International Journal of Emerging Trends in Engineering and Development Issue 3, Vol.2 (May 2013)
Available online on http://www.rspublication.com/ijeted/ijeted_index.htm ISSN 2249-6149

A COMMON FIXED POINT THEOREM FOR CYCLIC WEAK ∅-CONTRACTION IN L- FUZZY METRIC SPACE

Surjeet Singh Chauhan[1] (Gonder) and Kiran Utreja[2]

Deptt. Of Applied Science and Humanities, Chandigarh University, Gharuan[1]

Deptt. Of Applied Science and Humanities, GNIT, Mullana[2]

Abstract: In this paper we are proving a common fixed point theorem for cyclic weak ∅ -contraction for four mappings in $\mathcal{L}$-fuzzy metric space generalizing the result of D. Gopal et al. [9] for cyclic weak Ø-contraction in fuzzy metric space. Here we are introducing the concept of cyclic weak Ø- contraction for four operators in $\mathcal{L}$-fuzzy metric space to prove our result. We are giving the corollary in respect of the proved theorem.

Key Words: Fixed point, L-fuzzy metric space, Cyclic Weak ∅ Contraction

Mathematics Subject Classification: 46S40, 54H25, 54E35

1. Introduction: The notion of fuzzy sets was introduced by Zadeh [19] and various concepts of fuzzy metric space were considered in [2, 4, 7, 12, and 13]. Many authors have studied fixed point theory in fuzzy metric spaces as [3, 8, 11, and 14].In 1988 Grabeic [10] defined contraction and contractive mappings on fuzzy metric space and extended their results in such spaces. Continuing in this field Mihet [15, 16] proved fixed point theorems for various contraction mappings in fuzzy metric space. In 2010 Pacurar et al. [17] introduced the concept of cyclic ∅-contraction and proved a fixed point theorem for cyclic ∅ −contraction in complete metric space. Using his concept D. Gopal et al. [9] proved a fixed point theorem for cyclic weak ∅-contraction in fuzzy metric spaces. In this sequel we are extending the work on the space introduced by Saadati et al. [18] to prove a common fixed point theorem for cyclic ∅-contraction for four mappings in $\mathcal{L}$-fuzzy metric space

2. Prelimnaries*:*

Def.2.1. Complete Lattice is a partially ordered set (L,≤) in which all subsets have both supremum (join) and infimum (meet).

Def.2.2 [18] Let $\mathcal{L}$= $(L,\leq_L)$ be a complete lattice and U a non-empty set called universe. An $\mathcal{L}$-fuzzy set A on U is defined as a mapping A: U→ L. For each u in U, A (u) represents the degree (in L) to which u satisfies A

Lemma2.3 [5]: Consider the set L^* and operation $\leq_{L^*}$defined by

$L^* = \{(x_1, x_2): (x_1, x_2) \in [0,1]^2$ and $x_1 + x_2 \leq 1\}$, $(x_1, x_2) \leq_{L^*} (y_1, y_2)$ if and only if

$x_1 \leq y_1$ *and* $x_2 \geq y_2$ for every $(x_1, x_2), (y_1, y_2) \in L^*$. Then $(L^*, \leq_{L^*})$ is complete Lattice.

Def.2.4 [1].An intuitionistic fuzzy set $A_{\zeta,\eta}$ on a universe U is an object $A_{\zeta,\eta} = \{(\zeta_A(u), \eta_A(u);\ u \in U\}$ where for all u∈ U, $\zeta_A(u) \in [0, 1]$ and $\eta_A(u) \in [0, 1]$ are called the membership degree and the non-membership degree respectively of u in $A_{\zeta,\eta}$ and furthermore satisfy $\zeta_A(u) + \eta_A(u) \leq 1$.

A triangular norm Ton ([0, 1],≤) is defined as an increasing, commutative, associative mapping T: $[0,1]^2 \to [0, 1]$ satisfying T(1, x) = x $\forall x \in [0,1]$.

We define first $0_{\mathcal{L}} = \inf L$

$1_{\mathcal{L}} = \sup L$.

Def.2.5. A triangular norm (t-norm) on $\mathcal{L}$ is a mapping $\mathcal{J}$: $L^2 \to L$ satisfying the following conditions:

i. $(\forall x \in L)$ $(\mathcal{J}(x, 1_{\mathcal{L}}) = x)$; (boundary condition);
ii. $(\forall (x, y) \in L^2)$ $(\mathcal{J}(x, y) = \mathcal{J}(y, x))$; (commutativity);
iii. $(\forall (x, y\, z) \in L^3)$ $(\mathcal{J}(x, \mathcal{J}(y, z)) = \mathcal{J}(\mathcal{J}(x, y), z)$; (associativity);
iv. $(\forall (x, x', y, y') \in L^4)$ $(x \leq_L x'$ and $y \leq_L y') \Rightarrow \mathcal{J}(x, y) \leq_L \mathcal{J}(x', y')$ (monotonocity);
A t- norm can also be defined recursively as an (n+1) - ary operation (n ∈N \{0}) by $\mathcal{J}^1 = \mathcal{J}$ and $\mathcal{J}^n(x_{(1)}, x_{(2)}, \ldots\ldots\ldots, x_{(n+1)}) = \mathcal{J}\ (\mathcal{J}^{n-1}(x_{(1)}, x_{(2)}, \ldots, x_{(n)}), x_{(n+1)}))$ for $n \geq 2$ and $x_{(i)} \in L$.

Def.2.6. [6] A t –norm $\mathcal{J}$ on L^* is called t-representable iff there exists a t-norm T and a t-conorm S on [0, 1] such that ∀ x = (x_1, x_2), y= $(y_1, y_2) \in L^*$.

$\mathcal{J}(x, y) = \{T(x_1, y_1), S(x_2, y_2)\}$.

Def.2.7.A negation on $\mathcal{L}$ is any decreasing mapping N: L→L satisfying N $(0_{\mathcal{L}}) = 1_{\mathcal{L}}$ and

N $(1_{\mathcal{L}}) = 0_{\mathcal{L}}$. If N (N(x)) =x ∀ x ∈ L, then N is called an involutive negation.

If for all x ∈[0, 1], $N_s(x) = 1-x$, we say that N_s is the standard negation on ([0, 1],≤).

Def.2.8. The 3-tuple (X, $\mathcal{M}$, $\mathcal{J}$) is said to be an $\mathcal{L}$-fuzzy metric space if X is an arbitrary (non-empty)set, $\mathcal{J}$ is continuous t-norm on $\mathcal{L}$ and $\mathcal{M}$ is an $\mathcal{L}$-fuzzy set on $X^2 \times (0, +\infty)$ satisfying the following conditions for every x, y, z in X and t, s in (0, +∞)

i. $\mathcal{M}(x, y, t) >_L 0_{\mathcal{L}}$;
ii. $\mathcal{M}(x, y, t) = 1_{\mathcal{L}}$ for all t>0 if and only if x= y;
iii. $\mathcal{M}(x, y, t) = \mathcal{M}(y, x, t)$;
iv. $\mathcal{J}(\mathcal{M}(x, y, t), \mathcal{M}(y, z, s)) \leq_L \mathcal{M}(x, z, t+s)$;
v. $\mathcal{M}(x, y, .):]0, \infty[\to L$ is continuous.
In this case $\mathcal{M}$ is called an $\mathcal{L}$-fuzzy metric. If $\mathcal{M} = \mathcal{M}_{M,N}$ is an intuitionistic fuzzy set then the 3-tuple $(X, \mathcal{M}_{M,N}, \mathcal{J})$ is said to be an intuitionistic fuzzy metric space.

Example: Let (X, d) be a metric space. Set $\mathcal{J}$ (a, b) = $(a_1b_1, \min(a_2 + b_2, 1))$ for all

a= (a_1, a_2) and $b = (b_1, b_2)$ in $L^*$and let M and N be fuzzy sets on $X^2 \times (0, \infty)$defined as follows:

$\mathcal{M}_{M,N}$ (x, y, t) = (M (x, y, t), N(x, y, t)) = $\left(\frac{ht^n}{ht^n+md(x,y)}, \frac{md(x,y)}{ht^n+md(x,y)}\right)$ for all t, h, m, n$\in R^+$. Then (X,$\mathcal{M}_{M,N}$, $\mathcal{J}$) is an intuitionistic fuzzy metric space.

Lemma 2.9. [2]: Let (X, $\mathcal{M}$, $\mathcal{J}$) be an $\mathcal{L}$-fuzzy metric space .Then $\mathcal{M}$(x, y, t) is non-decreasing with respect to t for all x, y in X.

Proof: Let t, s $\in (0, +\infty)$ be such that t<s. Then k= s-t > 0 and $\mathcal{M}$(x, y, t) =$\mathcal{J}$ (M(x, y, t),$1_{\mathcal{L}}$)

=$\mathcal{J}$ (M(x, y, t), M(y, y, k)) $\leq_L$M(x, y, s).

Def.2.10. A sequence $\{x_n\}_{n\in N}$in an $\mathcal{L}$-fuzzy metric space (X, $\mathcal{M}$, $\mathcal{J}$) is called a Cauchy sequence, if for each ε$\in L\backslash\{0_{\mathcal{L}}\}$and t>0, there exists $n_o \in N$ such that $\forall\, m \geq n \geq n_o (n \geq m \geq n_o)$, $\mathcal{M}$ $(x_m, x_n, t) >_L$N (ε).The sequence $\{x_n\}_{n\in N}$ is said to be convergent to x∈ X in the $\mathcal{L}$-fuzzy metric space (X, $\mathcal{M}$, $\mathcal{J}$) (denoted by $x_n \to x$ in $\mathcal{M}$) if $\mathcal{M}$ (x_n, x, t)= $\mathcal{M}(x, x_n, t) \to 1_{\mathcal{L}}$whenever n$\to +\infty$for every t >0. A $\mathcal{L}$-fuzzy metric space is said to be complete if and only if every Cauchy sequence is convergent.

Thus $\mathcal{J}$ is a continuous t-norm on lattice $\mathcal{L}$ such that for every$\mu \in L\backslash\{0_{\mathcal{L}}, 1_{\mathcal{L}}\}$, there is a λ$\in L\backslash\{0_{\mathcal{L}}, 1_{\mathcal{L}}\}$ such that $\mathcal{J}^{n-1}(N(\lambda), \ldots \ldots, N(\lambda)) >_L$N($\mu$).

Lemma 2.11.[20] Let (X, $\mathcal{M}$, $\mathcal{J}$) be an $\mathcal{L}$-fuzzy metric space. Then $\mathcal{M}$ is continuous function on X$\times X \times]0, \infty[$.

Def.2.12.Let (X, $\mathcal{M}$, $\mathcal{J}$) be an $\mathcal{L}$-fuzzy metric space. Then $\mathcal{M}$ is said to be continuous on X$\times X \times]0, \infty[$ that is $\lim_{n\to\infty} \mathcal{M}(x_n, y_n, t_n)$=$\mathcal{M}$(x, y, t) whenever a sequence $\{(x_n, y_n, t_n)\}$ in X$\times X \times]0, \infty[$ converges to a point (x, y, t)∈ X$\times X \times]0, \infty[$that is $\lim_{n\to\infty} \mathcal{M}(x_n, x, t)$=$\lim_{n\to\infty} \mathcal{M}(y_n, y, t)$= $1_{\mathcal{L}}$and$\lim_{n\to\infty} \mathcal{M}(x, y, t_n)$=$\mathcal{M}$(x, y, t).

Lemma2.13 [2]: Let (X, $\mathcal{M}$, $\mathcal{J}$) be an $\mathcal{L}$-fuzzy metric space. Define $E_{\lambda,\mathcal{M}}$:$X^2 \to R^+ \cup \{0\}$ by $E_{\lambda,\mathcal{M}}$(x, y) = inf { t>0 : $\mathcal{M}$(x, y, t) $>_L$N(λ)}for each λ$\in L\backslash\{0_{\mathcal{L}}, 1_{\mathcal{L}}\}$and x, y ∈X. Then we have

i. For any μ∈ $L\backslash\{0_{\mathcal{L}}, 1_{\mathcal{L}}\}$there exists λ∈ $L\backslash\{0_{\mathcal{L}}, 1_{\mathcal{L}}\}$such that $E_{?,\mathcal{M}}(x_1, x_n) \leq E_{\lambda,\mathcal{M}}(x_1, x_2) + E_{\lambda,\mathcal{M}}(x_2, x_3) + \ldots\ldots\ldots\ldots E_{\lambda,\mathcal{M}}(x_{n-1}, x_n)$for any $x_1, \ldots\ldots\ldots\ldots\ldots, x_n \in X$.
ii. The sequence $\{x_n\}_{n\in N}$is convergent w.r.t $\mathcal{L}$-fuzzy metric $\mathcal{M}$ if and only if $E_{\lambda,\mathcal{M}}(x_n, x) \to 0$.Also the sequence $\{x_n\}_{n\in N}$is Cauchy w.r.t $\mathcal{L}$-fuzzy metric $\mathcal{M}$ if and only if it is Cauchy with$E_{\lambda,\mathcal{M}}$.

Lemma2.14: Let(X, $\mathcal{M}$, $\mathcal{J}$) be an $\mathcal{L}$-fuzzy metric space. If $\mathcal{M}$ $(x_n, x_{n+1}, t) \geq_L \mathcal{M}(x_0, x_1, k^n t)$ for some k>1 and n ∈N .then $\{x_n\}$ is a Cauchy sequence.

Proof: For every λ$\in L\backslash\{0_{\mathcal{L}}, 1_{\mathcal{L}}\}$ and $x_n \in$ X, we have

$E_{\lambda,\mathcal{M}}(x_{n+1}, x_n)$= inf {t> 0: $\mathcal{M}$ $(x_{n+1}, x_n, t) >_L$N (λ)}

$\leq$ inf {t>0: $\mathcal{M}(x_0, x_1, k^n t) >_L$ N (λ)}

= inf {$\frac{t}{k^n}$: $\mathcal{M}(x_0, x_1, t) >_L$ N (λ)}

=$\frac{1}{k^n}$ inf {t>0: $\mathcal{M}(x_0, x_1, t) >_L$ N (λ)}

=$\frac{1}{k^n} E_{\lambda,\mathcal{M}}(x_0, x_1)$.

From lemma 2.12, for every $\mu \in L\backslash\{0_{\mathcal{L}}, 1_{\mathcal{L}}\}$ there exists $\lambda \in L\backslash\{0_{\mathcal{L}}, 1_{\mathcal{L}}\}$, such that

$$E_{\mu,\mathcal{M}}(x_n, x_m) \leq E_{\lambda,\mathcal{M}}(x_n, x_{n+1}) + E_{\lambda,\mathcal{M}}(x_{n+1}, x_{n+2}) + \cdots + E_{\lambda,\mathcal{M}}(x_{m-1}, x_m)$$

$$\leq \frac{1}{k^n} E_{\lambda,\mathcal{M}}(x_0, x_1) + \frac{1}{k^{n+1}} E_{\lambda,\mathcal{M}}(x_0, x_1) + \cdots + \frac{1}{k^{m-1}} E_{\lambda,\mathcal{M}}(x_0, x_1)$$

$$= E_{\lambda,\mathcal{M}}(x_0, x_1) \sum_{j=n}^{m-1} \frac{1}{k^j} \to 0.$$

Hence the sequence $\{x_n\}$ is a Cauchy sequence.

Def.2.15. We say that $\mathcal{L}$-fuzzy metric space(X, $\mathcal{M}$, $\mathcal{I}$) has a property C, if it satisfies the following condition $\mathcal{M}$(x, y, t) = C for all t > 0 implies C = $1_{\mathcal{L}}$.

Def.2.16. Let (X, $\mathcal{M}$, $\mathcal{I}$) be an $\mathcal{L}$-fuzzy metric space. The $\mathcal{L}$-fuzzy metric is triangular if it satisfies the condition $(\frac{1}{\mathcal{M}(x,y,t)} - 1) \leq (\frac{1}{\mathcal{M}(x,z,t)} - 1) + (\frac{1}{\mathcal{M}(y,z,.t)} - 1)$

for every x, y, z ∈ X and every t >0.

Def.2.17. Let X be a non-empty set, m be a positive integer and f: X → X an operator. By definition, X = $\bigcup_{i=1}^{m} X_i$ is a cyclic representation of X with respect to f if

i. X_i , i = 1, 2, ……., m are nonempty stes;
ii. $f(X_1) \subset X_2$, .. ……….. $f(X_{m-1}) \subset X_m$, $f(X_m) \subset X_1$.

Inspired by D. Gopal et al. [9] we present the notion of cyclic weak Ø-contraction for four operators in $\mathcal{L}$- fuzzy metric space.

Def.2.18. Let (X, $\mathcal{M}$, $\mathcal{I}$) be an $\mathcal{L}$-fuzzy metric space, $A_1, A_2, \dots \dots \dots, A_m$ be closed subsets of X and Y = $\bigcup_{i=1}^{m} A_i$. Operators P, Q, S and T: Y→Y are called cyclic weak $\emptyset$-contraction if the following conditios hold:

i. Y = $\bigcup_{i=1}^{m} A_i$ is a cyclic representation of Y with respect to P, Q, S and T respectively
ii. There exists a continuous, non-decreasing function $\emptyset$: [0, +∞)→ [0, +∞)

With $\emptyset(r)$>0 for r >0 and $\emptyset$(0) = 0, such that

$$(\frac{1}{\mathcal{M}(Px, Qy, t)} - 1) \leq_L (\frac{1}{\mathcal{M}(Sx, Ty, kt)} - 1) - \emptyset(\frac{1}{\mathcal{M}(Px, Sx, kt)} - 1)$$

For any x ∈ A_i , y ∈ A_{i+1} (i =1, 2… m, where $A_{m+1} = A_1$ and each t > 0.

3. Main Theorem:

Theorem3.1. Let (X, $\mathcal{M}, \mathcal{I}$) be an $\mathcal{L}$-fuzzy metric space, $A_1, A_2, \dots \dots \dots, A_m$ be closed subsets of X and Y = $\bigcup_{i=1}^{m} A_i$ be complete. Suppose that $\emptyset: [0, +\infty) \to [0, +\infty)$ is continuous, non-decreasing function with $\emptyset(r)>0$ for r ∈ $(0, +\infty)$ and $\emptyset(0) = 0$.

If P, Q, S and T: Y→Y be a cyclic weak ∅ - contraction such that

i. P(X)∪Q(X)⊆ S(X)∩T(X),
ii. $\left(\frac{1}{\mathcal{M}(Px,\ Qy,\ t)} -1\right) \leq_L \left(\frac{1}{\mathcal{M}(Sx,\ Ty,\ kt)} -1\right) - \emptyset\left(\frac{1}{\mathcal{M}(Px,\ Sx,\ kt)} - 1\right)$.
Then P, Q, S and T has a unique fixed point z ∈ $\bigcap_{i=1}^{m} A_i$.

Proof: Since P(X) ∪Q(X)⊆ S(X)∩T(X) , thus we have P(X)⊆ T(X), therefore for any arbitrary point x_0 ∈Y = $\bigcup_{i=1}^{m} A_i$ there exists x_1 ∈Y such that Px_0= Tx_1

And Q(X) ⊆ S(X), for this point x_1, we can choose a point x_2 ∈Y such that Qx_1= Sx_2 and so on.

Thus by induction we can define a sequence $\{y_n\}$ such that y_{2n} = Tx_{2n+1} = Px_{2n}.

y_{2n+1}= Sx_{2n+2} = Qx_{2n+1} for every n ∈ N∪ {0}. Now to prove that $\{y_n\}$ is a cauchy sequence in X.

Put x =x_{2n}, y = x_{2n+1} in (ii)

$$\left(\frac{1}{\mathcal{M}(Px_{2n},Qx_{2n+1},t)} -1\right) \leq_L \left(\frac{1}{\mathcal{M}(Sx_{2n},Tx_{2n+1},kt)} -1\right) - \emptyset\left(\frac{1}{\mathcal{M}(Px_{2n},Sx_{2n},kt)} - 1\right).$$

$$\Longrightarrow \left(\frac{1}{\mathcal{M}(y_{2n},\ y_{2n+1},\ t)} -1\right) \leq_L \left(\frac{1}{\mathcal{M}(y_{2n-1},y_{2n},\ kt)} -1\right) - \emptyset\left(\frac{1}{\mathcal{M}(y_{2n},y_{2n-1},kt)} - 1\right).$$

$$\leq_L \left(\frac{1}{\mathcal{M}(y_{2n-1},y_{2n},\ kt)} -1\right).$$

$$\Longrightarrow \mathcal{M}\ (y_{2n}, y_{2n+1}, t) \geq_L \mathcal{M}\ (y_{2n-1}, y_{2n}, kt)\ \ \forall\ n \in N.$$

This implies $\mathcal{M}\ (y_{2n}, y_{2n+1}, t)$ is a non-decreasing sequence of positive real numbers in [0, 1].

Thus we have $\mathcal{M}\ (y_n, y_{n+1}, t) \geq_L \mathcal{M}\ (\ y_{n-1}, y_n, k\ t) \geq_L \dots \dots \dots .. \geq_L\ \mathcal{M}(y_0, y_1, k^n\ t)$.

Therefore by lemma 2.13 we have $\{y_n\}$ is a Cauchy sequence.

Since Y is complete, there exists a point z∈ Y such that $\lim_{n\to\infty} y_n$ = z.

Thus T$x_{2n+1}, Px_{2n}, Sx_{2n+2}$ and Qx_{2n+1} converge to z ∈ Y.

Since the mappings are ∅-contraction, thus the iterative sequence $\{y_n\}$ has an infinite number of terms in A_i for each i =1, 2…m. As Y is complete then from each A_i , i=1, 2,…m, one can extract a subsequence of $\{y_n\}$that converges to z. Also each A_i , i=1, 2…m, is closed ,we have that point y $\in \bigcap_{i=1}^{m} A_i$ and $\bigcap_{i=1}^{m} A_i \neq \emptyset$.

Next we are to prove that z is fixed point of P, Q, S and T.

As Q(X) ⊆ S(X) therefore there exists a point u∈Ysuch that Su= z. Now on putting x = u and y = x_{2n+1} in (ii) we get

$$\left(\frac{1}{\mathcal{M}(Pu, Qx_{2n+1}, t)} - 1\right) \leq_L \left(\frac{1}{\mathcal{M}(Su, Tx_{2n+1}, kt)} - 1\right) - \emptyset\left(\frac{1}{\mathcal{M}(Pu, Su, kt)} - 1\right).$$

Now on taking the limit as n→ ∞ we get

$$\left(\frac{1}{\mathcal{M}(Pu,\ z,\ t)} - 1\right) \leq_L \left(\frac{1}{\mathcal{M}(z, z, kt)} - 1\right) - \emptyset\left(\frac{1}{\mathcal{M}(Pu, z,\ kt)} - 1\right).$$

$\Longrightarrow \mathcal{M}$ (Pu, z, t)$\geq_L \mathcal{M}$(z, z, kt)=$1_{\mathcal{L}}$ as $\mathcal{M}$ is continuous.

Thus we have Pu=Su= z.

Now as P(X) ⊆ T(X), therefore there exists a point v $\in Y$ such that Tv= z. Now on putting x = x_{2n} and y =v in (ii)

we get $\left(\frac{1}{\mathcal{M}(Px_{2n}, Qv, t)} - 1\right) \leq_L \left(\frac{1}{\mathcal{M}(Sx_{2n},\ Tv,\ kt)} - 1\right) - \emptyset\left(\frac{1}{\mathcal{M}(Px_{2n}, Sx_{2n}, kt)} - 1\right)$.

Now on taking the limit as n → ∞ we get,

$$\Longrightarrow \left(\frac{1}{\mathcal{M}(z,\ Qv, t)} - 1\right) \leq_L \left(\frac{1}{\mathcal{M}(z,\ z,\ kt)} - 1\right) - \emptyset\left(\frac{1}{\mathcal{M}(z,\ z,\ kt)} - 1\right).$$

$\Longrightarrow \mathcal{M}$ (z,Qv, t)$\geq_L \mathcal{M}$(z, z, kt)=$1_{\mathcal{L}}$ as $\mathcal{M}$ is continuous.

Thus we have Qv=Tv= z.

⟹ Pu=Su= Qv=Tv= z.

Now we are to prove that Pz=z, for this put x=z and y =v in (ii) we get

$$\left(\frac{1}{\mathcal{M}(Pz, Qv, t)} - 1\right) \leq_L \left(\frac{1}{\mathcal{M}(Sz, Tv, kt)} - 1\right) - \emptyset\left(\frac{1}{\mathcal{M}(Pz, Sz, kt)} - 1\right).$$

$$\Longrightarrow\left(\frac{1}{\mathcal{M}(Pz,\ z,\ t)} - 1\right) \leq_L \left(\frac{1}{\mathcal{M}(Pz, z,\ kt)} - 1\right) - \emptyset\left(\frac{1}{\mathcal{M}(Pz, Pz, kt)} - 1\right) \leq_L \left(\frac{1}{\mathcal{M}(Pz, z,\ kt)} - 1\right)$$

$$\Longrightarrow \mathcal{M}(Pz, z, t) \geq_L \mathcal{M}(Pz, z, kt) \ldots \geq_L \mathcal{M}(Pz, z, k^n t)$$

But as $\mathcal{M}$(x, y, t) is non-decreasing with respect to t for all x, y in X. Hence $\mathcal{M}$ (Pz, z, t) = C for all t >0. Since (X, $\mathcal{M}$, $\mathcal{J}$) has property (C), it follows that C = $1_{\mathcal{L}}$ .Thus we get Pz=z, therefore Pz = Sz= z .Similarly we can prove that Qz = Tz =z.

Thus P, Q, S and T have a fixed point z$\in \bigcap_{i=1}^{m} A_i$.

Now to prove their uniqueness let us choose another point w$\in \bigcap_{i=1}^{m} A_i$.Therefore w is another common fixed point of P, Q, S and T that is Pw= Qw = Sw = Tw = w. Thus

$$(\frac{1}{\mathcal{M}(z,w,t)} -1) = (\frac{1}{\mathcal{M}(Pz,Qw,t)} -1) \leq_L (\frac{1}{\mathcal{M}(Sz,Tw,kt)} -1) - \emptyset(\frac{1}{\mathcal{M}(Pz,Sz,kt)} - 1).$$

$$\Rightarrow (\frac{1}{\mathcal{M}(z,w,t)} -1) = (\frac{1}{\mathcal{M}(z, w, t)} -1) \leq_L (\frac{1}{\mathcal{M}(z, w, kt)} -1) - \emptyset(\frac{1}{\mathcal{M}(z, z, kt)} - 1).$$

$$\leq_L (\frac{1}{\mathcal{M}(z, w, kt)} -1)$$

$$\Rightarrow \mathcal{M}(z, w, t) \geq_L \mathcal{M}(z, w, kt) \ldots\ldots \geq_L \mathcal{M}(z, w, k^n t).$$

Again as $\mathcal{M}$(x, y, t) is non-decreasing with respect to t for all x, y in X. Hence $\mathcal{M}$ (z, w, t) = C for all t >0. . Since (X, $\mathcal{M}$, $\mathcal{I}$) has property (C), it follows that C = $1_{\mathcal{L}}$ .Thus we get z= w .Thus z, is the unique common fixed point of maps P, Q, S and T.

Corollary 3.2. Let (X, $\mathcal{M}$, $\mathcal{I}$) be an $\mathcal{L}$-fuzzy metric space, $A_1, A_2, \ldots\ldots\ldots, A_m$ be closed subsets of X and Y = $\bigcup_{i=1}^{m} A_i$ be complete. Suppose that $\emptyset: [0, +\infty) \to [0, +\infty)$ is continuous, non-decreasing function with $\emptyset(r)$>0 for r $\in (0, +\infty)$ and $\emptyset$(0) = 0.

If P, Q, S and T: Y→Y be a cyclic weak $\emptyset$ - contraction such that

i. P(X)∪Q(X)⊆ S(X)∩T(X),
ii. $(\frac{1}{\mathcal{M}(Px, Qy, t)} -1) \leq_L (\frac{1}{\mathcal{M}(Sx, Ty, kt)} -1) - \emptyset(\frac{1}{\mathcal{M}(Px, Sx, kt)} - 1).$
Then P, Q, S and T has a unique fixed point z $\in \bigcap_{i=1}^{m} A_i$.

Proof: to prove the result replace Q by P in theorem 3.1.

References:

[1] A.T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets Syst. 20 (1986) 87–96.

[2] A. George, P. Veeramani, On some results in fuzzy metric spaces, Fuzzy Sets Syst. 64 (1994) 395–399.

[3] S.S.Chang , Y.J.Cho, B.S.Lee, J.S.Jung and S.M.Kang, Coincidence point and minimization theorems in fuzzy metric spaces, Fuzzy Sets Syst.88(1997),119-128.

[4] Z.K,Deng, Fuzzy pseudo-metric spaces, J.Math. Anal. Appl. 86 (1982), 74-95.

[5] G. Deschrijver, E.E. Kerre, On the relationship between some extensions of fuzzy set theory, Fuzzy Sets Syst. 33 (2003) 227–235.

[6] G.Deschrijver, C.Cornelis and E.E.Kerre, On the representation of intuitionistic fuzzy t-norms and t-conorms, IEEE Transactions on Fuzzy Systems, 12 (2004), 45-61.

[7] M.A.Erceg, Metric spaces in fuzzy ste theory, J.Math. Anal. Appl. 69 (1979), 205-230.

[8] J. Goguen, L-fuzzy sets, J. Math. Anal. Appl. 18 (1967) 145–174.

[9] D.Gopal, M.Imdad, C.Vetro , M. Hasan , Fixed point theory for cyclic weak ∅-contraction in fuzzy metric spaces, Journal of Nonlinear Analysis and Applications, Vol.2012

[10] M.Grabeic , Fixed points in fuzzy metric spaces, Fuzzy Sets Syst.27(1988),385-389.

[11] V.Gregori and A.Sapena, On fixed point theorem in fuzzy metric spaces, Fuzzy Sets Syst.125(2002),245-252.

[12] O.Kaleva and S.Seikkala, On fuzzy metric spaces, Fuzzy Stes Syst. 12 (1984), 215-229.

[13] I.Kramosil and J.Michalek, Fuzzy metric and statistical metric spaces, Kybernetica11 (1975), 326-334.

[14] D.Mihet, A Banach contraction theorem in fuzzy metric spaces, Fuzzy Sets Syst.144(2004),431-439.

[15] D.Mihet, On fuzzy contractive mappings in fuzzy metric spaces, Fuzzy Sets and Systems 158 (2007)915-921.

[16] D.Mihet, Aclass of contractions in fuzzy metric spaces, Fuzzy Sets and Systems 161(2010)1131-1137.

[17] M.Pacurar, I.A.Rus, Fixed point theory for ∅- contractions, Nonlinear Anal.72(2010)1181-1187.

[18] R. Saadati, A. Razani, H. Adibi, A Common fixed point theorem in L-fuzzy metric spaces, Chaos, Solitons& Fractals, doi:10.1016/ j.chaos.2006.01.023.

[19] L.A. Zadeh, Fuzzy sets, Inform. Control 8 (1965) 338–353.

[20] J.Rodriguez Lopez and S.Ramaguera, The Hausdroff Fuzzy metric on compact sets, Fuzzy Sets and Systems, 147(2004), 273-283.

[21] J.X.Fang, On fixed point theorem in fuzzy metric spaces, Fuzzy Sets Syst. 46 (1992), 107-113.

[22] C.Di Bari, C.Vetro, A fixed point theorem for a family of mappings in fuzzy metric space, Rend. Circ.Mat.Palermo.52(2003), 315-332.