differentiation. The boundary conditions are written in terms of ψ as

$$\left.\begin{aligned} &\psi_y = \psi_x = 0,\ T=0, \quad \text{at} \quad y = 1+\cos\lambda x \\ &\psi_y = \psi_x = 0,\ T=1, \quad \text{at} \quad y = 1+\cos(\lambda x+\theta) \end{aligned}\right\} \quad (41)$$

Solution of the First Order Equation

We next solve the problem by the method of perturbation. We assume that the solution is made up of a mean and a perturbed part, so that the velocity and temperature distribution taken the forms, respectively.

$$\left.\begin{aligned} &\psi(x,y,t) = \psi_0(y) + \psi_1(x,y,t), \\ &T(x,y,t) = T_0(y) + T_1(x,y,t), \quad \text{where,} \\ &\psi_1^{(i)}(x,y) = \varepsilon \exp[i(\lambda x)]\bar{\psi}_1^{(i)}(y), \\ &T_1^{(i)}(x,y) = \varepsilon \exp[i(\lambda x)]\bar{T}_1^{(i)}(y), \end{aligned}\right\}$$

Where i = 1, 2

and the perturbed quantities ψ_1 and T_1 are small compared to their mean quantities ψ_0 and T_0, respectively.

With the help of equations (37) - (40) and (41) and the conditions (42) and yield,

$$\left(\psi_0^{iv} - GT_0'\right)^{(1)} = 0 \quad (43)$$

$$\left(T_0''\right)^{(1)} = 0 \quad (44)$$

$$\left(\psi_0^{iv} - G\beta m^2 r^2 h^3 T_0'\right)^{(2)} = 0 \quad (45)$$

$$\left(T_0''\right)^{(2)} = 0 \quad (46)$$

$$\left.\begin{aligned} &\psi_0'=0, \quad \psi_0=0, \quad T_0=0, \quad \text{at } y=1 \\ &\psi_0'=0, \quad \psi_0=0, \quad T_0=1, \quad \text{at } y=-1 \end{aligned}\right\} \tag{47}$$

to the zeroth order, and

$$\left(\psi_1^{iv}-\psi_1^{ii}\left(iu_0\lambda+2\lambda^2\right)+\psi_1\left(u_0^{ii}i\lambda-u_0 i\lambda^3+\lambda^4\right)+GT_1^1\right)^{(1)}=0 \tag{48}$$

$$\left(T_1''-T_1\left(p_r u_0 i\lambda+\lambda^2\right)\right)^{(1)}=\left(ip_r\lambda T_0'\psi_1\right)^{(1)} \tag{49}$$

$$\left(\psi_1^{iv}-\psi_1^{ii}\left(iu_0\lambda+2\lambda^2\right)+\psi_1\left(u_0^{ii}i\lambda-u_0 i\lambda^3+\lambda^4\right)+G\beta m^2r^2h^3T_1^1\right)^{(2)}=0 \tag{50}$$

$$\left(T_1''-T_1\left(\frac{p_r}{km}u_0 i\lambda+\lambda^2\right)\right)^{(2)}=\left(\frac{p_r}{km}i\lambda T_0'\psi_1\right)^{(2)} \tag{51}$$

$$\frac{\partial\psi_1}{\partial y}(1)=\cos\lambda x\frac{\partial u_0}{\partial y}(1), \quad \bar{\psi}_1=0, \quad T_1^{(1)}=-\cos\lambda x\left(\frac{\partial T_0}{\partial y}\right)^{(1)}, \quad \text{at } y=1, \tag{52}$$

$$\frac{\partial\psi_1}{\partial y}(-1)=\frac{1}{h}\cos\left(\lambda x+\theta\right)\frac{\partial u_0}{\partial y}(-1) \quad , \quad \bar{\psi}_1=0, \quad T_1^{(1)}=-\frac{1}{h}\cos\left(\lambda x+\theta\right)\left(\frac{\partial T_0}{\partial y}\right)^{(1)} \tag{53}$$

at y = -1, to the first-order, where a prime denotes differentiation with respect to y.

We consider small values of λ and then expand ψ_1 and T_1 as

$$\bar{\psi}_1(\lambda,y)=\sum_{r=0}^{\infty}\lambda^r\bar{\psi}_{1r} \quad , \quad \bar{T}_1(\lambda,y)=\sum_{r=0}^{\infty}\lambda^r\bar{T}_{1r}\,. \tag{54}$$

Substituting (54) into (48) - (53), we obtain the following sets of ordinary differential equations and the boundary conditions, to the order of λ^2.

$$\left(\psi_{10}{}^{iv}-GT_{10}'\right)^{(1)}=0 \tag{55}$$

$$\left(T_{10}{}''\right)^{(1)}=0 \tag{56}$$

$$\left(\frac{d^2T_{11}}{dy^2}-iu_0p_rT_{10}-ip_r\psi_{10}T_0'\right)^{(1)}=0 \tag{57}$$

$$\left(\psi_{10}{}^{iv}-G\beta r^2m^2h^3T_{10}'\right)^{(2)}=0 \tag{58}$$

$$\left(T_{10}''\right)^{(2)} = 0 \qquad (59)$$

$$\left(\frac{d^4\psi_{11}}{dy^4} - iu_0\psi_{10}'' + iu_0''\psi_{10} + G\beta m^2 r^2 h^3 T_{11}'\right)^{(2)} = 0 \qquad (60)$$

$$\left(\frac{d^2T_{11}}{dy^2} - iu_0\frac{p_r}{km}T_{10} - i\frac{p_r}{km}\psi_{10}T_0'\right)^{(2)} = 0 \qquad (61)$$

and

$$\left.\begin{aligned}&\frac{\partial\psi_{10}^{(1)}}{\partial y}(1) = -\cos\lambda x\frac{\partial u_0}{\partial y}(1),\ \psi_{10}^{(1)}(1) = 0,\ \psi_{11}'^{(1)}(1) = 0,\ \psi_{11}^{(1)}(1) = 0\ \ T_{10}^{(1)} = -\cos\lambda x\left(\frac{\partial T_0}{\partial y}\right)^{(1)},\\ &T_{11}^{(1)}(1) = 0 \qquad \text{at } y=1\end{aligned}\right\} \qquad (62)$$

$$\left.\begin{aligned}&\frac{\partial\psi_{10}^{(2)}}{\partial y}(-1) = -\frac{1}{h}\cos(\lambda x+\theta)\frac{\partial u_0}{\partial y}(-1),\ \psi_{10}^{(2)}(-1) = 0\ ,\psi_{11}'^{(2)}(-1) = 0,\ \psi_{11}^{(2)}(-1) = 0\\ &T_{10}^{(2)} = -\left(\frac{\cos}{h}\right)(\lambda x+\theta)\left(\frac{\partial T_0}{\partial y}\right)^{(2)},\ T_{11}^{(2)}(-1) = 0 \quad \text{at } y=-1\end{aligned}\right\} \qquad (63)$$

$$\left.\begin{aligned}&\frac{\partial\psi_{10}^{(1)}}{\partial y} = \frac{1}{rmh}\left(\frac{\partial\psi_{10}}{\partial y}\right)^{(2)},\ \psi_{10}^{(1)} = \frac{1}{rmh}\psi_{10}^{(2)},\ \left(\frac{-\partial^2\psi_{10}}{\partial y^2}\right)^{(1)} = \frac{1}{rm^2h^2}\left(-\frac{\partial^2\psi_{10}}{\partial y^2}\right)^{(2)}\\ &\left(\frac{-\partial^3\psi_{10}}{\partial y^3} - GT_{10}\right)^{(1)} = \left(\frac{\partial^3\psi_{10}}{\partial y^3} - G\beta m^2r^2h^3\right)^{(2)},\quad \left(\frac{-\partial^3\psi_{10}}{\partial y^3} - GT_{10}\right)^{(1)} = \left(\frac{\partial^3\psi_{10}}{\partial y^3} - G\beta m^2r^2h^3\right)^{(2)}\\ &T_{10}^{(1)} = T_{10}^{(2)},\ \frac{dT_{10}^{(1)}}{dy} = \frac{k}{h}\frac{dT_{10}^{(2)}}{dy},\ \psi_{11}^{(1)} = \frac{1}{rmh}\psi_{11}^{(2)},\\ &\left(\frac{-\partial^2\psi_{11}}{\partial y^2}\right)^{(1)} = \frac{1}{rm^2h^2}\left(-\frac{\partial^2\psi_{11}}{\partial y^2}\right)^{(2)},\ T_{11}^{(1)} = T_{11}^{(2)}\\ &\left(-iu_0\lambda\frac{d\psi_{10}}{dy} + iu_0'\psi_{10} + \frac{d^3\psi_{11}}{dy^3} - GT_{11}\right)^{(1)} = \left(-iu_0\lambda\frac{d\psi_{10}}{dy} + iu_0'\psi_{10} + \frac{d^3\psi_{11}}{dy^3} - G\beta m^2r^2h^3T_{11}\right)^{(2)}\\ &\qquad\qquad\qquad\qquad\qquad\qquad \text{at } y=0\end{aligned}\right\} \qquad (64)$$

4.1. The zeroth-order solution (mean part)

The solution for zeroth-order stream function u_0 and the zeroth- order temperature T_0 satisfying the ordinary differential equation (43) - (47) are obtained as

$$u_0^{(1)} = l_1 y^3 + l_2 y^2 + d_1 y + d_2 \tag{65}$$

$$T_0^{(1)} = c_1 y + c_2 \tag{66}$$

$$u_0^{(2)} = l_3 y^3 + l_4 y^2 + d_3 y + d_4 \tag{67}$$

$$T_0^{(2)} = c_3 y + c_4 \tag{68}$$

The expression for ψ_0 and T_0 for various values of y are numerically evaluated for several sets of the parameters G, m

The set of equations (65) - (68), subject to the conditions (62) - (64) are solved exactly for ψ_{10} and T_{10}.They are.

$$\psi_{10}^{(1)} = l_5 y^4 + \frac{d_5}{6} y^3 + \frac{d_6}{2} y^2 + d_7 y + d_8 \tag{69}$$

$$T_{10}^{(1)} = c_5 y + c_6 \tag{70}$$

$$\psi_{10}^{(2)} = l_6 y^4 + \frac{d_9}{6} y^3 + \frac{d_{10}}{2} y^2 + d_{11} y + d_{12} \tag{71}$$

$$T_{10}^{(2)} = c_7 y + c_8 \tag{72}$$

4.2. The first-order solution (Perturbed part)

The set of equations (69) - (72), subject to the conditions (62) - (64) are solved exactly ψ_{1r} and $t_{1r}\left(r=1,2\right)$ and the solutions are not presented here for brevity.

The equations

$$\bar{\psi}_1 = \sum_{r=0}^{2} \lambda^r \bar{\psi}_{1r} \quad , \qquad \bar{T}_1 = \sum_{r=0}^{2} \lambda^r \bar{T}_{1r} \tag{73}$$

used along with (42) to calculate the expression for the perturbed quantities ψ_1 and T_1 which after simplification take the forms,

$$\psi_1\left(x,y,t\right) = \varepsilon\left[\bar{\psi}_{1r}\left(y\right)\cos\left(\lambda x\right) - \bar{\psi}_{1i}\left(y\right)\sin\left(\lambda x+\theta\right)\right], \tag{74}$$

$$T_1\left(x,y,t\right) = \varepsilon\left[\bar{T}_{1r}\left(y\right)\cos\left(\lambda x\right) - \bar{T}_{1i}\left(y\right)\sin\left(\lambda x+\theta\right)\right], \tag{75}$$

$$\bar{\psi}_1 = \bar{\psi}_{1r} + i\bar{\psi}_{1i} \qquad \text{and} \qquad \bar{T}_1 = \bar{T}_{1r} + i\bar{T}_{1i}. \tag{76}$$

The solution takes the form for first order equations.

$$\psi_{11}^{(1)} = m_7 y^9 + m_8 y^8 + m_9 y^7 + m_{10} y^6 + m_{11} y^5 + m_{12} y^4 + \frac{d_{13}}{6} y^3 + \frac{d_{14}}{2} y^2 + d_{15} y + d_{16} \tag{77}$$

$$T_{11}^{(1)} = e_1 y^6 + e_2 y^5 + e_3 y^4 + e_4 y^3 + e_5 y^2 + c_9 y + c_{10} \tag{78}$$

$$\psi_{11}^{(2)} = S_7 y^9 + S_8 y^8 + S_9 y^7 + S_{10} y^6 + S_{11} y^5 + S_{12} y^4 + \frac{d_{17}}{6} y^3 + \frac{d_{18}}{2} y^2 + d_{19} y + d_{20} \tag{79}$$

$$T_{11}^{(2)} = f_1 y^6 + f_2 y^5 + f_3 y^4 + f_4 y^3 + f_5 y^2 + c_{11} y + c_{12} \tag{80}$$

Expression for $\psi_{11}^{(1)}$, $\psi_{11}^{(2)}$, $T_{11}^{(1)}$, and $T_{11}^{(2)}$ are called first-order solution or the disturbed (or perturbed) part. In the similar way, the total stream function ψ and the total temperature T are obtained, but the sake of brevity, they are not presented here. For several sets of values if the non-dimensional parameters G, M, Pr, ε , λ , and θ .

From these solutions the first order quantities can be put in the forms,

$$\left.\begin{aligned} u_1 &= \psi_i' \sin \lambda x - \psi_r' \cos \lambda x, \\ v_1 &= -\lambda \psi_r \sin \lambda x - \lambda \psi_i \cos \lambda x, \\ T_1 &= T_r \cos \lambda x - T_i \sin \lambda x , \end{aligned}\right\} \tag{81}$$

Where

$$\psi = \psi_r + i\psi_i = \sum_{i=0}^{2} \lambda^i \psi_i \ ,$$

$$T = T_r + iT_i = \sum_{i=0}^{2} \lambda^i \psi_{i,} \ ,$$

We obtain the expression for real part of u_1 , v_1 and T_1 by using (77) - (80) and (81),

Skin Friction and Nusselt Number

The shearing stress τ_{xy} at any point in the fluid is given, in non-dimensional form, by

$$\tau_{xy} = \left(\frac{d^2}{\rho \upsilon^2} \right) \bar{\tau}_{xy} =$$

$$= u_0'(y) + \varepsilon e^{i\lambda x} u_1'(y) + i\varepsilon\lambda e^{i\lambda x} \bar{v}_1(y),$$

τ_{xy} at the wavy wall $(y = \varepsilon \cos \lambda x), (y = \varepsilon \cos(\lambda x + \theta))$ and at the flat wall y = 1 y = -1, are given by

$$\tau_w^{(1)} = \tau_0^{(1)} + \varepsilon \operatorname{Re}\left[e^{i\lambda x} \left(u_{10}'''^{(1)}(1) + u_{11}'^{(1)}(1) \right) \right],$$

$$\tau_w^{(2)} = \tau_0^{(2)} + \varepsilon \operatorname{Re}\left[e^{i\lambda x} \left(u_{10}'''^{(2)}(-1) + u_{11}'^{(2)}(-1) \right) \right],$$

Where Re represent the real part of

$$\tau_0^{(1)} = c_1 ,$$

$$\tau_w^{(1)} = 3l_1 + 2l_2 + d_1 + \varepsilon\left[\begin{array}{l}\cos\lambda x 6l_1 + \cos\lambda x 2l_2 + \\ \varepsilon\left(\cos\lambda x\cdot\lambda\sin\lambda x\left(72m_7 + 56m_8 + 42m_9 + 30m_{10} + 20m_{11} + 12m_{12} + d_{13} + d_{14}\right)\right) \\ +\cos\lambda x\cdot\lambda\cos\lambda x\left(-d_{13} - d_{14}\right) + \cos\lambda x\cdot\cos\lambda x\left(-12l_5 - d_5 - d_6\right)\end{array}\right]$$

$$Nu_w^{(1)} = c_1 + \varepsilon\left[\begin{array}{l}\left(\cos\lambda x\cdot -\varepsilon\lambda\sin\lambda x\cdot c_{9r}\right) - \left(\cos\lambda x\cdot\varepsilon\lambda\cos\lambda x\right) \\ \left(6e_1 + 5e_2 + 4e_3 + 3e_4 + 2e_5 + c_{9i}\right)\end{array}\right],$$

$$\tau_0^{(2)} = c_3,$$

$$\tau_w^{(2)} = 3l_1 - 2l_2 + d_1 + \varepsilon\left[\begin{array}{l}-\cos\left(\lambda x + \theta\right)6l_3 + \cos\left(\lambda x + \theta\right)4l_4 + \\ \varepsilon\left(\begin{array}{l}\cos\left(\lambda x + \theta\right)\cdot\lambda\sin\left(\lambda x + \theta\right) \\ \left(-72s_7 + 56s_8 - 42s_9 + 30s_{10} - 20s_{11} + 12s_{12} - d_{17} + d_{18}\right)\end{array}\right) \\ +\cos\left(\lambda x + \theta\right)\cdot\lambda\cos\left(\lambda x + \theta\right)\left(d_{17} - d_{18}\right) + \\ \cos\left(\lambda x + \theta\right)\cdot\cos\left(\lambda x + \theta\right)\left(-12l_6 + d_9 - d_{10}\right)\end{array}\right]$$

$$Nu_w^{(2)} = c_3 + \varepsilon\left[\begin{array}{l}\left(\cos\left(\lambda x + \theta\right)\cdot -\varepsilon\lambda\sin\left(\lambda x + \theta\right)\cdot c_{11r}\right) - \left(\cos\left(\lambda x + \theta\right)\cdot\varepsilon\lambda\cos\left(\lambda x + \theta\right)\right) \\ \left(-6f_1 + 5f_2 - 4f_3 + 3f_4 - 2f_5 + c_{11i}\right)\end{array}\right],$$

Results and Discussion

Analytical solutions for the stands mixed convection of two immiscible viscous fluids in a vertical wavy channels, the non linear equations are solved by linearization techniques where in the flow is assumed to be in two parts, a mean part and a perturbed part exact solutions are obtained for the mean part and the perturbed part is solved using long wave approximation.

Zeroth order solution

The solution of zeroth order velocity U_0 and the zeroth order temperature T_0 are applicable to the case of whose walls are flat, but with constant wall temperature. The solutions for the mean and perturbed parts are evaluated numerically and represented graphically for various governing parameters in Figs. (all graphs). In all graphs the parameters such as Prandtl number Grashof number, viscosities ratio, width ratio, conductivity ratio & thermal temperature are fixed as 0.7, 5, 1, 1, 1, & 0.785398 respectively for all graphs except for the varying one among them.

The behaviors of the non-dimensional zeroth order velocity with different Gr shown in (Fig. 2). It is seen that the zeroth order velocity U_0 (see Fig. (Gr on U_0 ,) with the increase in Gr.

The effect of viscosity ratio is similar as width ratio, that is m on zeroth order velocity U_0 is not significant in region I compared to region II whereas as velocity ratio increases in region II and decreases in region I (i.e., from $\left(-1.0 \le y \le 1.0\right)$. The effect of viscosity ratio (m) on U_{01} is observed in (fig. m on U01).

First order solution and Total solution

The behavior of the non-dimensional first order velocities with different G_r are shown in fig (Gr on u10 u11). It is shown that with the increases in Gr the first order velocity Ψ_{10} increases in region I $\left(y=-1 \le y \le 0\right)$ decreases in region II $\left(0 \ge y \ge 1.0\right)$ approximately.

The effect of increasing Grashof number Gr is to decrease the fluid motion for first order velocity of T_{11} as shown in Fig. (gr on t11). It is also observed that in region I the variation of Gr values is gradually increasing the value $\left(-0.25 \le y \le 1.0\right)$ approximately, hence velocity is reduced in region I when compared to region II.

For first order non-linear equation for temperature of T_{11} (Fig. on T_{11}) it up of parabolic form at r = -0.1 it will not effect to first order equation in region II and slight increment in region I (0 to 0.3) from the graphs it is seen that increasing the density it will decrease in region I and increases in region I for higher value (R = 3.0) if increases in region II then increases in region I.

The effect of Prandtl number $\left(c_p\, \mu/k\right)$ on first order velocity (Fig. P_r on ψ_{11}). T first order velocity ψ_{11} decreases while P_r increases from y = 0.044 to 7 (-1 to -10) approximately onwards seen in Fig. (p_r on ψ_{11}). The viscous velocity ψ_{11} increases in region I near the left wavy wall and increases in interface conditions as shown in Fig.The effect of Prandtl number on first order temperature steadily decrease in region II for increasing the Prandtl number and steadily increasing in region I $\left(y=-1.0 \le y \le 1.0\right)$. It depict the behavior of perturbed (first order solution) quantities. When r = h = m = k = 1 and when Prandtl number is 0.7 and 7 from this we observe that in presence of viscous fluid the velocity is decreasing steadily for a fixed y up to y = -0.55 approximately i.e., in the first half of the channel. While in the other half of the channels ψ_{11} is a increasing function of y we notice that when $\lambda > 0$ an increase in the frequency parameter λ is constant.

Total Solution

The behavior of viscosity ratio $m\left(=\mu^{(1)}/\mu^{(2)}\right)$ on the total solution (considering only real part and we ignored imaginary part) of U_1 and V_1 as shown in Fig. (U_{1r} and V_{1r} of m). As the viscosity ratio m increases on V_1 for higher value it is more effective in region II for lower value of viscosity i.e., (0.1, 1, 2) is a approximately equal. Similarly for region I there is no effect. It is a steady motion of the viscous fluid from (y = -1 to 1) approximately for higher value of the viscosity ratio steadily decrease (y = 25 to -0.5) in total solution of V_1 in region I and constant flow of viscous fluid in region I.The velocity profile is similar to those of the viscosity ratio when m up on small values is approximately equal flow in region I and region II but for the higher value of viscosity on U_1 i.e., (y = 3 and above) it is steadily increase in region II (y = -15 to 4) approximately and slight decrease in region I at (y is 0 to 1.0).The velocity profile for conductivity ratio $k\left(=k^{(2)}/k^{(1)}\right)$ from the Fig. (U_1 and V_1) It is shown that when the conductivity ratio k increases for total solution U_1 for smaller value it decrease from (-0.3 to 0.15) at y and for higher value it is significantly decrease from (0.4 to -0.5) on left wall (region II) similarly in region I steadily decreasing from higher value to smaller value for right wall (region I). Totally the conductivity ratio on U_{1r} is highly increases in region II and highly decreases in region I. Similarly, the conductivity for total solution V_1 it is totally different from U_1. In total solution of V_1 we find that in region I while increasing conductivity ratio the effect on region II is steadily decreasing for higher conductivity ratio $\left(0.2 \le y \le 0.0\right)$ and for the value k = 0.1 it is highly decreases i.e. (0.55 to 0.0 approximately) at the left wall (region II) but in right wall it constant flow of viscous fluid in region I (overlap on each other)

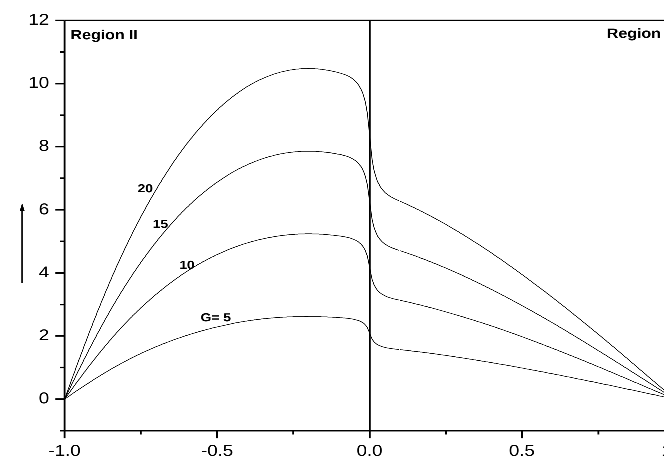

fig 2:Effect of Grashof number U_{01}

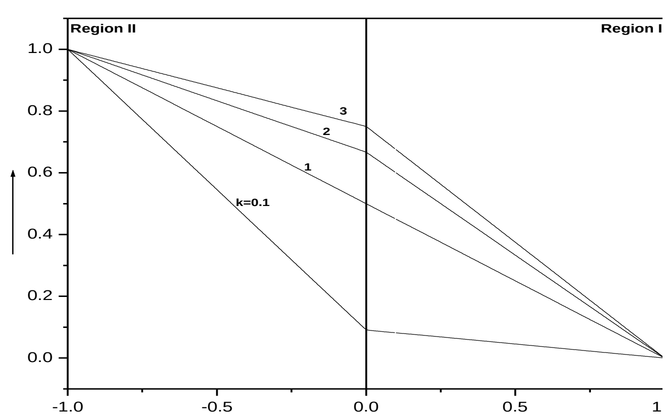

Fig 3 : Effect of Conductivity ratio on T_{01}

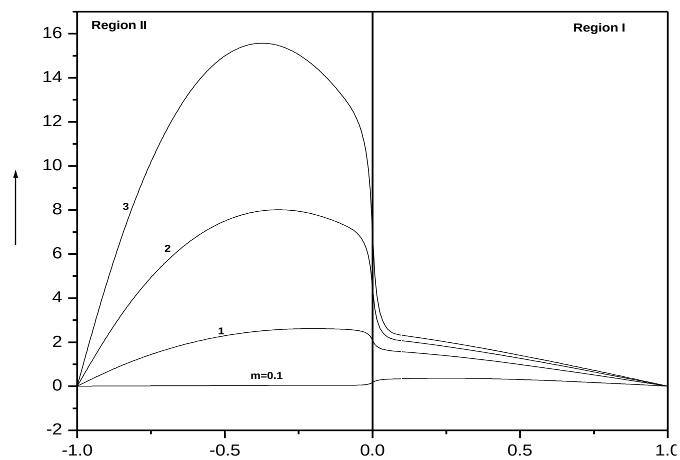

Fig 6 : Viscosity ratio m on U_{10}

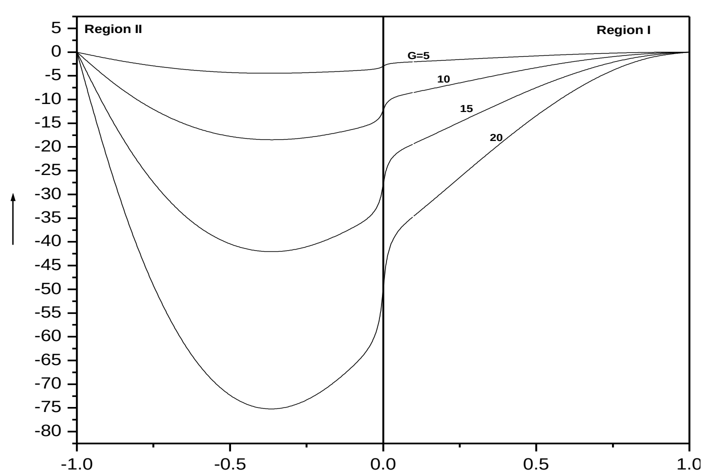

Fig 8 : Effect of Grashof number on Ψ_{11}

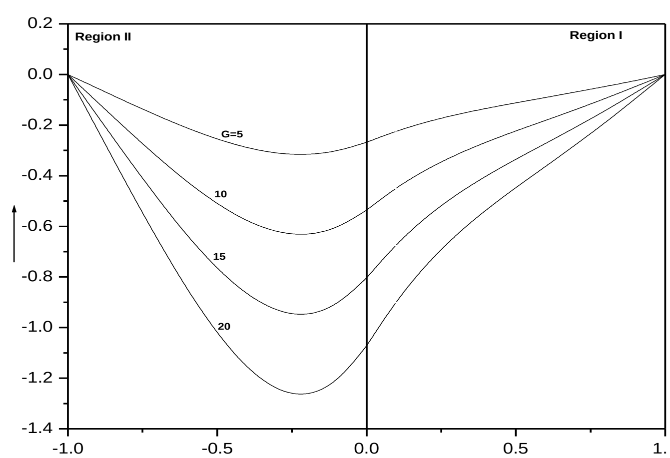

Fig 9 : Effect of Grashof number on T_{11}

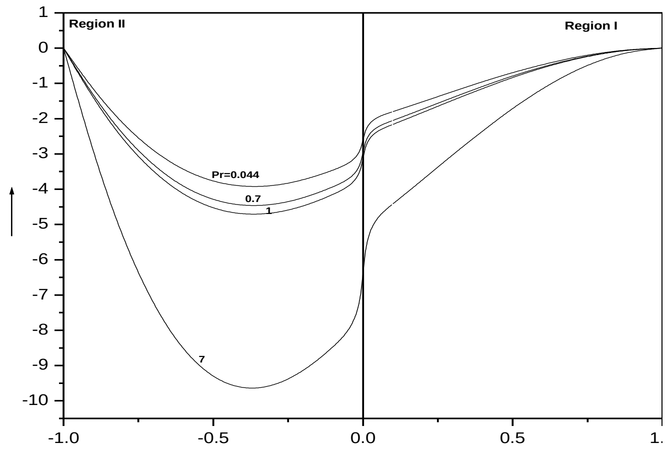

Fig 18 : Effect of P_r on ψ_{11}

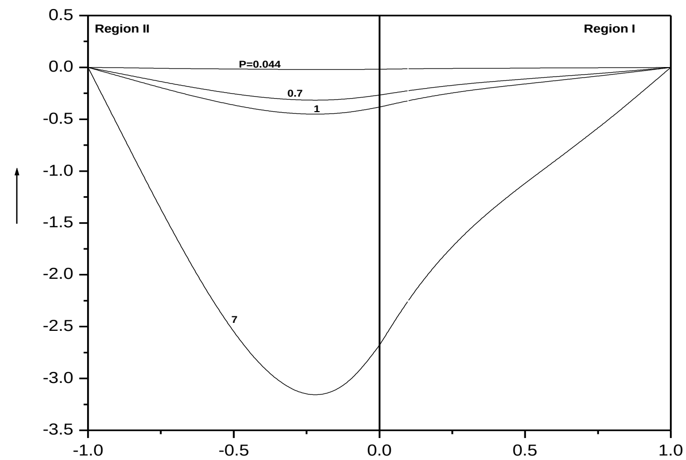

Fig 19 : Effect of P_r on T_{11}

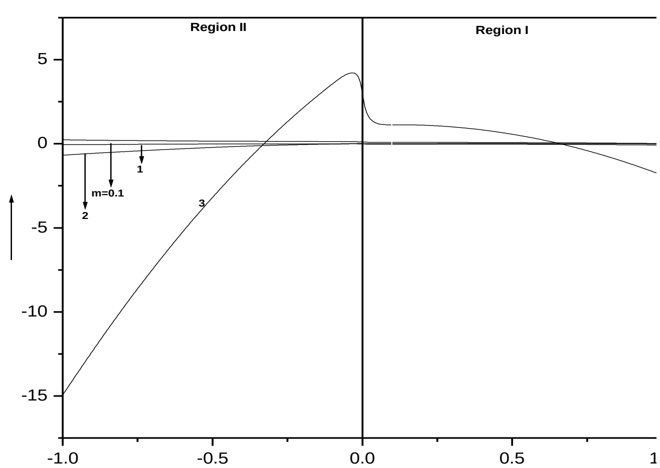

Fig 20 :Effect of Viscosity ratio on U1R

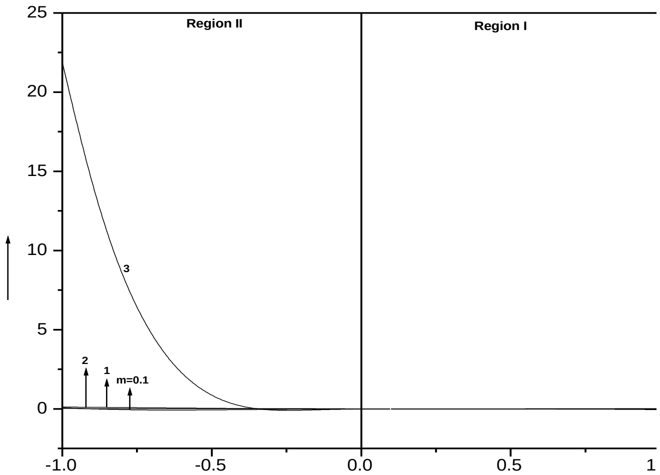

Fig 21 : Effect of Viscocity ratio on v1R

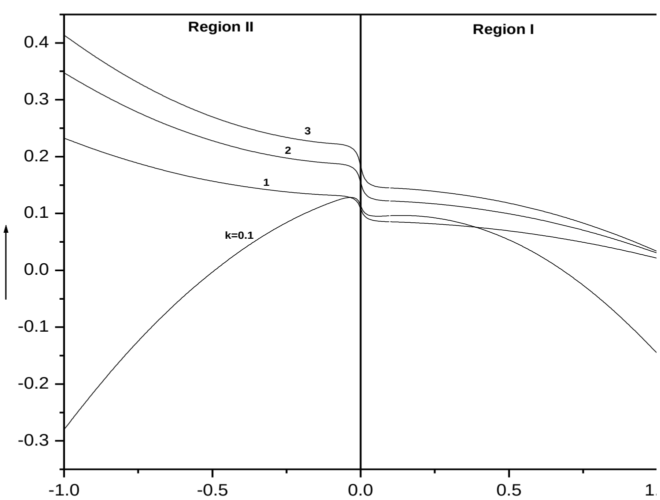

Fig 22 : Effect of Conductivity ratio on U1R

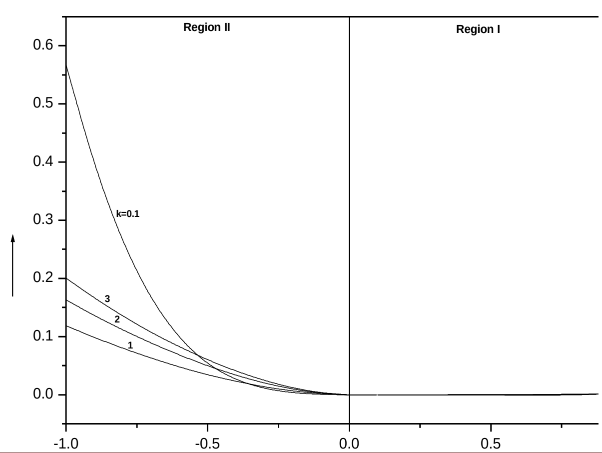

Fig 23 : Effect of Conductivity ratio on V1R

Table 1: G=5; k =m= Sig= $\alpha = \beta$ =0.1

y	T_0				T_{10}				T_{11}			
	h=0.1	h=1	h=2	h=3	h=0.1	h=1	h=2	h=3	h=0.1	h=1	h=2	h=3
+0.1	0	0	0	0	0.90909	0.5	0.33333	0.25	0	0	0	0
+0.5	0.45455	0.25	0.1667	0.125	0.4955	0.37489	0.27773	0.21872	-0.2059	-0.11157	0.20389	0.95106
0	0.90909	0.5	0.33333	0.25	0.08191	0.24978	0.22212	0.18744	-0.12884	-0.2678	0.3584	1.57223
0	0.90909	0.5	0.33333	0.25	0.08191	0.24978	0.22212	0.18744	-0.12884	-0.2678	0.3584	1.57223
-0.5	0.95455	0.75	0.66667	0.625	0.04055	0.12466	0.11091	0.09361	-0.06314	-0.25623	0.2863	1.3779
-1	1	1	1	1	-8.1E-4	-4.5E-4	-3E-4	-2.2E-4	0	0	0	0

Table 2: G=5; h=m=Sih= $\alpha = \beta$ =0.1

y	T_0				T_{10}				T_{11}			
	k=0.1	k=1	k=2	k=3	k=0.1	k=1	k=2	k=3	k=0.1	k=1	k=2	k=3
+0.1	1	0	0	0	0.90901	0.5	0.66667	0.75	0	0	0	0
+0.5	0.45455	0.25	0.33333	0.375	0.08674	0.37489	0.44435	0.46867	-0.18235	-0.11157	-0.02807	0.02598
0	0.90901	0.5	0.66667	0.75	0.08257	0.24978	0.22202	0.18733	-0.36603	-0.2678	-0.15655	-0.89
0	0.90901	0.5	0.66667	0.75	0.08257	0.24978	0.22202	0.18733	-0.36603	-0.2678	-0.15655	-0.89
-0.5	0.54545	0.75	0.83333	0.875	0.04088	0.12466	0.11086	0.09355	-0.94814	-0.25623	-0.13679	-0.0789
-1	1	1	1	1	-8.1E-4	-4.5E-4	-3E-4	-2.2E-4	0	0	0	0

Table 3: G=5; h=m=Sih= $\alpha = \beta$ =0.1

y	U_0				ψ_{10}				ψ_{11}			
	h=0.1	h=1	h=2	h=3	h=0.1	h=1	h=2	h=3	h=0.1	h=1	h=2	h=3
+0.1	0	0	0	0	0	0	0	0	0	0	0	0
+0.5	0.36897	098958	1.67824	2.34375	-1.56751	1.02437	2.57713	10.79034	0.96626	-0.78573	5.00905	37.02087
0	0.16977	1.66667	3.14815	4.53125	-3.68069	4.12224	0.46475	6.48175	3.2493	-2.35732	14.8756	111.4449
0	0.02546	2.5	9.44444	20.39063	0.48646	-6.5552	11.62652	38.21303	0.4874	-3.53598	44.62682	501.5021
-0.5	0.01407	2.30469	12.22222	33.92578	0.15302	-2.00286	3.42982	11.6906	0.71901	-4.32143	53.28053	600.5313
-1	0	0	-1E-5	-2E-5	0	0	0	0	0	0	-2E-5	-2.4E-4

Table 4: k=h=m=Sih= $\alpha = \beta$ =0.1

y	U_0				ψ_{10}				ψ_{11}			
	G=5	G=10	G=15	G=20	G=5	G=10	G=15	G=20	G=5	G=10	G=15	G=20
+0.1	0	0	0	0	0	0	0	0	0	0	0	0
+0.5	0.98958	1.97917	2.96875	3.95833	1.02437	2.04875	3.07312	4.097494	-0.78573	-3.26758	-7.44555	-13.31965
0	1.66667	333333	5	6.66667	4.12224	8.24449	12.36673	16.48898	-2.35732	-9.7721	-22.24435	-39.77407
0	2.5	5	7.5	10	-6.5552	-13.11041	-19.66561	-26.22082	-3.53598	-14.65816	-33.36654	-59.66111
-0.5	2.30469	4.60937	6.91406	9.21875	-2.00286	-4.00573	-6.00859	-8.01145	-4.32143	-17.91499	-40.78067	-72.91848
-1	0	0	0	0	0	0	0	0	0	1E-5	1E-5	3E-5

Table 5: G=5:=m=Sih= $\alpha = \beta$ =0.1

y	U_{1r}				V_{1r}			
	h=0.1	h=1	h=2	h=3	h=0.1	h =1	h=2	h =3
+0.1	-0.11607	0.02125	-0.78987	-54.76411	0.01588	0.00211	-0.0531	-0.25082
+0.5	-0.02614	0.0696	0.19945	15.29557	0.00137	-4E-5	-0.00456	-0.02286
0	-0.00599	0.08678	0.36743	30.4341	0	0	0	0
0	-9E-4	0.13016	1.10229	136.9534	0	0	0	0
-0.5	-0.02296	0.15653	-1.26376	-62.1159	0.0079	0.03418	-0.0916	37.35529
-1	-0.08882	0.23245	-5.42186	0	0.03161	0.11859	4.22099	6661.1129

Table 6: G=5:=m=Sih= $\alpha = \beta$ =0.1

y	U_{1r}				V_{1r}			
	K=0.1	K=1	K=2	k=3	K=0.1	h =1	K=2	K=3
+0.1	-0.14596	0.02125	0.03022	0.03345	0.00182	0.00211	0.00264	0.00309
+0.5	0.05471	0.0696	0.09979	0.11848	1.7E-4	-4E-5	-3.4E-4	-5.8E-4
0	0.09114	0.08678	0.12374	0.14677	0	0	0	0
0	0.13672	0.13016	0.18561	0.22015	0	0	0	0
-0.5	-0.00208	0.15653	0.22726	0.26934	0.05162	0.03418	0.04954	0.05982
-1	-0.27995	0.23245	0.34743	0.41363	0.5671	0.11859	0.16327	0.20052

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