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FOURIER ANALYSIS ON THE ELECTROCARDIOGRAM

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ABSTRACT

In order to develop the control signal for electrocardiograph (ECG), the property, forms, size and numeric parameters of the control signal shall be identified. In doing so, a mathematical processing and analysis were done on the electrocardiograms used as the input signal of the ECG because we supposed that the signal for checking normal condition of the equipment shall have the same rate and the same property with the equipment's input signal .

We aimed to select the most suitable option of control signals by making calculations using Fourier's transformation.

Up to date, the researches and studies on the electrocardiograms have been done only for clinical analysis, but it is insufficient today as the development of equipment and technology has reached much higher level. Thus, a feature-length study of electrocardio signals using special software has been started.

In this study, while performing mathematical processing of the electrocardiograms, we used MatCAD for Fourier's transformation.

KEYWORDS:

Electrocardiogram, fourier, signal, tools and analysis.

1. INTRODUCTION

In order to reveal the general features of the signals of electrocardiograms, an issue of the development of a mathematical model of the signals has arisen. Let's suppose the electrocardiograms are analog signals and demonstrate a whole period of it by the value of $y = f(x)$ function on the finite points of $[a,b]$ interval. For this, some methods of function interpolarization were reviewed using the principles of approaching or approach building.

Also, in case, where the function $y = f(x)$ is given as an analog function, but the calculation of its values is complicated, we need to replace the function with another function that is simple to solve.

Let's see the approaching method where the given function is replaced with the simpler one that is similar to it for certain values. Here we decide what function will most suit to approach to the given one depending on the properties and characteristic features of the input data.

There are three classes of approaching commonly used in digital analysis. They are:

- Class of "n rate algebraic polynominals" $\{P_n(x)\}$ or approaching as the linear mix of functions x, $x^2, \ldots, x^n$. Most approaches of classical digital analysis are based on the approaching method because such analytic actions as the integration and differentiation are simple for polynominals. However, approaching some functions; for instance, polynominals; can be improper.

- Class of trigonometric polynominals or {cosα_ix, sinα_ix} functions. This class is related to unpacking functions to the Fourier's sequence and the Fourier's integral, and is used to approach laminated functions or functions with strong fluctuation.
- Class of {exp(-α_I x)} functions. Collapse and accumulation principle of any process is expressed by such a function. An approach function can be a linear mix of functions of above three classes. Also, it is useful, sometimes, to approach with a ratio of two polynominals or by a rational function. Especially, it is used for ap[proaching the functions that take endless meanings at the finite point.

If consider the values of amplitudes in a whole period of a bioelectrocardiogram as the values of function on the n_i points that are placed in [n_1,n_k] interval and differ from each others, then:

$$y_i = f(n_i) \qquad\qquad i = 0,1,\ldots k \tag{1}$$

If $\varphi(n)$ is an approach function, then it can be described as a linear mix of nonlinearity functions in the following form:

$$f(n) \approx \varphi(n) = \sum_{i=0}^{k} c_i \varphi_i(n) \tag{2}$$

Here, the $\varphi_i(n)$ is a function of the above three classes. The Fourier's direct transformation method were used to calculate the approaching coefficient c_j of (2) approach. In doing so, the coefficient c_j of approaching interpolation function $\varphi(n)$ which must coincide with the values on n_i points of the interpolation node of the function was estimated.

2. THE FOURIER'S TRANSFORMATION

It is impossible to get immediate information related to the time of a frequency of a function transformed by Fourier's transformation. The time data hides in sine and cosine declination phases and appear only the phases are joined.

In many researches like the analysis on the signals of electrocardiograms, the information about the frequency and its relationship with time shall be displayed simultaneously. This failure can be eliminated by using the Fourier's transformation. The Fourier's express transformation is based on the principle to collapse the signal into intervals, to provide calculations within these intervals using Fourier's classic transformation and to develop data on the frequency matches. Thus, the time is identified definitely by certain interval. The area for analysis is moved along with the signal until the signal is fully analized in the whole. The opaque between the time and frequency precision can be explained by a vague relation defined by Verner-Heizenberg. When a given signal in a selected area is analized using the Fourier's express transformation, the opaque between the time and frequency precision is constant, i.e a human predetermines (by the value of frequency determined) how high will be the accuracy of the analisys.

In the Fourier's express transformation, the signal is multiplied by relatively short, in comparison with the signal length, value γ (discreting value N_0). Fourier's window transformation will be:

$$F_x^{\gamma}(\tau, f) = \int_{-x}^{+x} x(t) \cdot \gamma^*(t-\tau) \cdot \exp(-j2\pi f t)dt \tag{3}$$

This shows the Fourier's window transformation is a two-dimensional function that depends on the τ (time movement) and f (frequency). If $\tau = \tau_0$, the Fourier's transformation will have a form like:

$$X(r,n) = \sum_{k=0}^{N-1} x(r+k)\gamma^*(k)\exp(-j2\pi\frac{kn}{N}) \qquad (4)$$
$$n = 0....N, r = 0...N_1 - N$$

For the Fourier's discrete window transformation, signal $x(n)$ and window $\gamma(n)$ are divided with the same frequency f_A. Here, the N_0 is the number of points to divide the signal $x(n)$ and the N is the number of points to divide the window. The transporm is estimated and the number of N is important for the frequency precision of the transformation result.

$$X(r,n) = \sum_{k=0}^{N-1} x(r+k)\gamma^*(k)\exp(-j2\pi\frac{kn}{N}) \qquad (5)$$
$$n = 0....N, r = 0...N_1 - N$$

By this method, to analyze the given electrocardigrams, the electrocardiograms were recorded as a series of data. The recorded data was processed on using software system of MatCad. The series of data was recorded as shown in the Figure 1.

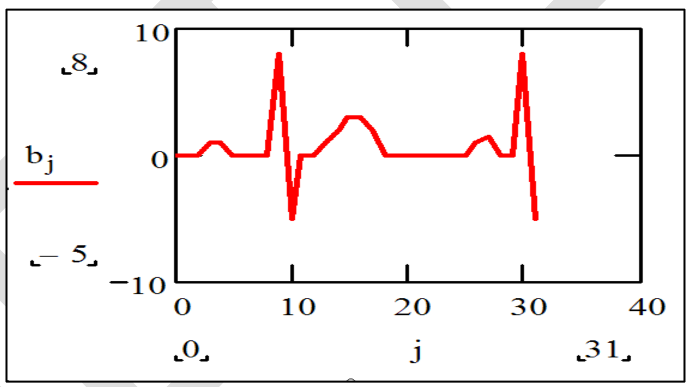

Figure 1. *An* electrocardiogram

Coefficient "c" (5) of Fourier's sequence was calculated and the result of the calculation was presented as modules in Figure 2 .

$$c_p = \frac{1}{\sqrt{N_0}} \cdot \sum_k \left[v_k \cdot e^{l_i\left(\frac{2\cdot\pi\cdot p}{N_0}\right)\cdot k} \right] \qquad (5)$$

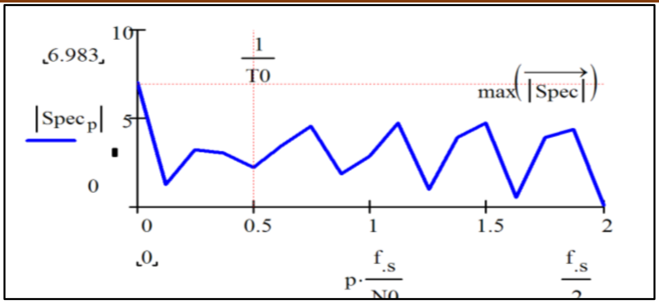

Figure 2. Coefficient module of Fourier's sequence

An equation with 16 coefficients in total is written to describe one whole period of a electrocardiogram, and how the final result c_j was approached to the initial data b_j function is shown on one coordinate in the graph in Figure 3.

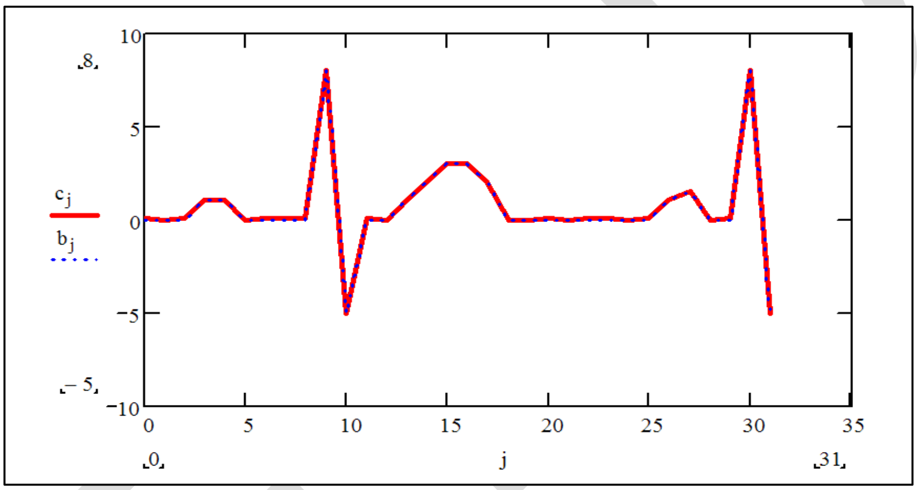

Figure 3. Comparison of the original function and the result of transformation

As a result of the approach, the errors arised between the original function and the approached function were calculated and shown as a graph in Figure 4.

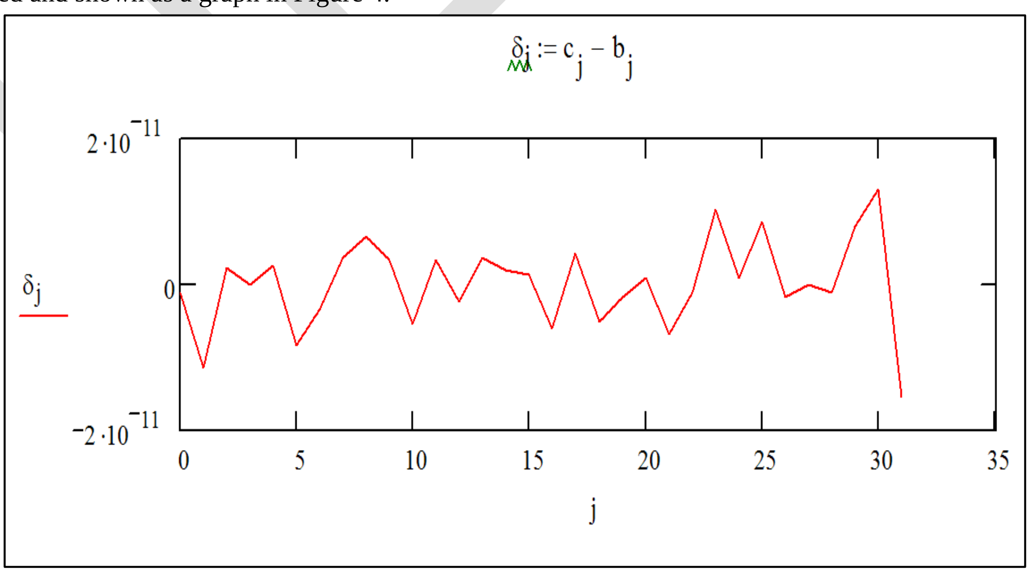

Figure 4. Approach errors

CONCLUSION

The Fourier's sequence divides the original signal into fixed ranges and calculates the corresponding coefficients. However, every part of the electrocardiogram signal changes independently, not corresponding the values of fixed ranges of the Fourier's sequence and dividing itself into fixed zones. This confirms that the Fourier's sequence can not be used for composing a mathematic model directly, as well as shows that it requires more researches using other ways.

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