STOCHASTIC NON-NEWTONIAN LUBRICATION WITH THERMAL EFFECT

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ABSTRACT: In this paper a Generalized Reynolds equation for power-law lubricant is derived and is applied to the squeeze film of parallel plates using stochastic theory. The roughness is taken to follow normal distribution. The lubricant consistency is assumed to vary due to the thermal effects. q is introduced for this purpose. The expected values of load capacity and time of squeezing for parallel plates are obtained analytically for both transversal and longitudinal roughness surface. It is deduced for the transversal roughness. These parameters increase as the standard deviation parameter increases. The effect of q is to decrease the expected value of load capacity and squeezing time.

Key words: Flow behavior index, Thermal effect, Standard deviation

1 INTRODUCTION

When we study in many papers the characteristics of power-law lubricants in various lubricated systems, it has been assumed that the bearing surfaces are smooth. However, in reality bearing surfaces would always have some roughness and it is very natural to study such effects on various bearing characteristics. Several attempts have been made to study such effects in bearing systems by both deterministic and stochastic approach.

In the deterministic approach the profile or roughness asperities are represented by a given shape function such as sine or cosine wave and thus modifying the film thickness in the usual study of the bearing characteristics. Using this approach, some studies have been conducted. It has been pointed out that the load capacity, frictional forces etc. are different from the corresponding case of smooth surfaces and they mainly depend upon the amplitude and the wave lengths of the representing the roughness surfaces. Recently a new deterministic theory has been proposed by Shukla when the mean height of the asperities is of the same order as the minimum film thickness.

In another method, known as stochastic approach, the film thickness is assumes to be a stochastic or random function and hence the corresponding Reynolds equation becomes a stochastic differential equation. To study various characteristics, this equation is solved by taking the mean or the average of the stochastic variables involved. This concept has been used by Tzeng and Saibel to study the effect of surface roughness in an infinite slider bearings and short journal bearings. Later Christensen and Tonder derived generalized form of Reynolds equation for stochastic lubrication. Further refinement of this theory has been given by Christensen, Shukla and Kumar. Since then several models including the effects of viscosity variation have been proposed and their applicability to various bearing systems have been investigated.

It may be noted that in the above mentioned studies, the lubricant is characterized by a Newtonian model. As pointed our earlier when additives of higher concentration are added to the base lubricant, it becomes Non-Newtonian and the simplest of these is the power-law model which describes the characteristics of several polymer solutions and silicone fluids. In the last two dacades several attempts have been made to study the behavior of power-law lubricant in different bearing systems, but no attempt has been made to study the effects of surface roughness on bearing characteristics using such lubricants.

Keeping the above in view, in this chapter the generalized form of one dimensional stochastic form of Reynolds equation has been derived following Christensen, for power-law lubricant applicable for two symmetric rough surfaces both in the case of longitudinal and transversal roughness. The case of squeeze film between two parallel plates have been discussed. It may be noted that the analysis presented here is valid when the mean height of the asperities is much smaller than the minimum film thickness.

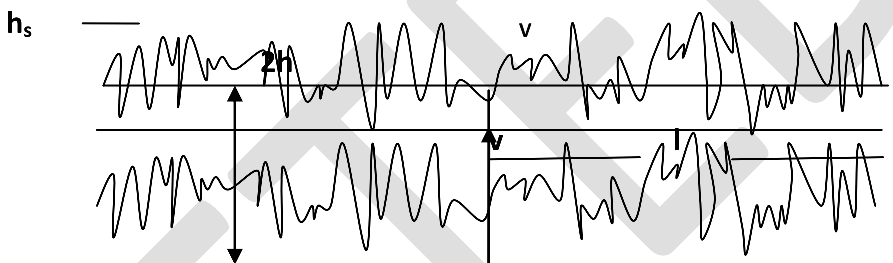

Fig (1): Squeezing between two rough parallel plates

2 BASIC EQUATION

Consider the flow of a power-law lubricant with constant consistency between two symmetrically rolling and squeezing rough surfaces. The equation governing the pressure in the film is given from equations

$$\frac{d}{dx}\left[\frac{n}{2n+1} H^{\frac{2n+1}{n}} \left(-\frac{1}{m}\frac{dp}{dx}\right)^{1/n}\right] = -\frac{dH}{dt} - U\frac{dH}{dx} \tag{1}$$

In the region $-X_1 \le X \le X_2$, $y > 0$, $\frac{dp}{dx} \le 0$ and

$$\frac{d}{dx}\left[\frac{n}{2n+1} H^{\frac{2n+1}{n}} \left(\frac{1}{m}\frac{dp}{dx}\right)^{1/n}\right] = \frac{dH}{dt} + U\frac{dH}{dx} \tag{2}$$

In the region $- \le X \le X_1$, $y > 0$, $\frac{dp}{dx} \ge 0$

where 2H is the total stochastic film thickness , U,V are the rolling and squeezing velocities respectively. Since, the film thickness 2H is a stochastic variables, equations (1) and (2) are stochastic differential equations for pressure.

Taking the stochastic mean of equation (1) and (2) we get,

$$\frac{d}{dx}\left[\frac{n}{2n+1} E\left\{H^{\frac{2n+1}{n}} \left(-\frac{1}{m}\frac{dp}{dx}\right)^{1/n}\right\}\right] = -E\frac{dH}{dt} - UE\frac{dH}{dx} \tag{3}$$

For $\frac{dp}{dx} < 0$ and

$$\frac{d}{dx}\left[\frac{n}{2n+1}\, E\left\{H^{\frac{2n+1}{n}}\left(-\frac{1}{m}\frac{dp}{dx}\right)^{1/n}\right\}\right] = \mathrm{E}\,\frac{dH}{dt} + UE\,\frac{dH}{dx} \tag{4}$$

For $\frac{dp}{dx} > 0$

Where

$$\mathrm{E(S)} = \int_{-\infty}^{\infty} sf(s)\, ds \tag{5}$$

and f(s) , is the distribution of the stochastic variable s. The film thickness function H is given by

$$H = h(x,z) + h_s(x,z,\xi) \tag{6}$$

Where 2h is the nominal film thickness and $2h_s$ is the part of the film thickness due to surface roughness measured from the nominal level . It is assumed that h_s is a function of the random variable ξ , the mean value of which over the bearing surface is zero.

To evaluate the terms on the left hand side of equations (3) and (4) we apply the same postulates as proposed by Christensen i.e.,

(i) Let S_1 be the direction parallel to the roughness and S_2 the direction perpendicular to it. The pressure gradient in the roughness direction is assumed to be a stochastic variable with zero or negligible variance.

(ii) In the direction perpendicular to the direction of roughness the flux is assumed to have zero or negligible variance.

Using the above two postulates the Reynolds equations in the following cases are derived.

2.1 LONGITUDINAL ONE-DIMENSIONAL ROUGHNESS

In this type of roughness the narrow ridges and valleys of the asperities run parallel to the direction of motion . The film thickness in this case is given by

$$H = h(x,z,t) + h_s(z, \xi) \tag{7}$$

Since by first postulate, $\frac{dp}{dx}$ has zero or negligible variance $\left(\frac{dp}{dx}\right)^{1/n}$ is also a stochastic variable having zero or negligible variance and

$$\mathrm{E}\left[\left(\frac{dp}{dx}\right)^{1/n}\right] = \left[E\left(\frac{dp}{dx}\right)\right]^{1/n} \tag{8}$$

Since $\left(\frac{dp}{dx}\right)^{1/n}$, $H^{\frac{2n+1}{n}}$ are stochastically independent, we get from equation (3) and (8)

$$\frac{d}{dx}\left[\frac{n}{2n+1}\, E\left\{H^{\frac{2n+1}{n}}\left(\frac{d\bar{p}}{dx}\right)^{1/n}\right\}\right] = -\frac{dh}{dt} - U\,\frac{dh}{dx} \tag{9}$$

Where $\bar{P} = E(P)$, E(H) = h.

Similarly for the region $\frac{dp}{dx} > 0$ we get the corresponding Reynolds equation for the longitudinal roughness which can be written from equation (4) as follows

$$\frac{d}{dx}\left[\frac{n}{2n+1}\, E\left\{H^{\frac{2n+1}{n}}\left(\frac{1}{m}\frac{d\bar{p}}{dx}\right)^{1/n}\right\}\right] = \frac{dh}{dt} + U\,\frac{dh}{dx} \tag{10}$$

Equations (9) and (10) are generalized one dimensional form of Reynolds equation applicable for two symmetric rough rollers having longitudinal roughness using power-law lubricant.

2.2 TRANSVERSE ONE-DIMENSIONAL ROUGHNESS

In this type of roughness, it is noted that the narrow ridges and valleys run perpendicular to the direction of motion. The film thickness function in this case is given by,

$$H = h(x,z,t) + h_s(x, \xi)$$

To evaluate the term on the left hand side of equation (3) let us define the flux as follows:

$$q = UH + \frac{n}{2n+1}\, H^{\frac{2n+1}{n}} \left\{ \left(-\frac{1}{m}\,\frac{d\bar{p}}{dx}\right)^{1/n} \right\} \tag{11}$$

which can also be written as

$$\frac{q}{H^{\frac{2n+1}{n}}} = \frac{U}{H^{\frac{n+1}{n}}} + \frac{n}{2n+1}\left(-\frac{1}{m}\,\frac{dp}{dx}\right)^{1/n} \tag{12}$$

Using postulate (ii) and taking the expectation of equation (12) we get,

$$E(q)\, E\left(\frac{1}{H^{\frac{2n+1}{n}}}\right) = U\, E\left(\frac{1}{H^{\frac{n+1}{n}}}\right) + \frac{n}{2n+1}\, E\left(-\frac{1}{m}\,\frac{dp}{dx}\right)^{1/n} \tag{13}$$

Taking again expectation of equation (11) and using equation (13) we get,

$$E\left[UH + \frac{n}{2n+1}\, H^{2n+1}\left(-\frac{1}{m}\,\frac{dp}{dx}\right)^{1/n}\right] = \frac{UE\left(\frac{1}{H^{\frac{n+1}{n}}}\right)}{E\left(\frac{1}{H^{\frac{2n+1}{n}}}\right)} + \frac{n}{2n+1}\,\frac{E\left(-\frac{dp}{dx}\right)^{1/n}}{E\left(\frac{1}{H^{\frac{2n+1}{n}}}\right)} \tag{14}$$

Which on substituting in equation (3) gives ,

$$\frac{d}{dx}\left[\frac{n}{2n+1}\,\frac{E\left(-\frac{1}{m}\frac{dp}{dx}\right)^{1/n}}{E\left(\frac{1}{H^{\frac{2n+1}{n}}}\right)}\right] = -\frac{dh}{dx} - U\,\frac{d}{dx}\left[\frac{E\left(\frac{1}{H^{\frac{n+1}{n}}}\right)}{E\left(\frac{1}{H^{\frac{2n+1}{n}}}\right)}\right] \tag{15}$$

Similarly for the region $\frac{dp}{dx} > 0$, we get

$$\frac{d}{dx}\left[\frac{n}{2n+1}\,\frac{E\left(\frac{1}{m}\frac{dp}{dx}\right)^{1/n}}{E\left(\frac{1}{H^{\frac{2n+1}{n}}}\right)}\right] = \frac{dh}{dx} + U\,\frac{d}{dx}\left[\frac{E\left(\frac{1}{H^{\frac{n+1}{n}}}\right)}{E\left(\frac{1}{H^{\frac{2n+1}{n}}}\right)}\right] \tag{16}$$

Equations (15) and (16) are generalized one dimensional form of Reynolds equation applicable for symmetric rough surfaces in the case of transverse roughness.

In equation (15) and (16) the terms $E\left(-\frac{dp}{dx}\right)^{1/n}$ is yet to be expressed in terms of $\frac{d\bar{p}}{dx}$. This cannot be done exactly as $\frac{dp}{dx}$ is a stochastic function. To get an approximate relation we proceed as follows. By keeping U = 0 in equation (11) we get,

$$q = \frac{n}{2n+1}\, H^{\frac{2n+1}{n}}\left(-\frac{1}{m}\,\frac{dp}{dx}\right)^{1/n} \tag{17}$$

or

$$\frac{q^n}{H^{2n+1}} = \left(\frac{n}{2n+1}\right)^n H^{2n+1}\left(-\frac{1}{m}\,\frac{dp}{dx}\right) \tag{18}$$

Taking expectation of equation (17) and (18) , and using

$E(q^n) = [E(q)]^n$

We get

$$E\left(\frac{dp}{dx}\right)^{1/n} = \frac{E\left[\frac{1}{H^{\frac{2n+1}{n}}}\right]}{E\left[\frac{1}{H^{2n+1}}\right]^{1/n}}\left(\frac{d\bar{p}}{dx}\right)^{1/n} \tag{19}$$

Using equation (19) in equations (15) and (16) we get the approximate Reynolds equation in the case of transverse roughness.

In the following we discuss the squeezing between two symmetric rough bearings.

3 PARALLEL PLATES (SQUEEZE FILMS)

Consider squeezing flow between symmetric parallel plates as shown in fig 1. Here we discuss the following cases:

3.1 LONGITUDINAL ONE-DIMENSIONAL ROUGHNESS

In this type of roughness it may be noted again that the narrow ridges and valleys run in the direction of the flow of the lubricant . The equation governing the mean pressure can be written from equation (9) by putting U=0 as follows:

$$\frac{d}{dx}\left[\frac{n}{2n+1}\, E\left\{H^{\frac{2n+1}{n}}\left(-\frac{1}{m}\frac{d\bar{p}}{dx}\right)^{1/n}\right\}\right] = V \tag{20}$$

Where

$$V = \frac{dh}{dt}$$

Integrating equation(20) and using

$$\frac{d\bar{p}}{dx} = 0 \text{ at } x = 0$$
$$\bar{p} = 0 \text{ at } x = l \tag{21}$$

We get

$$\bar{p} = m\left(\frac{2n+1}{n}\right)^{n}\left[\frac{V}{E\left(H^{\frac{2n+1}{n}}\right)}\right]^{n}\frac{l^{n+1}-x^{n+1}}{n+1} \tag{22}$$

Where 2l is the length of the bearing .

The mean load capacity W_r is defined as

$$E(W) = W_r = 2b\int_0^l \bar{p}\, dx \tag{23}$$

Which on using equation(22) gives,

$$W_r = 2bm\left(\frac{2n+1}{n}V\right)^{n}\frac{l^{n+2}}{n+2}\left[\frac{1}{E\left(H^{\frac{2n+1}{n}}\right)}\right]^{n} \tag{24}$$

Where b is the width of the bearing

The time of squeezing , t_r , for the surface to approach from an initial film thickness, 2hi to a final film thickness $2h_f$ is given by,

$$t_r = \left(\frac{2n+1}{n}\right)\left(\frac{2bm}{W_r}\frac{l^{n+2}}{n+2}\right)^{1/n}\int_{h_f}^{h_i}\frac{dh}{E\left(H^{\frac{2n+1}{n}}\right)} \tag{25}$$

Substituting the value of $E\left(H^{\frac{2n+1}{n}}\right)$ from equation (appendix) in equation (24) and (25) we can write W_r and t_r approximately as follows:

$$W_r = 2bm\left(\frac{2n+1}{n}V\right)^{n}\frac{l^{n+2}}{n+2}\frac{1}{\left[1+\left(\frac{2n+1}{n}\right)\left(\frac{n+1}{n}\right)\frac{1}{2}\frac{\sigma^2}{h^2}\right]^{n}h^{2n+1}} \tag{26}$$

$$t_r = \left(\frac{2bm}{W_r}\frac{l^{n+2}}{n+2}\right)^{1/n}\left(\frac{2n+1}{n}\right)\int_{h_f}^{h_i}\frac{dh}{\left[1+\left(\frac{2n+1}{n}\right)\left(\frac{n+1}{n}\right)\frac{1}{2}\frac{\sigma^2}{h^2}\right]h^{\frac{2n+1}{n}}} \tag{27}$$

By keeping $\sigma = 0$ in equation (26) and (27) we get the case of smooth surfaces. If W_s and t_s are the load capacity and the squeezing time respectively for this case we can write,

$$W_s = 2bm\left(\frac{2n+1}{n}V\right)^{n}\frac{l^{n+2}}{n+2}\frac{1}{h^{2n+1}}$$

Where m = $m_1\left(\frac{h}{h_i}\right)^q$

$$\frac{W_{r,q}}{W_{s,0}} = \bar{w} = \left(\frac{h}{h_i}\right)^q \frac{1}{\left[1+\left(\frac{2n+1}{n}\right)\left(\frac{n+1}{n}\right)\frac{1}{2}\frac{\sigma^2}{h^2}\right]^n} \tag{28}$$

$$t_r = \left(\frac{2bm}{W_r}\ \frac{l^{n+2}}{n+2}\right)^{1/n}\left(\frac{2n+1}{n}\right)\int_{h_f}^{h_i}\frac{dh}{h^{\frac{2n+1}{n}}}$$

$$\frac{t_{r,q}}{t_{s,0}} = \bar{t} = \left[\frac{1}{1-\bar{h}_f^{\,-\left(\frac{n+1}{n}\right)}}\right]\int_{\bar{h}_f}^{1}\frac{d\bar{h}}{\left[1+\left(\frac{2n+1}{n}\right)\left(\frac{n+1}{n}\right)\frac{1}{2}\frac{\sigma^2}{hi^2}\frac{1}{\bar{h}^2}\right]\bar{h}^{\frac{2n+1}{n}-q}} \tag{29}$$

$$\bar{h}_f = \frac{h_f}{h_i}\ ,\bar{h} = \frac{h}{h_i} \tag{30}$$

Equation (28) and (29) are evaluated numerically and graphs of $\frac{W_{r,q}}{W_{s,0}}$, $\frac{t_{r,q}}{t_{s,0}}$ are plotted with σ for various values in fig.

3.2 TRANSVERSE ONE-DIMENSIONAL ROUGHNESS

In this type of roughness it may be noted again that the narrow ridges and valleys run perpendicular to the direction of the flow of the lubricant. The one dimensional equation governing the pressure in this case can be written, from equation (15) and (19) as follows:

$$\frac{d}{dx}\left[\frac{n}{2n+1}\ \frac{\left(-\frac{1}{m}\frac{d\bar{p}}{dx}\right)^{1/n}}{\left[E\left(\frac{1}{H^{2n+1}}\right)\right]^{1/n}}\right] = V \tag{31}$$

Integrating the equation (31) and using boundary conditions (21) we get,

$$\bar{p} = m\left(\frac{2n+1}{n}\ V\right)^n E\left(\frac{1}{H^{2n+1}}\right)\frac{l^{n+1}-x^{n+1}}{n+1} \tag{32}$$

The mean load capacity W_r and squeezing time t_r can be written by using equations (23) and (32) as follows:

$$W_r = 2bm\left(\frac{2n+1}{n}\ V\right)^n\ E\left(\frac{1}{H^{2n+1}}\right)\frac{l^{n+2}}{n+2} \tag{33}$$

$$t_r = \left(\frac{2n+1}{n}\right)\left(\frac{2bm}{W_r}\ \frac{l^{n+2}}{n+2}\right)^{1/n}\int_{h_f}^{h_i}\left[E\left(\frac{1}{H^{2n+1}}\right)\right]^{1/n} \tag{34}$$

Substituting the values of $E\left(\frac{1}{H^{2n+1}}\right)$ from equations (appendix) the values of W_r and t_r can be written approximately as follows:

By keeping $\sigma = 0$ in equation (35) and (36) we get the case of smooth surfaces. If W_s and t_s are the load capacity and the squeezing time respectively in this case of smooth surface, then we can write,

$$W_s = 2bm\left(\frac{2n+1}{n}\ V\right)^n \frac{l^{n+2}}{n+2}\ \frac{1}{h^{2n+1}}$$

Where m = $m_1\left(\frac{h}{h_i}\right)^q$

$$\frac{W_{r,q}}{W_{s,0}} = \bar{w} = \left(\frac{h}{h_i}\right)^q\left[1+(n+1)(2n+1)\frac{\sigma^2}{h^2}\right] \tag{35}$$

$$t_s = \left(\frac{2n+1}{n}\right)\left(\frac{2bm}{W_r}\ \frac{l^{n+2}}{n+2}\right)^{1/n}\int_{h_f}^{h_i}\left[\frac{1}{h^{\frac{2n+1}{n}}}\right]dh$$

$$\frac{t_{r,q}}{t_{s,0}} = \bar{t} = \left[\frac{1}{1-\bar{h}_f^{\frac{n+1}{n}}}\right] \int_{\bar{h}_f}^{1} \frac{\int_{h_f}^{h_i}\left[\left[1+(n+1)(2n+1)\frac{\sigma^2}{h_i^2}\frac{1}{h^2}\right]\right]^{1/n}}{\bar{h}^{\frac{2n+1}{q}}-q}\, dh \tag{36}$$

Equation (28), (29), (35) and (36) are evaluated numerically and graphs of are plotted for various parameters.

4 RESULTS AND DISCUSSION

Here the results are shown for the graphs from (2) to (13)

Fig (2) is plotted for load capacity with $\frac{\sigma}{h_i}$ for different values of q in the case of longitudinal roughness. From this we can say with the increase of $\frac{\sigma}{h_i}$ the load capacity decreases also decreases with the increase of q. Fig (3) is plotted for load capacity with $\frac{\sigma}{h_i}$ for different values of q in the case of transversal roughness. From this we can say with the increase of $\frac{\sigma}{h_i}$ the load capacity increases and decreases with the increase of q.

Fig (4) is plotted for load capacity with $\frac{\sigma}{h_i}$ for different values of flow behavior index n in the case of longitudinal roughness. From this we can say with the increase of $\frac{\sigma}{h_i}$ the load capacity decreases also decreases with the increase of n. Fig (5) is plotted for load capacity with $\frac{\sigma}{h_i}$ for different values of n in the case of transversal roughness. From this we can say with the increase of $\frac{\sigma}{h_i}$ the load capacity increases and increases with the increase of q.

Fig (6) is plotted for load capacity with q for different values of $\boldsymbol{\frac{\sigma}{h_i}}$ in the case of longitudinal roughness. From this we can say with the increase of q the load capacity decreases and decreases with the increase of $\boldsymbol{\frac{\sigma}{h_i}}$**.**

Fig (7) is plotted for squeezing time with $\frac{\sigma}{h_i}$ for different values of flow behavior index n in the case of longitudinal roughness. From this we can say with the increase of $\frac{\sigma}{h_i}$ the squeezing time decreases also decreases with the increase of n. Fig (8) is plotted for squeezing time with $\frac{\sigma}{h_i}$ for different values of flow behavior index n in the case of transversal roughness. From this we can say with the increase of $\frac{\sigma}{h_i}$ the squeezing time increases also increases with the increase of n.

Fig (9) is plotted for squeezing time with $\frac{\sigma}{h_i}$ for different values of $\bar{h}_f$ in the case of longitudinal roughness. From this we can say with the increase of $\frac{\sigma}{h_i}$ the squeezing time decreases also decreases with the increase of $\bar{h}_f$. Fig (10) is plotted for squeezing time with $\frac{\sigma}{h_i}$ for different values of $\bar{h}_f$ in the case of transversal roughness. From this we can say with the increase of $\frac{\sigma}{h_i}$ the squeezing time increases also increases with the increase of $\bar{h}_f$.

5 GRAPHS

Case of longitudinal Roughness

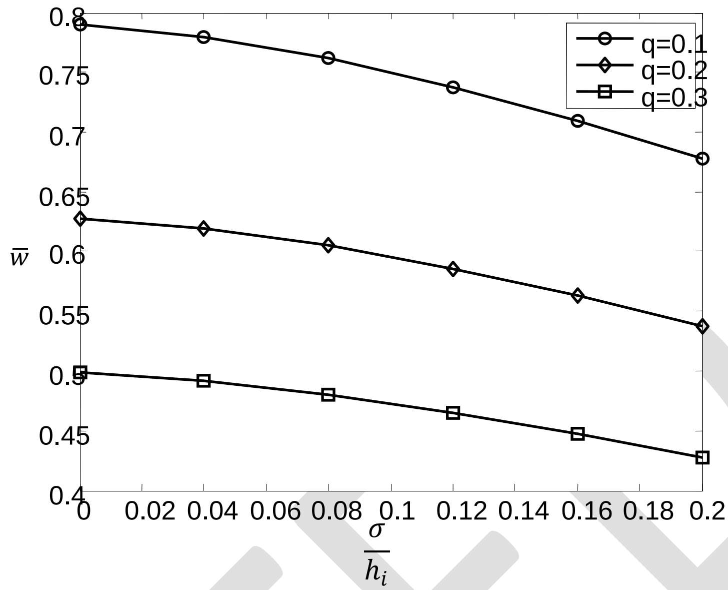

Fig (2): Load capacity Vs $\frac{\sigma}{h_i}$ for various q

Case of transversal Roughness

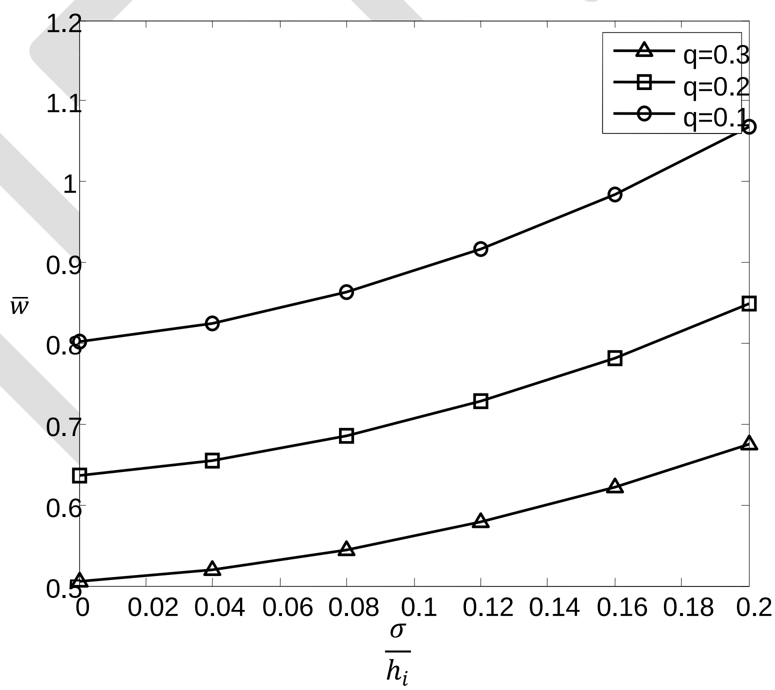

Fig (3): Load capacity Vs $\frac{\sigma}{h_i}$ for various q

Case of longitudinal Roughness

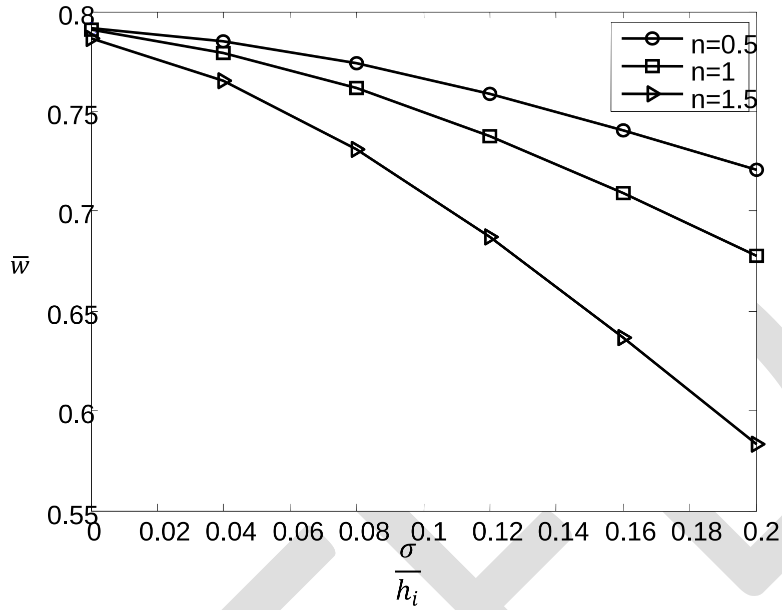

Fig (4): Load capacity Vs $\frac{\sigma}{h_i}$ for various n

Case of transversal Roughness

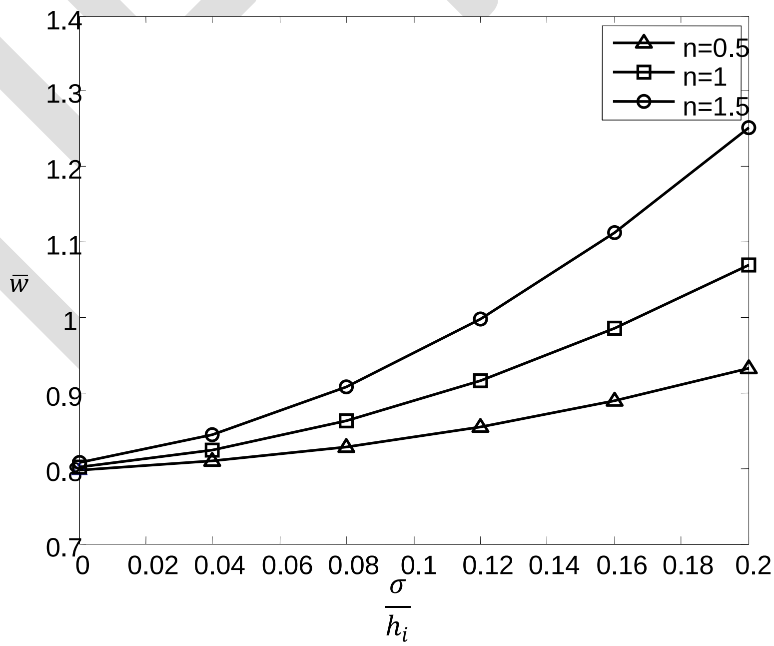

Fig (5): Load capacity Vs $\frac{\sigma}{h_i}$ for various n

Case of longitudinal Roughness

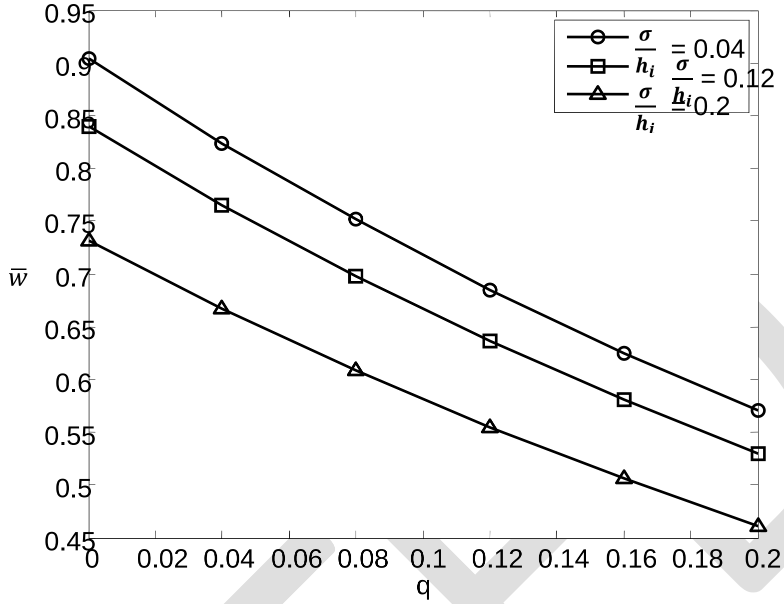

Fig (6): Load capacity Vs q for various $\frac{\sigma}{h_i}$

Case of longitudinal Roughness

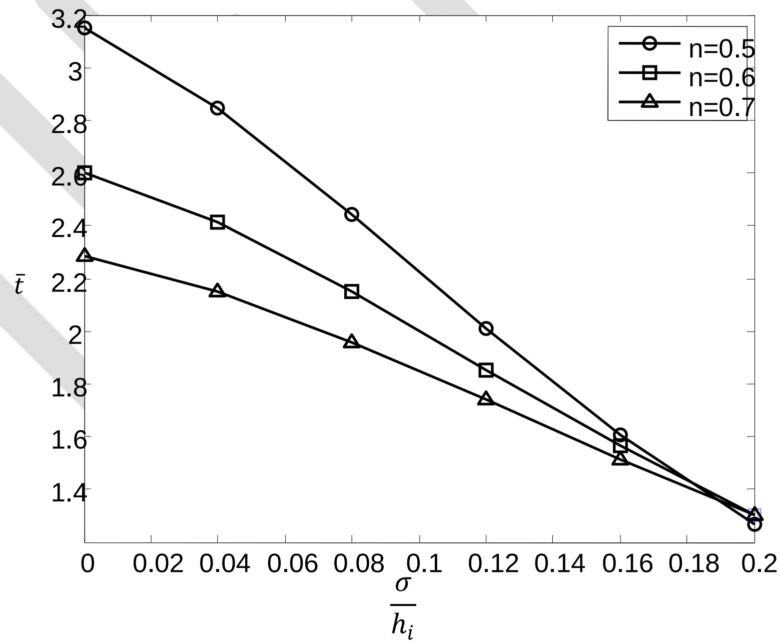

Fig (7): Squeezing time Vs $\frac{\sigma}{h_i}$ for various n

Case of transversal Roughness

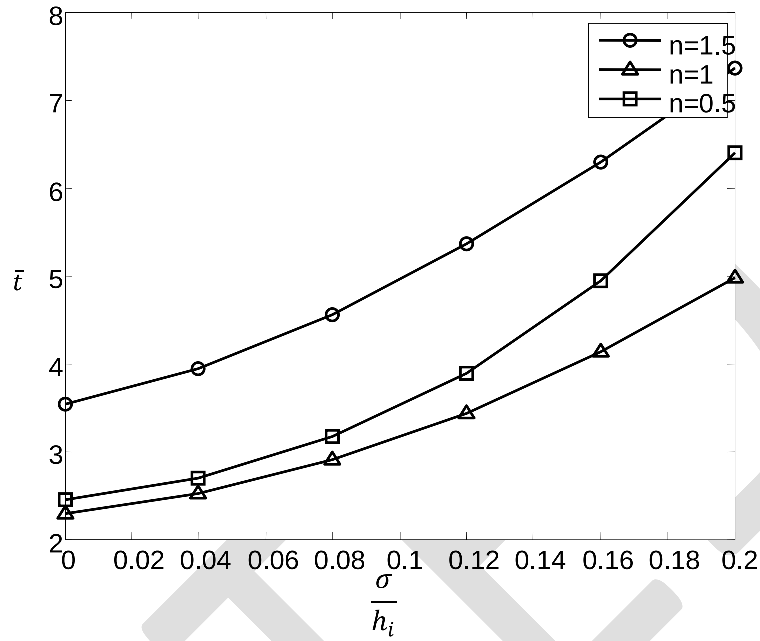

Fig (8): Squeezing time Vs $\frac{\sigma}{h_i}$ for various n

Case of longitudinal Roughness

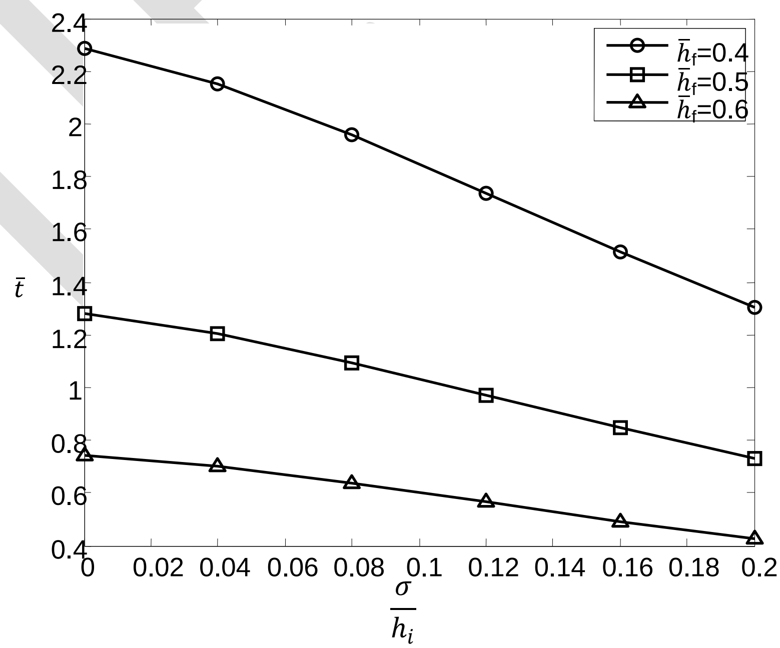

Fig (9): Squeezing time Vs $\frac{\sigma}{h_i}$ for various $\bar{h}_f$

Case of transversal Roughness

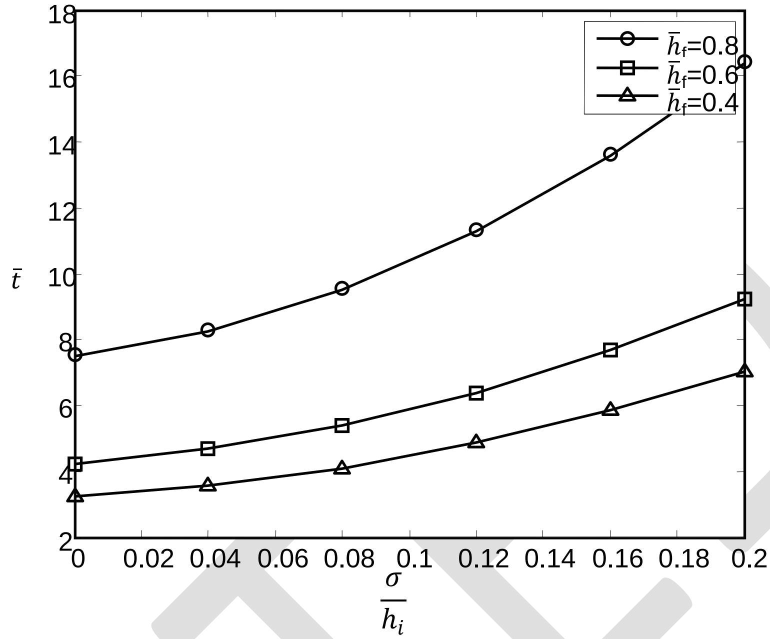

Fig (10): Squeezing time Vs $\frac{\sigma}{h_i}$ for various $\bar{h}_f$

6 CONCLUSION

In this chapter a generalized form of one dimensional Stochastic Reynolds equation by considering thermal effect is applicable for rough surfaces has been derived for transverse and longitudinal roughness using power law fluid as a lubricant. The cases of squeeze films have been investigated for parallel plates.

It is noted that for all n the load capacity and time of squeezing increase as the surface roughness increases in the case of transverse roughness and this increase is enhanced as the flow behavior index increases. However in the case of longitudinal roughness the load capacity and time of squeezing decreases as surface roughness increases for all values of the flow behavior index.

NOMENCLATURE

b	Width of the bearing
c	Half range random film thickness variable
E	Expectation operator
H	Half film thickness random variable
2h	Deterministic part of film thickness (nominal film thickness)
$2h_i$	Initial film thickness
$2h_s$	Random part of the film thickness

2	Length of the bearing
m	Consistency of the fluid
n	Flow behavior index
p	Hydrodynamic pressure, a random variable
$\bar{p}$	Mean Hydrodynamic pressure
2q	Flow flux
x,y,z	Coordinate system

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