

## Semi symmetric non-metric $S$ -connection on an integrated contact metric structure manifold

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### ABSTRACT

The purpose of the present paper is to study some properties of semi-symmetric non-metric  $S$ -connection on an integrated contact metric structure manifold [16] and also the form of curvature tensor  $R$  of the manifold relative to this connection has been derived. It has been shown that if an integrated contact metric structure manifold admits a semi-symmetric non-metric  $S$ -connection whose curvature tensor is locally isometric to the unit sphere  $S^{(n)}(1)$ , then the Conformal and Con-harmonic curvature tensors with respect to the Riemannian connection are identical iff  $n - \frac{a^2}{c}(n+2) = 0$ . Also it has been shown that if an integrated contact metric structure manifold admits a semi-symmetric non-metric  $S$ -connection whose curvature tensor is locally isometric to the unit sphere  $S^{(n)}(1)$ , then the Con-circular curvature tensor coincides with the Riemannian connection iff  $n - \frac{a^2}{c}(n+2) = 0$ . Some other useful results and theorem have been obtained, which are of great geometrical importance.

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**Keywords and Phrases:**  $C^\infty$ -manifold, integrated contact metric structure manifold, semi-symmetric non-metric  $S$ -connection, curvature tensors.

### 1. INTRODUCTION:

The idea of semi-symmetric linear connection in a differentiable manifold has been introduced by Friedman and Schouten [1]. Hayden [4] has introduced the idea of metric connection with torsion in a Riemannian manifold. The properties of semi-symmetric metric connection in a Riemannian manifold have been studied by Yana [7] and others [14] [18]. Various properties of such connection have been studied by many geometers. Agashe and Chafle

[10] defined and studied a semi-symmetric non-metric connection in a Riemannian manifold. This was further developed by Agashe and Chafle [11], Prasad [3], De and Kamilya [19], Tripathi and Kakkar [9], Pandey and Ojha [8], Chaturvedi and Pandey [2] and several other geometers. Sengupta, De and Binh [5], De and Sengupta [20] defined new types of semi-symmetric non-metric connections on a Riemannian manifold and studied some geometrical properties with respect to such connections. Ojha and Choubey [15] defined semi-symmetric non-metric connection in an almost contact metric manifold.

## 2. PRELIMINARIES:

Let  $M_n$  be a differentiable manifold of differentiability class  $C^\infty$ . Let there exist in  $M_n$  a vector valued  $C^\infty$ -linear function  $\Phi$ , a  $C^\infty$ -vector field  $\eta$  and a  $C^\infty$ -one form  $\xi$  such that

$$(2.1) \quad \Phi^2(X) = a^2 X - c\xi(X)\eta$$

$$(2.2) \quad \bar{\eta} = 0,$$

$$(2.3) \quad G(\bar{X}, \bar{Y}) = a^2 G(X, Y) - c\xi(X)\xi(Y)$$

Where  $\Phi(X) = \bar{X}$ ,  $a$  is a nonzero complex number and  $c$  is an integer.

Let us agree to say that  $\Phi$  gives to  $M_n$  a differentiable structure define by algebraic equation (2.1). We shall call  $(\Phi, \eta, a, c, \xi)$  as an integrated contact structure.

**Remark 2.1:** The manifold  $M_n$  equipped with an integrated contact structure  $(\Phi, \eta, a, c, \xi)$  will be called an integrated contact structure manifold.

**Remark 2.2:** The  $C^\infty$ -manifold  $M_n$  satisfying (2.1), (2.2) and (2.3) is called an integrated contact metric structure manifold  $(\Phi, \eta, a, c, G, \xi)$

**Agreement 2.1:** All the equations which follow will hold for arbitrary vector field  $X, Y, Z, \dots$  etc.

It is easy to calculate in  $M_n$  that

$$(2.4) \quad \xi(\eta) = \frac{a^2}{c}$$

$$(2.5) \quad G(X, \eta) \underline{\underline{\text{def}}} \xi(X)$$

and

$$(2.6) \quad \Phi(\bar{X}) = 0$$

**Remark 2.3:** The integrated contact metric structure manifold  $(\Phi, \eta, a, c, G, \xi)$  gives an almost Norden contact metric manifold [13], Lorentzian Para-contact manifold [6] or an almost Para-contact Riemannian manifold [17] according as  $(a^2 = -1, c = 1)$ ,  $(a^2 = 1, c = -1)$  or  $(a^2 = 1, c = 1)$

**Agreement 2.2:** An integrated contact metric structure manifold will be denoted by  $M_n$ .

**Definition 2.1:** A  $C^\infty$ -manifold  $M_n$ , satisfying

$$(2.7) \quad D_X \eta = \Phi(X) \underline{\underline{\text{def}}} \bar{X}$$

will be denoted by  $M_n^*$

In  $M_n^*$ , we can easily shown that

$$(2.8) \quad (D_X \xi)(Y) = \Phi(X, Y) = (D_Y \xi)(X)$$

where

$$(2.9) \quad \Phi(X, Y) \underline{\underline{\text{def}}} G(\bar{X}, Y) = G(X, \bar{Y})$$

**Definition 2.2:** An affine connection  $B$  is said to be metric if

$$(2.10) \quad B_X G = 0$$

The affine metric connection  $B$  satisfying

$$(2.11) \quad (B_X \Phi)(Y) = \xi(Y)X - G(X, Y)\eta$$

is called metric  $S$ -connection

A metric  $S$ -connection  $B$  is called semi-symmetric non-metric  $S$ -connection iff

$$(2.12) \quad B_X Y = D_X Y - \xi(Y)X - G(X, Y)\eta$$

Where  $D$  is the Riemannian connection.

Also

$$(B_X G)(Y, Z) = 2\xi(Y)G(X, Z) + 2\xi(Z)G(X, Y)$$

which implies

$$(2.13) \quad S(X, Y) = \xi(Y)X - \xi(X)Y$$

where  $S$  is the torsion tensor of connection  $B$ .

The curvature tensor with respect to the semi-symmetric non-metric connection is defined as

$$(2.14) \quad \tilde{R}(X, Y, Z) \underline{\underline{\text{def}}} B_X B_Y Z - B_Y B_X Z - B_{[X, Y]}Z$$

Using (2.12) in (2.14), we get

$$(2.15) \quad \begin{aligned} \tilde{R}(X, Y, Z) = & K(X, Y, Z) - \beta(X, Z)Y + \beta(Y, Z)X \\ & - G(Y, Z)(D_X \eta - \xi(X)\eta) + G(X, Z)(D_Y \eta - \xi(Y)\eta) \end{aligned}$$

Where

$$(2.16) \quad \beta(X, Y) = (D_X \xi)(Y) + \xi(X)\xi(Y) + G(X, Y)\xi(\eta)$$

and

$$(2.17) \quad K(X, Y, Z) \underline{\underline{\text{def}}} D_X D_Y Z - D_Y D_X Z - D_{[X, Y]}Z$$

where  $\tilde{R}$  and  $K$  be the curvature tensors with respect to the connection  $B$  and  $D$  respectively.

Using (2.7) in (2.15), we get

$$(2.18) \quad \begin{aligned} \tilde{R}(X, Y, Z) = & K(X, Y, Z) - \beta(X, Z)Y + \beta(Y, Z)X \\ & - G(Y, Z)(\bar{X} - \xi(X)\eta) + G(X, Z)(\bar{Y} - \xi(Y)\eta) \end{aligned}$$

If  $\tilde{R}(X, Y, Z) = 0$  then above equation becomes

$$(2.19) \quad K(X, Y, Z) - \beta(X, Z)Y + \beta(Y, Z)X - G(Y, Z)(\bar{X} - \xi(X)\eta) + G(X, Z)(\bar{Y} - \xi(Y)\eta) = 0$$

Contracting above equation with respect to  $X$ , we get

$$(2.20) \quad Ric(Y, Z) - \beta(Y, Z) + n\beta(Y, Z) + \frac{a^2}{c}G(Y, Z) + G(\bar{Y}, Z) - \xi(Y)\xi(Z) = 0$$

Using (2.16) in (2.20), we get

$$(2.21) \quad cRic(Y, Z) + c(n-1) \left[ \Phi(Y, Z) + \xi(Y)\xi(Z) + \frac{a^2}{c}G(Y, Z) \right] + G(\bar{Y}, \bar{Z}) + cG(\bar{Y}, Z) = 0$$

Contracting above equation with respect to  $Z$ , we get

$$(2.22) \quad rY + n \left( \frac{a^2}{c}Y + \bar{Y} \right) + (n-2)\xi(Y)\eta = 0$$

Contracting above equation with respect to  $Y$ , we get

$$(2.23) \quad R = -\frac{a^2}{c}(n+2)(n-1)$$

Where  $Ric$  and  $R$  are Ricci tensor and scalar curvature of the manifold respectively.

The Projective curvature tensor  $W$ , Conformal curvature tensor  $Q$ , Con-harmonic curvature tensor  $L$  and Con-circular curvature tensor  $C$  in a Riemannian manifold are given by [12]

$$(2.24) \quad W(X, Y, Z) = K(X, Y, Z) - \frac{1}{(n-1)} [Ric(Y, Z)X - Ric(X, Z)Y]$$

$$(2.25) \quad Q(X, Y, Z) = K(X, Y, Z) - \frac{1}{(n-2)} [Ric(Y, Z)X - Ric(X, Z)Y + G(Y, Z)r(X) - G(X, Z)r(Y)] + \frac{R}{(n-1)(n-2)} [G(Y, Z)X - G(X, Z)Y]$$

$$(2.26) \quad L(X, Y, Z) = K(X, Y, Z) - \frac{1}{(n-2)} [Ric(Y, Z)X - Ric(X, Z)Y + G(Y, Z)r(X) - G(X, Z)r(Y)]$$

and

$$(2.27) \quad C(X, Y, Z) = K(X, Y, Z) - \frac{R}{n(n-2)} [G(Y, Z)X - G(X, Z)Y]$$

where

$$(2.28) \quad W(X, Y, Z, T) \stackrel{\text{def}}{=} G(W(X, Y, Z), T)$$

$$(2.29) \quad Q(X, Y, Z, T) \stackrel{\text{def}}{=} G(Q(X, Y, Z), T)$$

$$(2.30) \quad L(X, Y, Z, T) \stackrel{\text{def}}{=} G(L(X, Y, Z), T)$$

$$(2.31) \quad C(X, Y, Z, T) \stackrel{\text{def}}{=} G(C(X, Y, Z), T)$$

### 3. CURVATURE TENSORS:

**Theorem 3.1:** If an integrated contact metric structure manifold admits a semi-symmetric non-metric  $S$ -connection whose curvature tensor is locally isometric to the unit sphere  $S^{(n)}(1)$ , then the Conformal

and Con-harmonic curvature tensors with respect to the Riemannian connection are identical iff

$$n - \frac{a^2}{c}(n+2) = 0$$

**Proof:** If the curvature tensor with respect to the semi-symmetric non metric  $S$  - connection is locally isometric to the unit sphere  $S^{(n)}(1)$ , then

$$(3.1) \quad \tilde{R}(X, Y, Z) = G(Y, Z)X - G(X, Z)Y$$

Using (3.1) in (2.18), we get

$$(3.2) \quad G(Y, Z)X - G(X, Z)Y = K(X, Y, Z) - \beta(X, Z)Y + \beta(Y, Z)X \\ - G(Y, Z)(\bar{X} - \xi(X)\eta) + G(X, Z)(\bar{Y} - \xi(Y)\eta)$$

Contracting above with respect to  $X$  and using (2.4) and (2.9), we get

$$(3.3) \quad cRic(Y, Z) = c(n-1) \left[ G(Y, Z) - \Phi(Y, Z) - \xi(Y)\xi(Z) - \frac{a^2}{c}G(Y, Z) \right] \\ - G(\bar{Y}, \bar{Z}) - cG(\bar{Y}, Z)$$

Contracting above equation with respect to  $Z$ , we get

$$(3.4) \quad cr = -cn(\bar{Y} - Y) - (n-2)c\xi(Y)\eta - (a^2n + c)Y$$

Contracting above equation with respect to  $Y$ , we get

$$(3.5) \quad R = (n-1) \left[ n - \frac{a^2}{c}(n+2) \right]$$

Where  $Ric$  and  $R$  are Ricci tensor and scalar curvature of the manifold respectively.

From (3.5), (2.25) and (2.26), we obtain the necessary part of the theorem. Converse part is obvious from (2.25) and (2.26).

Now, using (2.19) and (2.21) in (2.24), we get

$$(3.6) \quad W(X, Y, Z) = \beta(X, Z)Y - \beta(Y, Z)X + G(Y, Z)\bar{X} - G(Y, Z)\xi(X)\eta \\ - G(X, Z)\bar{Y} + G(X, Z)\xi(Y)\eta + \xi(Y)\xi(Z)X - \xi(X)\xi(Z)Y \\ + \frac{n}{(n-1)}[G(\bar{Y}, Z)X - G(\bar{X}, Z)Y] + \frac{1}{c(n-1)}[G(\bar{Y}, \bar{Z})X - G(\bar{X}, \bar{Z})Y] \\ + \frac{a^2}{c}[G(Y, Z)X - G(X, Z)Y]$$

Now operating  $G$  on both the sides of above equation and using (2.5) and (2.28), we get

$$(3.7) \quad W(X, Y, Z, T) = \beta(X, Z)G(Y, T) - \beta(Y, Z)G(X, T) + G(Y, Z)G(\bar{X}, T) \\ - G(Y, Z)\xi(X)\xi(T) - G(X, Z)G(\bar{Y}, T) + G(X, Z)\xi(Y)\xi(T) + \xi(Y)\xi(Z)G(X, T) \\ - \xi(X)\xi(Z)G(Y, T) + \frac{n}{(n-1)}[G(\bar{Y}, Z)G(X, T) - G(\bar{X}, Z)G(Y, T)] \\ + \frac{1}{c(n-1)}[G(\bar{Y}, \bar{Z})G(X, T) - G(\bar{X}, \bar{Z})G(Y, T)]$$

$$+\frac{a^2}{c}\left[G(Y,Z)G(X,T)-G(X,Z)G(Y,T)\right]$$

**Theorem 3.2:** On a  $C^\infty$ -manifold  $M_n$ , we have

$$(3.8a) \quad \mathcal{W}(X,Y,Z,\eta) = \beta(X,Z)\xi(Y) - \beta(Y,Z)\xi(X)$$

$$+\frac{n}{(n-1)}\left[\Phi(Y,Z)\xi(X) - \Phi(X,Z)\xi(Y)\right]$$

$$+\frac{1}{c(n-1)}\left[G(\bar{Y},\bar{Z})\xi(X) - G(\bar{X},\bar{Z})\xi(Y)\right]$$

$$(3.8b) \quad \mathcal{W}(\eta,Y,Z,\eta) = \beta(\eta,Z)\xi(Y) - \frac{a^2}{c}\beta(Y,Z)$$

$$+\frac{a^2}{c}\left(\frac{n}{n-1}\right)G(\bar{Y},Z) + \frac{a^2}{c^2}\left(\frac{n}{n-1}\right)G(\bar{Y},\bar{Z})$$

$$(3.8c) \quad \mathcal{W}(\bar{X},\bar{Y},Z,\eta) = 0$$

$$(3.8d) \quad \mathcal{W}(X,Y,\eta,\eta) = \beta(X,\eta)\xi(Y) - \beta(Y,\eta)\xi(X)$$

$$(3.8e) \quad \mathcal{W}(\eta,Y,Z,T) = \beta(\eta,Z)G(Y,T) - \beta(Y,Z)\xi(T) - \xi(Z)G(\bar{Y},T)$$

$$+2\xi(Z)\xi(Y)\xi(T) - \frac{2a^2}{c}G(Y,T)\xi(Z) + \frac{n}{(n-1)}G(\bar{Y},Z)\xi(T)$$

$$-\frac{1}{c(n-1)}G(\bar{Y},\bar{Z})\xi(T)$$

$$(3.8f) \quad \mathcal{W}(\eta,Y,Z,\eta) = \beta(\eta,Z)\xi(Y) - \frac{a^2}{c}\beta(Y,Z) + \frac{a^2}{c^2(n-1)}\left[ncG(\bar{Y},Z) + G(\bar{Y},Z)\right]$$

**Proof:** Replacing  $T$  by  $\eta$  in (3.7) and using (2.4), (2.5), (2.6) and (2.9) we get (3.8a).

Replacing  $X$  by  $\eta$  in (3.8a) and using (2.2), (2.4), (2.5), (2.6) and (2.9) we get (3.8b).

Replacing  $X$  by  $\bar{X}$  and  $Y$  by  $\bar{Y}$  in (3.8a) and using (2.6), we get (3.8c).

Replacing  $Z$  by  $\eta$  in (3.8a) and using (2.2), (2.6) and (2.9), we get (3.8d).

Replacing  $X$  by  $\eta$  in (3.7) and using (2.2), (2.4), (2.5), we get (3.8e).

Replacing  $T$  by  $\eta$  in (3.8e) and using (2.4), (2.5) and (2.6), we get (3.8f).

**Theorem 3.3:** If an integrated contact metric structure manifold admits a semi-symmetric non metric  $S$ -connection whose curvature tensor is locally isometric to the unit sphere  $S^{(n)}(1)$ , then the Con-circular

curvature tensor coincides with respect to the Riemannian connection iff  $n - \frac{a^2}{c}(n+2) = 0$

**Proof:** Using (3.5) in (2.27), we get

$$(3.9) \quad C(X,Y,Z) = K(X,Y,Z) - \frac{\left[n - \frac{a^2}{c}(n+2)\right]}{n} [G(Y,Z)X - G(X,Z)Y]$$

which is required proves of the theorem.

Now, using (2.19) and (2.23) in (2.27), we get

$$(3.10) \quad C(X, Y, Z) = \beta(X, Z)Y - \beta(Y, Z)X + G(Y, Z)(\bar{X}) - G(Y, Z)\xi(X)\eta \\ - G(X, Z)\bar{Y} + G(X, Z)\xi(Y)\eta + \frac{a^2}{c} \left( \frac{n+2}{n} \right) [G(Y, Z)X - G(X, Z)Y]$$

Operating  $G$  on both the sides of above equation and using (2.5), (2.9) and (2.31), we get

$$(3.11) \quad \nabla C(X, Y, Z, T) = \beta(X, Z)G(Y, T) - \beta(Y, Z)G(X, T) + G(Y, Z)\Phi(X, T) \\ - G(Y, Z)\xi(X)\xi(T) - G(X, Z)\Phi(Y, T) + G(X, Z)\xi(Y)\xi(T) \\ + \frac{a^2}{c} \left( \frac{n+2}{n} \right) [G(Y, Z)G(X, T) - G(X, Z)G(Y, T)]$$

**Theorem 3.4:** On  $C^\infty$ -manifold we have

$$(3.12a) \quad \nabla C(X, Y, Z, \eta) = \beta(X, Z)\xi(Y) - \beta(Y, Z)\xi(X) - \frac{a^2}{c}G(Y, Z)\xi(X) \\ + \frac{a^2}{c}G(X, Z)\xi(Y) + \frac{a^2}{c} \left( \frac{n+2}{n} \right) [G(Y, Z)\xi(X) - G(X, Z)\xi(Y)]$$

$$(3.12b) \quad \nabla C(\eta, Y, Z, T) = \beta(\eta, Z)G(Y, T) - \beta(Y, Z)\xi(T) - \frac{a^2}{c}G(Y, Z)\xi(T) \\ - \xi(Z)G(\bar{Y}, T) + \xi(Y)\xi(Z)\xi(T) + \frac{a^2}{c} \left( \frac{n+2}{n} \right) [G(Y, Z)\xi(T) - G(Y, T)\xi(Z)]$$

$$(3.12c) \quad \nabla C(\eta, Y, Z, \eta) = \beta(\eta, Z)\xi(Y) - \frac{a^2}{c}\beta(Y, Z) - \frac{a^4}{c^2}G(Y, Z) \\ + \frac{a^2}{c}\xi(Y)\xi(Z) + \frac{a^2}{c} \left( \frac{n+2}{n} \right) \left[ \frac{a^2}{c}G(Y, Z) - \xi(Y)\xi(Z) \right]$$

$$(3.12d) \quad \nabla C(X, Y, \eta, \eta) = \beta(X, \eta)\xi(Y) - \beta(Y, \eta)\xi(X)$$

$$(3.12e) \quad \nabla C(\bar{X}, \bar{Y}, Z, \eta) = 0$$

$$(3.12f) \quad \nabla C(\eta, Y, \bar{Z}, \bar{T}) = \beta(\eta, \bar{Z})G(Y, \bar{T})$$

**Proof:** Replacing  $T$  by  $\eta$  in (3.8) and using (2.4), (2.5), (2.6) and (2.9), we get (3.12a).

Replacing  $X$  by  $\eta$  in (3.8) and using (2.2), (2.4), (2.5), (2.6) and (2.9), we get (3.12b)

Replacing  $T$  by  $\eta$  in (3.12b) and using (2.4) and (2.5), we get (3.12c).

Replacing  $Z$  by  $\eta$  in (3.12a) and using (2.5), we get (3.12d).

Replacing  $X$  by and  $Y$  by in (3.12a) and using (2.4), we get (3.12e).

Replacing  $Z$  by and  $T$  by in (3.12b) and using (2.4), we get (3.12f).

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