vg -CLOSED MAPPINGS

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Abstract: The aim of this paper is to introduce and study the concepts of *vg*-closed and almost *vg*-closed mappings and the interrelationship between other almost closed maps.
Keywords: *vg*-closed set, almost *vg*-closed map.
AMS Classification: 54C10, 54C08, 54C05

§1. INTRODUCTION:

Mappings plays an important role in the study of modern mathematics, especially in Topology and Functional analysis. Closed mappings are one such mappings which are studied for different types of closed sets by various mathematicians for the past many years. N.Biswas, discussed about semiopen mappings in the year 1970, A.S.Mashhour, M.E.Abd El-Monsef and S.N.El-Deeb studied preopen mappings in the year 1982 and S.N.El-Deeb, and I.A.Hasanien defind and studied about preclosed mappings in the year 1983. Further Asit kumar sen and P. Bhattacharya discussed about pre-closed mappings in the year 1993. A.S.Mashhour, I.A.Hasanein and S.N.El-Deeb introduced α-open and α-closed mappings in the year in 1983, F.Cammaroto and T.Noiri discussed about semipre-open and semipre-clsoed mappings in the year 1989 and G.B.Navalagi further verified few results about semipreclosed mappings. M.E.Abd El-Monsef, S.N.El-Deeb and R.A.Mahmoud introduced β-open mappings in the year 1983 and Saeid Jafari and T.Noiri, studied about β-closed mappings in the year 2000. In the year 2010, S. Balasubramanian and P.A.S.Vyjayanthi introduced *v*-open mappings and in the year 2011 they further defined almost *v*-open mappings. In the last year S. Balasubramanian and P.A.S.Vyjayanthi introduced *v*-closed and Almost *v*-closed mappings. Author of the present paper studied *vg*-open mappings in the year 2011. In the present paper author tried to study a new variety of closed map called *vg*-closed and almost *vg*-closed map. Throughout the paper *X*, *Y* means topological spaces (*X*, τ) and (*Y*,σ) on which no separation axioms are assured.

§2. PRELIMINARIES:

Definition 2.1: $A \subseteq X$ is said to be

a) regular open[pre-open; semi-open; α-open; β-open] if A= int(cl(A)) [$A \subseteq$int(cl(A)); $A \subseteq$ cl(int(A)); $A \subseteq$ int(cl(int(A))); $A \subseteq$ cl(int(cl(A)))] and regular closed[pre-closed; semi-closed; α-closed; β-closed] if A = cl(int(A))[cl(int(A)) $\subseteq$ A; int(cl(A))$\subseteq$ A; cl(int(cl(A)))$\subseteq$ A; int(cl(int(A)))$\subseteq$ A]
b) *v*-open if there exists a r-open set U such that $U \subseteq A \subseteq cl(U)$.
c) g-closed[rg-closed] if cl(A)⊂U[rcl(A)⊂U] whenever A⊂U and U is open[r-open] in X.
d) sg-closed[gs-closed] if scl(A)⊂U whenever A⊂U and U is s-open[open] in X.
e) pg-closed[gp-closed] if pcl(A)⊂U whenever A⊂U and U is p-open[open] in X.
f) αg-closed[gα-closed; rgα-closed] if αcl(A)⊂U whenever A⊂U and U is α-open[open; rα-open] in X.
g) βg-closed[gβ-closed] if βcl(A)⊂U whenever A⊂U and U is β-open[open] in X.
h) *vg*-closed if *vcl*(A)⊂U whenever A⊂U and U is *v*-open in X.
i) g-open[rg-open; sg-open; gs-open; pg-open; gp-open; *vg*-open; αg-open; gα-open; rgα-open; βg-open; gβ-open] if its complement X - A is g-closed[rg-closed; sg-closed; gs-closed; pg-closed; gp-closed; *vg*-closed; αg-closed; gα-closed; rgα-closed; βg-closed; gβ-closed].

Remark 1: We have the following implication diagrams for closed sets.

g.closed   gs.closed
↓   ↓
rαg.closed → rgα.closed → rg.closed → ***vg*.closed** ← sg.closed ← βg.closed
↑   ↑   ↑   ↑   ↑
→ → rα.closed → → → *v*.closed → → → →
↑   ↓ ↑   ↑
r.closed → π.closed → closed → α.closed → s.closed → β.closed
↓   ↓   ↓
πg.closed   p.closed → ω.closed   gα.closed
↓   ↓
gp.closed ← pg.closed   rω.closed

Definition 2.2: A function $f:X\to Y$ is said to be

a) continuous [resp: semi-continuous, r-continuous, *v*-continuous] if the inverse image of every open set is open [resp: semi open, regular open, *v*-open].
b) irresolute [resp: r-irresolute, *v*-irresolute] if the inverse image of every semi open [resp: regular open, *v*-open] set is semi open [resp: regular open, *v*-open].
c) open[resp: r-open, semi-open, pre-open, α-open, β-open, rα-open] if the image of every open set is open[resp: regular-open, semi-open, pre-open, α-open, β-open, rα-open].
d) g-continuous [resp: rg-continuous] if the inverse image of every closed set is g-closed [resp: rg-closed].
e) g-open[resp: rg-open, sg-open, pg-open, αg-open, βg-open, rαg-open, rgα-open, gs-open, gp-open, gα-open] if the image of every open set is g-open[resp: rg-open, sg-open, pg-open, αg-open, βg-open, rαg-open, rgα-open, gs-open, gp-open, gα-open].

Definition 2.3: *X* is said to be

(i) *v*-regular space (or *v*-T_3 space) if for a open set F and a point $x\notin F$, there exists disjoint *v*-open sets G and H such that $F\subseteq G$ and $x\in H$.

(ii) *vg*-regular space (or *vg*-T_3 space) if for a open set F and a point $x\notin F$, there exists disjoint *v*-open sets G and H such that $F\subseteq G$ and $x\in H$.

Definition 2.4: *X* is said to be $T_{1/2}$[r-$T_{1/2}$] if every (regular) generalized closed set is (regular) closed.

§3. *vg*-CLOSED MAPPINGS:

Definition 3.1: A function $f: X\to Y$ is said to be *vg*-closed if the image of every closed set in *X* is *vg*-closed in *Y*.

Theorem 3.1: We have the following implication diagrams for closed mappings.

(i)
g.closed   gs.closed
↓   ↓
rαg.closed → rgα.closed → rg.closed → ***vg*.closed** ← sg.closed ← βg.closed
↑   ↑   ↑   ↑   ↑
→ → rα.closed → → *v*.closed →
↑   ↓ ↑   ↑
r.closed → π.closed → closed → α.closed → s.closed → β.closed
↓   ↓   ↓
πg.closed   p.closed → ω.closed   gα.closed
↓   ↓
gp.closed ← pg.closed   rω.closed   None is reversible

(ii) If *v*GO(Y) = RO(Y), then the reverse relations hold for all closed maps.

g.closed   gs.closed
↑↓   ↑↓
rαg.closed ↔ rgα.closed ↔ rg.closed ↔ ***vg*.closed** ↔ sg.closed ↔ βg.closed
↑↓   ↑↓   ↑↓   ↑↓   ↑↓
↔ ↔ rα.closed ↔ ↔ ↔ *v*.closed ↔
↑↓   ↑↓   ↑↓   ↑↓
r.closed ↔ π.closed ↔ closed ↔ α.closed ↔ s.closed ↔ β.closed

Example 1: Let $X = Y = \{a, b, c\}$; $\tau = \{\varphi, \{a\}, \{a, b\}, X\}$; $\sigma = \{\varphi, \{a, c\}, Y\}$. Let $f{:}X\rightarrow Y$ be defined $f(a) = c$, $f(b) = b$ and $f(c) = a$. Then f is *vg*-closed.and *v*-closed but not *v*-closed.

Example 2: Let $X = Y = \{a, b, c\}$; $\tau = \{\varphi, \{a\}, \{b, c\}, X\}$; $\sigma = \{\varphi, \{a, c\}, Y\}$. Let $f{:}X\rightarrow Y$ be defined $f(a) = b$, $f(b) = c$ and $f(c) = a$. Then f is *vg*-closed but not *v*-closed.

Theorem 3.2:
(i) If (Y, σ) is discrete, then f is closed of all types.
(ii) If f is closed[*r*-closed] and g is *vg*-closed then gof is *vg*-closed.
(iii) If f and g are *r*-closed then gof is *vg*-closed.

Corollary 3.1: If f is closed[r-closed] and g is *g*-[*rg-; sg-; gs-;* βg-; $r\alpha g$-; $rg\alpha$-; $r\alpha$-; α-; *s-; p-;* β-; *v-;* π-; r-] closed then gof is *vg*-closed.

Corollary 3.2: If f is almost closed[almost r-closed] and g is *g*-[*rg-; sg-; gs-;* βg-; $r\alpha g$-; $rg\alpha$-; $r\alpha$-; α-; *s-; p-;* β-; *v-;* π-; r-] closed then gof is *vg*-closed.

Theorem 3.3: If f: $X\rightarrow Y$ is *vg*-closed, then $vg(cl\{f(A)\}) \subset f(cl\{A\})$
Proof: Let $A\subset X$ be closed and f: $X\rightarrow Y$ is *vg*-closed gives $f(cl\{A\})$ is *vg*-closed in Y and $f(A) \subset f(cl\{A\})$ which in turn gives $vg(cl\{f(A)\}) \subset vgcl\{(f(cl\{A\}))\}$- - - - - (1)
Since $f(cl\{A\})$ is *vg*-closed in Y, $vgcl\{(f(cl\{A\}))\} = f(cl\{A\})$ - - - - - - (2)
From (1) and (2) we have $vg(cl\{f(A)\}) \subset (f(cl\{A\}))$ for every subset A of X.

Remark 2: Converse is not true in general.

Corollary 3.3: If f: $X\rightarrow Y$ is *g*-[*rg-; sg-; gs-;* βg-; $r\alpha g$-; $rg\alpha$-; $r\alpha$-; α-; *s-; p-;* β-; *v-;* π-; r-] closed, then $vg(cl\{f(A)\}) \subset f(cl\{A\})$

Theorem 3.4: If $f{:}X\rightarrow Y$ is *vg*-closed and $A\subseteq X$ is closed, $f(A)$ is τ_{vg}-closed in Y.
Proof: Let $A\subset X$ be closed and f: $X\rightarrow Y$ is *vg*-closed implies $vg(cl\{f(A)\}) \subset f(cl\{A\})$ which in turn implies $vg(cl\{f(A)\}) \subset f(A)$, since $f(A) = f(cl\{A\})$. But $f(A) \subset vg(cl\{f(A)\})$. Combaining we get $f(A) = vg(cl\{f(A)\})$. Hence $f(A)$ is τ_{vg}-closed in Y.

Theorem 3.5: If $f{:}X\rightarrow Y$ is *g*-[*rg-; sg-; gs-;* βg-; $r\alpha g$-; $rg\alpha$-; $r\alpha$-; α-; *s-; p-;* β-; *v-;* π-; r-]closed and $A\subseteq X$ is closed, $f(A)$ is τ_{vg}-closed in Y.
Proof: For $A\subset X$ is closed and $f{:}X\rightarrow Y$ is *rg*-closed, $f(A)$ is τ_{rg}-closed in Y and so $f(A)$ is τ_{vg}-closed in Y. [since *g*-closed set is *vg*-closed]. Similarly we can preove the remaining results.

Theorem 3.6: If $vg(cl\{A\}) = r(cl\{A\})$ for every $A\subset Y$ and X is discrete space, then the following are equivalent:
a) f: $X\rightarrow Y$ is *vg*-closed map
b) $vg(cl\{f(A)\}) \subset f(cl\{A\})$
Proof: (a) $\Rightarrow$ (b) follows from theorem 3.3
(b) $\Rightarrow$ (a) Let A be any closed set in X, then $f(A) = f(cl\{A\}) \supset vg(cl\{f(A)\})$ by hypothesis. We have $f(A) \subset vg(cl\{f(A)\})$. Combining we get $f(A) = vg(cl\{f(A)\}) = r(cl\{f(A)\})$[by given condition] which implies $f(A)$ is *r*-closed and hence *vg*-closed. Thus f is *vg*-closed.

Theorem 3.7: If $v(cl\{A\}) = r(cl\{A\})$ for every $A\subset Y$ and X is discrete space, then the following are equivalent:
c) f: $X\rightarrow Y$ is *vg*-closed map
d) $vg(cl\{f(A)\}) \subset f(cl\{A\})$
Proof: (a) $\Rightarrow$ (b) follows from theorem 3.3
(b) $\Rightarrow$ (a) Let A be any closed set in X, then $f(A) = f(cl\{A\}) \supset vg(cl\{f(A)\})$ by hypothesis. We have $f(A) \subset vg(cl\{f(A)\})$. Combining we get $f(A) = vg(cl\{f(A)\}) = r(cl\{f(A)\})$[by given condition] which implies $f(A)$ is *r*-closed and hence *vg*-closed. Thus f is *vg*-closed.

Theorem 3.8: $f{:}X\rightarrow Y$ is *vg*-closed iff for each subset S of Y and each $U\in RO(X, f^{-1}(S))$, there is an *vg*-closed set V of Y such that $S\subseteq V$ and $f^{-1}(V) \subseteq U$.

Proof: Let S$\subseteq$*Y* and U$\in$RO(X, f^{1}(S)). Then V = *f*(U) is *vg*-closed in *Y* as *f* is *vg*-closed. f^{-1}(S)$\subseteq$U $\Rightarrow$ S$\subseteq$*f*(U) = V and f^{-1}(V) = f^{-1}(*f*(U)) = U

Conversely Let U be r-closed in *X*. Then by hypothesis there exists an *vg*-closed set V of *Y*, such that f^{-1}(V) $\subseteq$U and so V $\subseteq$ *f* (U). Thus *f*(U) is *vg*-closed in *Y*. Hence *f* is *vg*-closed.

Remark 3: Composition of two *vg*-closed maps is not *vg*-closed in general.

Theorem 3.9: Let *X*, *Y*, *Z* be topological spaces and every *vg*-closed set is *r*-closed[closed] in *Y*. Then the composition of two *vg*-closed maps is *vg*-closed.
Proof: (a) Let *f* and g be *vg*-closed maps. Let A be any closed set in *X* $\Rightarrow$ *f*(A) is *r*-closed[closed] in *Y* (by assumption) $\Rightarrow$ *g*(*f* (A)) = *g o f*(A) is *vg*-closed in *Z*. Hence *gof* is *vg*-closed.

Theorem 3.10: Let *X*, *Y*, *Z* be topological spaces and every *v*-closed set is closed [*r*-closed] in *Y*. Then the composition of two *v*-closed[*r*-closed] maps is *vg*-closed.
Proof: (a) Let *f*, g be *v*-closed maps. Let A be closed in *X* $\Rightarrow$ *f*(A) is *v*-closed and so closed in *Y* (by assumption) $\Rightarrow$ *g*(*f* (A)) = *gof*(A) is *v*-closed in *Z*. Hence *gof* is *vg*-closed [since every *v*-closed set is *vg*-closed].

Theorem 3.11: Let *X*, *Y*, *Z* be topological spaces and every *g*-[*rg-; sg-; gs-; βg-; rαg-; rgα-*; *rα-*; *α-; s-; p-*; *β-;* π-]closed set is closed [*r*-closed] in *Y*. Then the composition of two *g*-[*rg-; sg-; gs-; βg-; rαg-; rgα-*; *rα-*; *α-; s-; p-*; *β-; v-;* π-; r-]closed maps is *vg*-closed.
Proof: Let A be closed set in X, then *f*(A) is *sg*-closed in Y and so closed in *Y* (by assumption) $\Rightarrow$ *g*(*f* (A)) = *gof*(A) is *sg*-closed in *Z*. Hence *gof* is *vg*-closed [since every *sg*-closed set is *vg*-closed].

Corollary 3.4: Let *X*, *Y*, *Z* be topological spaces and every *g*-[*rg-; sg-; gs-; βg-; rαg-; rgα-*; *rα-*; *α-; s-; p-*; *β-;* π-]closed set is *r*-closed in *Y*. Then the composition of two al-*g*-[al-*rg-;* al-*sg-;* al-*gs-;* al-*βg-;* al-*rαg-;* al-*rgα-*; al-*r-*; al-*rα-*; al-*α-;* al-*s-;* al-*p-*; al-*β-;* al-*v-;* al-π-]closed maps is *vg*-closed.
Proof: Let A be *r*-closed set in X, then *f*(A) is *sg*-closed in Y and so *r*-closed in *Y* (by assumption) $\Rightarrow$ *g*(*f* (A)) = *gof*(A) is *sg*-closed in *Z*. Hence *gof* is *vg*-closed [since every *sg*-closed set is *vg*-closed].

Example 3: Let *X* = *Y* = Z = {a, b, c}; τ = {φ, {a}, {a, b}, *X*}; σ = {φ, {a, c}, *Y*} and η = {φ, {a}, {b, c}, *Z*}. *f:X*→*Y* be defined *f*(a) = c, *f*(b) = b and *f*(c) = a and *g:Y*→*Z* be defined *g*(a) = b, *g*(b) = a and *g*(c) = c, then *g, f* and *g o f* are *vg*-closed.

Theorem 3.12: If *f:X*→*Y* is g-closed[rg-closed], g:*Y*→*Z* is *vg*-closed and *Y* is $T_{½}$ [*r*-$T_{½}$] then *gof* is *vg*-closed.
Proof: Let A be closed in *X*. Then *f*(A) is *g*-closed and so closed in *Y* as *Y* is $T_{½}$ $\Rightarrow$ *g*(*f* (A)) = *gof* (A) is *vg*-closed in *Z* (since *g* is *vg*-closed). Hence *gof* is *vg*-closed.

Theorem 3.13: If *f:X*→*Y* is *g*-closed[*rg*-closed], *g*:*Y*→*Z* is *g*-[*rg-; sg-; gs-; βg-; rαg-; rgα-*; *rα-*; *α-; s-; p-*; *β-;* π-]closed and *Y* is $T_{½}$[*r*-$T_{½}$], then *gof* is *vg*-closed.
Proof: Let A be closed set in X, then *f*(A) is *g*-closed in Y and so closed in *Y* (by assumption) $\Rightarrow$ *g*(*f* (A)) = *gof*(A) is *gs*-closed in *Z*. Hence *gof* is *vg*-closed [since every *gs*-closed set is *vg*-closed].

Corollary 3.5: If *f:X*→*Y* is *g*-closed[*rg*-closed], *g*:*Y*→*Z* is *g*-[*rg-; sg-; gs-; βg-; rαg-; rgα-*; *rα-*; *α-; s-; p-*; *β-; v-;* π-; r-]closed and *Y* is $T_{½}$ [*r*-$T_{½}$] then *gof* is *vg*-closed.
Proof: Let A be closed in *X*. Then *f*(A) is *g*-closed and so closed in *Y* as *Y* is $T_{½}$ $\Rightarrow$ *g*(*f* (A)) = *gof* (A) is *v*-closed in *Z* (since *g* is *v*-closed). Hence *gof* is *vg*-closed[since every *v*-closed set is *vg*-closed].

Theorem 3.14: If *f:X*→*Y*, *g*:*Y*→*Z* be two mappings such that *gof* is *vg*-closed [*r*-closed] then the following statements are true.
a) If *f* is continuous [*r*-continuous] and surjective then *g* is *vg*-closed.
b) If *f* is g-continuous[resp: rg-continuous], surjective and *X* is $T_{½}$ [resp: *r*-$T_{½}$] then *g* is *vg*-closed.
Proof: (a) For A closed in *Y*, f^{1}(A) closed in *X* $\Rightarrow$ (*g o f*)(f^{1}(A)) = *g*(A) *vg*-closed in *Z*. Hence *g* is *vg*-closed.Similarly one can prove the remaining parts and hence omitted.

Corollary 3.6: If *f:X*→*Y*, g:*Y*→*Z* be two mappings such that *gof* is -[*rg-; sg-; gs-; βg-; rαg-; rgα-*; *rα-*; *α-; s-; p-*; *β-; v-;* π-; r-]closed then the following statements are true.

a) If *f* is continuous [*r*-continuous] and surjective then *g* is *vg*-closed.
b) If *f* is g-continuous[rg-continuous], surjective and *X* is $T_{½}$ [r-$T_{½}$] then *g* is *vg*-closed.

Theorem 3.15: If X is *vg*-regular, *f:* X→ Y is *r*-closed, nearly-continuous, *vg*-closed surjection and $\bar{A}$ = A for every *vg*-closed set in Y, then Y is *vg*-regular.
Proof: Let $p \in U \in vGO(Y)$. Then there exists a point $x \in X$ such that *f*(x) = p as *f* is surjective. Since *X* is *vg*-regular and *f* is r-continuous there exists $V \in RO(X)$ such that $x \in V \subseteq \overline{V} \subseteq f^{1}(U)$ which implies $p \in f(V) \subseteq f(\overline{V}) \subseteq f(f^{-1}(U)) = U \rightarrow (1)$
Since *f* is *vg*-closed, $f(\overline{V}) \subseteq U$, By hypothesis $\overline{f(\overline{V})} = f(\overline{V})$ *and* $\overline{f(\overline{V})} = \overline{f(V)} \rightarrow (2)$
By (1) & (2) we have $p \in f(V) \subseteq f(\overline{V}) \subseteq U$ and *f*(*V*) is *vg*-open. Hence *Y* is *vg*-regular.

Corollary 3.7: If X is *vg*-regular, *f:* X→ Y is *r*-closed, nearly-continuous, *vg*-closed surjection and $\bar{A}$ = A for every r-closed set in *Y* then *Y* is *vg*-regular.

Theorem 3.16: If $f:X \rightarrow Y$ is *vg*-closed and $A \in RO(X)$, then $f_A:(X,\tau(A)) \rightarrow (Y,\sigma)$ is *vg*-closed.
Proof: Let F be an closed set in *A*. Then F = A∩E for some closed set E of *X* and so F is closed in $X \Rightarrow f(A)$ is *vg*-closed in *Y*. But $f(F) = f_A(F)$. Hence f_A is *vg*-closed.

Theorem 3.17: If $f:X \rightarrow Y$ is *vg*-closed, *X* is $rT_{½}$ and A is rg-closed set of *X* then $f_A:(X,\tau(A)) \rightarrow (Y,\sigma)$ is *vg*-closed.
Proof: Let F be r-closed set in *A*. Then F = A∩E for some r-closed set E of *X* and so F is r-closed in *X* $\Rightarrow f(A)$ is *vg*-closed in *Y*. But $f(F) = f_A(F)$. Hence f_A is *vg*-closed.

Corollary 3.8: If $f:X \rightarrow Y$ is *g*-[*rg-; sg-; gs-;* β*g-;* $r\alpha g$*-;* $rg\alpha$-; *r-;* $r\alpha$-; α-; *s-; p-;* β-; *v-;* π-] closed and $A \in RO(X)$, then $f_A:(X,\tau(A)) \rightarrow (Y,\sigma)$ is *vg*-closed.
Proof: Let F be closed in *A*. Then F = A∩E for some closed set E of *X* and so F is closed in $X \Rightarrow f(A)$ is $r\alpha g$-closed in *Y*. But $f(F) = f_A(F)$. Hence f_A is *vg*-closed[since every $r\alpha g$-closed set is *vg*-closed].

Corollary 3.9: If $f:X \rightarrow Y$ is al-*g*-[al-*rg-;* al-*sg-;* al-*gs-;* al-β*g-;* al-$r\alpha g$*-;* al-$rg\alpha$-; al-*r*-; al-$r\alpha$-; al-α-; al-*s-;* al-*p*-; al-β-; al-*v-;* al-π-] closed and $A \in RO(X)$, then $f_A:(X,\tau(A)) \rightarrow (Y,\sigma)$ is *vg*-closed.
Proof: Let F be r-closed in *A*. Then F = A∩E for some r-closed set E of *X* and so F is r-closed in $X \Rightarrow f(A)$ is $rg\alpha$-closed in *Y*. But $f(F) = f_A(F)$. Hence f_A is *vg*-closed[since every $rg\alpha$-closed set is *vg*-closed].

Theorem 3.18: If $f_i:X_i \rightarrow Y_i$ be *vg*-closed for i =1, 2. Let $f: X_1 x X_2 \rightarrow Y_1 x Y_2$ be defined as $f(x_1,x_2) = (f_1(x_1), f_2(x_2))$. Then $f:X_1 x X_2 \rightarrow Y_1 x Y_2$ is *vg*-closed.
Proof: Let $U_1 x U_2 \subseteq X_1 x X_2$ where U_i is closed in X_i for i = 1,2. Then $f(U_1 x U_2) = f_1(U_1)$ x $f_2(U_2)$ is *vg*-closed set in $Y_1 x Y_2$. Hence *f* is *vg*-closed.

Corollary 3.10: If $f_i:X_i \rightarrow Y_i$ be al-*g*-[al-*rg-;* al-*sg-;* al-*gs-;* al-β*g-;* al-$r\alpha g$*-;* al-$rg\alpha$-; al-*r*-; al-$r\alpha$-; al-α-; al-*s-;* al-*p*-; al-β-; al-*v-;* al-π-]closed for i =1, 2. Let $f:X_1 x X_2 \rightarrow Y_1 x Y_2$ be defined as $f(x_1,x_2) = (f_1(x_1), f_2(x_2))$, then $f:X_1 x X_2 \rightarrow Y_1 x Y_2$ is *vg*-closed.
Proof: Let $U_1 x U_2 \subseteq X_1 x X_2$ where U_i is r-closed in X_i for i = 1,2. Then $f(U_1 x U_2) = f_1(U_1)$ x $f_2(U_2)$ is *vg*-closed set in $Y_1 x Y_2$ [since every α-closed set is *vg*-closed]. Hence *f* is *vg*-closed.

Corollary 3.11: If $f_i:X_i \rightarrow Y_i$ be *g*-[*rg-; sg-; gs-;* β*g-;* $r\alpha g$*-;* $rg\alpha$-; *r-;* $r\alpha$-; α-; *s-; p-;* β-; *v-;* π-]closed for i =1, 2. Let $f:X_1 x X_2 \rightarrow Y_1 x Y_2$ be defined as $f(x_1,x_2) = (f_1(x_1), f_2(x_2))$, then $f:X_1 x X_2 \rightarrow Y_1 x Y_2$ is *vg*-closed.
Proof: Let $U_1 x U_2 \subseteq X_1 x X_2$ where U_i is closed in X_i for i = 1,2. Then $f(U_1 x U_2) = f_1(U_1)$ x $f_2(U_2)$ is *vg*-closed set in $Y_1 x Y_2$ [since every α-closed set is *vg*-closed]. Hence *f* is *vg*-closed.

Theorem 3.19: Let $h:X \rightarrow X_1 x X_2$ be *vg*-closed. Let $f_i:X \rightarrow X_i$ be defined as $h(x) = (x_1,x_2)$ and $f_i(x) = x_i$. Then $f_i: X \rightarrow X_i$ is *vg*-closed for i =1, 2.
Proof: Let U_1 be closed in X_1, then $U_1 x X_2$ is closed in $X_1 x\ X_2$, and $h(U_1 x\ X_2)$ is *vg*-closed in X. But $f_1(U_1) = h(U_1 x\ X_2)$, Hence f_1 is *vg*-closed. Similarly we can show that f_2 is also *vg*-closed and thus $f_i:X \rightarrow X_i$ is *vg*-closed for i = 1, 2.

Corollary 3.12: Let h: $X \to X_1 x X_2$ be al-g-[al-rg-; al-sg-; al-gs-; al-βg-; al-$r\alpha g$-; al-$rg\alpha$-; al-r-; al-$r\alpha$-; al-α-; al-s-; al-p-; al-β-; al-v-; al-π-] closed. Let f_i: $X \to X_i$ be defined as h(x) = (x_1, x_2) and f_i(x) = x_i. Then f_i: $X \to X_i$ is vg-closed for i =1,2.
Proof: Let $U_1 x U_2 \subseteq X_1 x X_2$ where U_i is r-closed in X_i for i = 1,2. Then $f(U_1 x U_2) = f_1(U_1)$ x $f_2(U_2)$ is vg-closed set in $Y_1 x Y_2$ [since every β-closed set is vg-closed]. Hence f is vg-closed.

Corollary 3.13: Let h: $X \to X_1 x X_2$ be g-[rg-; sg-; gs-; βg-; $r\alpha g$-; $rg\alpha$-; r-; $r\alpha$-; α-; s-; p-; β-; v-; π-] closed. Let f_i: $X \to X_i$ be defined as h(x) = (x_1, x_2) and f_i(x) = x_i. Then f_i: $X \to X_i$ is vg-closed for i =1,2.
Proof: Let $U_1 x U_2 \subseteq X_1 x X_2$ where U_i is closed in X_i for i = 1,2. Then $f(U_1 x U_2) = f_1(U_1)$ x $f_2(U_2)$ is vg-closed set in $Y_1 x Y_2$ [since every s-closed set is vg-closed]. Hence f is vg-closed.

§4. ALMOST *vg*-CLOSED MAPPINGS:

Definition 4.1: A function f: $X \to Y$ is said to be almost vg-closed if the image of every r-closed set in X is vg-closed in Y.

Theorem 4.1: Every vg-closed map is almost vg-closed but not conversely.

Example 1: Let $X = Y$ = {a, b, c}; τ = {φ, {a}, {a, b}, X}; σ = {φ, {a, c}, Y}. Let f:$X \to Y$ be defined f(a) = c, f(b) = b and f(c) = a. Then f is almost vg-closed.and almost v-closed but not v-closed.

Example 2: Let $X = Y$ = {a, b, c}; τ = {φ, {a}, {b, c}, X}; σ = {φ, {a, c}, Y}. Let f:$X \to Y$ be defined f(a) = b, f(b) = c and f(c) = a. Then f is almost vg-closed but not almost v-closed.

Theorem 4.2:
(i) If (Y, σ) is discrete, then f is almost closed of all types.
(ii) If f is almost closed[almost r-closed] and g is vg-closed then gof is almost vg-closed.
(iii) If f and g are almost r-closed then gof is almost vg-closed.

Note 1: We have the following implication diagrams for almost closed mappings.

(i) al.g.closed al.gs.closed
↓ ↓
al.rαg.closed → al.rgα.closed → al.rg.closed → al.***vg*.closed** ← al.sg.closed ← al.βg.closed
↑ ↑ ↑ ↑ ↑
→ → al.rα.closed → → al.*v*.closed →
↑ ↓ ↑ ↑
al.r.closed → al.π.closed → al.closed → al.α.closed → al.s.closed → al.β.closed
↓ ↓ ↓
al.πg.closed al.p.closed → al.ω.closed al.gα.closed
↓ ↓
al.gp.closed ← al.pg.closed al.rω.closed None is reversible

(ii) g.closed gs.closed
↓ ↓
rαg.closed → rgα.closed → rg.closed → al.***vg*.closed** ← sg.closed ← βg.closed
↑ ↑ ↑ ↑ ↑
→ → rα.closed → → *v*.closed →
↑ ↓ ↑ ↑
r.closed → π.closed → closed → α.closed → s.closed → β.closed None is reversible

(iii) If vGO(Y) = RO(Y), then the reverse relations hold for all almost closed maps.
al.g.closed al.gs.closed
↑↓ ↑↓
al.rαg.closed ↔ al.rgα.closed ↔ al.rg.closed ↔ al.***vg*.closed** ↔ al.sg.closed ↔ al.βg.closed
↑↓ ↑↓ ↑↓ ↑↓ ↑↓
↔ ↔ al.rα.closed ↔ ↔ ↔ al.*v*.closed ↔
↑↓ ↑↓ ↑↓ ↑↓
al.r.closed ↔ al.π.closed ↔ al.closed ↔ al.α.closed ↔ al.s.closed ↔ al.β.closed

Corollary 4.1: If f is almost closed[almost r-closed] and g is g-[rg-; sg-; gs-; βg-; $r\alpha g$-; $rg\alpha$-; $r\alpha$-; α-; s-; p-; β-; v-; π-; r-] closed then gof is almost vg-closed.

Corollary 4.2: If f is closed[r-closed] and g is g-[rg-; sg-; gs-; βg-; $r\alpha g$-; $rg\alpha$-; $r\alpha$-; α-; s-; p-; β-;π-] closed then gof is almost vg-closed.

Theorem 4.3: If f: X$\rightarrow$ Y is almost vg-closed, then vg(cl{f(A)}) $\subset$ f(cl{A})
Proof: Let A$\subset$X be r-closed and f: X$\rightarrow$ Y is vg-closed gives f(cl{A}) is vg-closed in Y and f(A) $\subset$ f(cl{A}) which in turn gives vg(cl{f(A)}) $\subset$ vgcl{(f(cl{A}))}- - - - - (1)
Since f(cl{A}) is vg-closed in Y, vgcl{(f(cl{A}))} = f(cl{A}) - - - - - - (2)
From (1) and (2) we have vg(cl{f(A)}) $\subset$ (f(cl{A})) for every subset A of X.

Remark 2: Converse is not true in general.

Corollary 4.3: If f: X$\rightarrow$ Y is al-g-[al-rg-; al-sg-; al-gs-; al-βg-; al-$r\alpha g$-; al-$rg\alpha$-; al-r-; al-$r\alpha$-; al-α-; al-s-; al-p-; al-β-; al-v-; al-π-] closed, then vg(cl{f(A)}) $\subset$ f(cl{A})

Theorem 4.4: If f:$X$$\rightarrow$$Y$ is almost vg-closed and A$\subseteq$$X$ is r-closed, f(A) is τ_{vg}-closed in Y.
Proof: Let A$\subset$X be r-closed and f: X$\rightarrow$ Y is vg-closed implies vg(cl{f(A)}) $\subset$ f(cl{A}) which in turn implies vg(cl{f(A)}) $\subset$ f(A), since f(A) =f(cl{A}). But f(A) $\subset$ vg(cl{f(A)}). Combaining we get f(A) = vg(cl{f(A)}). Hence f(A) is τ_{vg}-closed in Y.

Corollary 4.4: If f:$X$$\rightarrow$$Y$ is al-g-[al-rg-; al-sg-; al-gs-; al-βg-; al-$r\alpha g$-; al-$rg\alpha$-; al-r-; al-$r\alpha$-; al-α-; al-s-; al-p-; al-β-; al-v-; al-π-]closed, then f(A) is τ_{vg} closed in Y if A is r-closed set in X.

Theorem 4.5: If f:$X$$\rightarrow$$Y$ is g-[rg-; sg-; gs-; βg-; $r\alpha g$-; $rg\alpha$-; $r\alpha$-; α-; s-; p-; β-; v-; π-; r-]closed and A$\subseteq$$X$ is r-closed, f(A) is τ_{vg}-closed in Y.
Proof: For A$\subset$X is r-closed and f:X$\rightarrow$Y is rg-closed, f(A) is τ_{rg}-closed in Y and so f(A) is τ_{vg}-closed in Y. [since rg-closed set is vg-closed]. Similarly we can preove the remaining results.

Theorem 4.6: If vg(cl{A}) = r(cl{A}) for every A$\subset$Y and X is discrete space, then the following are equivalent:
a) f: X$\rightarrow$ Y is almost vg-closed map
b) vg(cl{f(A)}) $\subset$ f(cl{A})
Proof: (a) $\Rightarrow$ (b) follows from theorem 3.3
(b) $\Rightarrow$ (a) Let A be any r-closed set in X, then f(A) = f(cl{A})$\supset$ vg(cl{f(A)}) by hypothesis. We have f(A) $\subset$ vg(cl{f(A)}). Combining we get f(A) = vg(cl{f(A)}) = r(cl{f(A)})[by given condition] which implies f(A) is r-closed and hence vg-closed. Thus f is almost vg-closed.

Theorem 4.7: If v(cl{A}) = r(cl{A}) for every A$\subset$Y and X is discrete space, then the following are equivalent:
a) f: X$\rightarrow$ Y is almost vg-closed map
b) vg(cl{f(A)}) $\subset$ f(cl{A})
Proof: (a) $\Rightarrow$ (b) follows from theorem 3.3
(b) $\Rightarrow$ (a) Let A be any r-closed set in X, then f(A) = f(cl{A})$\supset$ vg(cl{f(A)}) by hypothesis. We have f(A) $\subset$ vg(cl{f(A)}). Combining we get f(A) = vg(cl{f(A)}) = r(cl{f(A)})[by given condition] which implies f(A) is r-closed and hence vg-closed. Thus f is almost vg-closed.

Theorem 4.8: f:$X$$\rightarrow$$Y$ is almost vg-closed iff for each subset S of Y and each U$\in$RO(X, f^{-1}(S)), there is an vg-closed set V of Y such that S$\subseteq$V and f^{1}(V) $\subseteq$ U.
Proof: Let S$\subseteq$$Y$ and U$\in$RO(X, f^{1}(S)). Then V = f(U) is vg-closed in Y as f is almost vg-closed. f^{-1}(S)$\subseteq$U $\Rightarrow$ S$\subseteq$$f$(U) = V and f^{-1}(V) = f^{-1}(f(U)) = U
Conversely Let U be r-closed in X. Then by hypothesis there exists a vg-closed set V of Y, such that f^{-1}(V) $\subseteq$U and so V $\subseteq$ f (U). Thus f(U) is vg-closed in Y. Hence f is almost vg-closed.

Remark 3: Composition of two almost vg-closed maps is not almost vg-closed in general.

Theorem 4.9: Let *X*, *Y*, *Z* be topological spaces and every *vg*-closed set is *r*-closed in *Y*. Then the composition of two almost *vg*-closed maps is almost *vg*-closed.
Proof: (a) Let *f* and g be almost *vg*-closed maps. Let A be any r-closed set in $X \Rightarrow f(A)$ is *r*-closed in *Y* (by assumption) $\Rightarrow g(f(A)) = g \circ f(A)$ is *vg*-closed in *Z*. Hence *gof* is almost *vg*-closed.

Theorem 4.10: Let *X*, *Y*, *Z* be topological spaces and every *v*-closed set is closed [*r*-closed] in *Y*. Then the composition of two *v*-closed[*r*-closed] maps is almost *vg*-closed.
Proof: (a) Let *f*, g be *v*-closed maps. Let A be r-closed in $X \Rightarrow f(A)$ is *v*-closed and so closed in *Y* (by assumption) $\Rightarrow g(f(A)) = gof(A)$ is *v*-closed in *Z*. Hence *gof* is almost *vg*-closed [since every *v*-closed set is *vg*-closed].

Theorem 4.11: Let *X*, *Y*, *Z* be topological spaces and every *g*-[*rg-; sg-; gs-;* β*g-;* $r\alpha g$-; $rg\alpha$-; $r\alpha$-; α-; *s-; p*-; β-; π-]closed set is *r*-closed in *Y*. Then the composition of two al-*g*-[al-*rg-;* al-*sg-;* al-*gs-;* al-βg-; al-$r\alpha g$-; al-$rg\alpha$-; al-*r*-; al-$r\alpha$-; al-α-; al-*s-;* al-*p*-; al-β-; al-*v-;* al-π-]closed maps is almost *vg*-closed.
Proof: Let A be *r*-closed in X, then *f*(A) is *sg*-closed in Y and so *r*-closed in *Y*(by assumption) $\Rightarrow g(f(A)) = gof(A)$ is *sg*-closed in *Z*. Hence *gof* is almost *vg*-closed [since every *sg*-closed set is *vg*-closed].

Corollary 4.5: Let *X*, *Y*, *Z* be topological spaces and every *g*-[*rg-; sg-; gs-;* β*g-;* $r\alpha g$-; $rg\alpha$-; $r\alpha$-; α-; *s-; p*-; β-; π-]closed set is closed [*r*-closed] in *Y*. Then the composition of two *g*-[*rg-; sg-; gs-;* β*g-;* $r\alpha g$-; $rg\alpha$-; $r\alpha$-; α-; *s-; p*-; β-; *v-;* π-; r-]closed maps is almost *vg*-closed.
Proof: Let A be r-closed set in X, then *f*(A) is *sg*-closed in Y and so closed in *Y* (by assumption) $\Rightarrow g(f(A)) = gof(A)$ is *sg*-closed in *Z*. Hence *gof* is almost *vg*-closed [since every *sg*-closed set is *vg*-closed].

Example 3: Let $X = Y = Z = \{a, b, c\}$; $\tau = \{\varphi, \{a\}, \{a, b\}, X\}$; $\sigma = \{\varphi, \{a, c\}, Y\}$ and $\eta = \{\varphi, \{a\}, \{b, c\}, Z\}$. $f:X \to Y$ be defined $f(a) = c$, $f(b) = b$ and $f(c) = a$ and $g:Y \to Z$ be defined $g(a) = b$, $g(b) = a$ and $g(c) = c$, then *g*, *f* and $g \circ f$ are almost *vg*-closed.

Theorem 4.12: If $f:X \to Y$ is almost g-closed[almost rg-closed], g:$Y \to Z$ is *vg*-closed and *Y* is $T_{½}$ [r-$T_{½}$] then *gof* is almost *vg*-closed.
Proof: (a) Let A be r-closed in *X*. Then *f*(A) is *g*-closed and so closed in *Y* as *Y* is $T_{½} \Rightarrow g(f(A)) = gof(A)$ is *vg*-closed in *Z* (since *g* is *vg*-closed). Hence *gof* is almost *vg*-closed.

Theorem 4.13: If $f:X \to Y$ is *g*-closed[*rg*-closed], $g:Y \to Z$ is *g*-[*rg-; sg-; gs-;* β*g-;* $r\alpha g$-; $rg\alpha$-; $r\alpha$-; α-; *s-; p*-; β-; π-]closed and *Y* is $T_{½}$[r-$T_{½}$], then *gof* is almost *vg*-closed.
Proof: Let A be r-closed set in X, then *f*(A) is *g*-closed in Y and so closed in *Y* (by assumption) $\Rightarrow g(f(A)) = gof(A)$ is *gs*-closed in *Z*. Hence *gof* is almost *vg*-closed [since every *gs*-closed set is *vg*-closed].

Corollary 4.6: If $f:X \to Y$ is almost *g*-closed[almost *rg*-closed], $g:Y \to Z$ is *g*-[*rg-; sg-; gs-;* β*g-;* $r\alpha g$-; $rg\alpha$-; $r\alpha$-; α-; *s-; p*-; β-; *v-;* π-; r-]closed and *Y* is $T_{½}$ [r-$T_{½}$] then *gof* is almost *vg*-closed.
Proof: (a) Let A be r-closed in *X*. Then *f*(A) is *g*-closed and so closed in *Y* as *Y* is $T_{½} \Rightarrow g(f(A)) = gof(A)$ is *v*-closed in *Z* (since *g* is *v*-closed). Hence *gof* is almost *vg*-closed[since every *v*-closed set is *vg*-closed].

Theorem 4.14: If $f:X \to Y$, $g:Y \to Z$ be two mappings such that *gof* is almost *vg*-closed [almost *r*-closed] then the following statements are true.
a) If *f* is continuous [*r*-continuous] and surjective then *g* is almost *vg*-closed.
b) If *f* is g-continuous[resp: rg-continuous], surjective and *X* is $T_{½}$ [resp: r-$T_{½}$] then *g* is almost *vg*-closed.
Proof: (a) For A r-closed in *Y*, $f^{-1}(A)$ closed in $X \Rightarrow (g \circ f)(f^{-1}(A)) = g(A)$ *vg*-closed in *Z*. Hence *g* is almost *vg*-closed. Similarly one can prove the remaining parts and hence omitted.

Corollary 4.7: If $f:X \to Y$, g:$Y \to Z$ be two mappings such that *gof* is -[*rg-; sg-; gs-;* β*g-;* $r\alpha g$-; $rg\alpha$-; $r\alpha$-; α-; *s-; p*-; β-; *v-;* π-; r-]closed then the following statements are true.
a) If *f* is continuous [*r*-continuous] and surjective then *g* is almost *vg*-closed.
b) If *f* is g-continuous[rg-continuous], surjective and *X* is $T_{½}$ [r-$T_{½}$] then *g* is almost *vg*-closed.

Theorem 4.15: If X is *vg*-regular, *f:* X→ Y is *r*-closed, nearly-continuous, almost *vg*-closed surjection and $\bar{A}$ = A for every *vg*-closed set in Y, then Y is *vg*-regular.

Proof: Let $p \in U \in vGO(Y)$. Then there exists a point $x \in X$ such that $f(x) = p$ as *f* is surjective. Since *X* is *vg*-regular and *f* is r-continuous there exists $V \in RO(X)$ such that $x \in V \subseteq \bar{V} \subseteq f^{1}(U)$ which implies $p \in f(V) \subseteq f(\bar{V}) \subseteq f(f^{-1}(U)) = U \to (1)$

Since *f* is *vg*-closed, $f(\bar{V}) \subseteq U$, By hypothesis $\overline{f(\bar{V})} = f(\bar{V})$ *and* $\overline{f(\bar{V})} = \overline{f(V)} \to (2)$

By (1) & (2) we have $p \in f(V) \subseteq f(\bar{V}) \subseteq U$ and $f(V)$ is *vg*-open. Hence *Y* is *vg*-regular.

Corollary 4.8: If X is *vg*-regular, *f:* X→ Y is *r*-closed, nearly-continuous, almost *vg*-closed surjection and $\bar{A}$ = A for every r-closed set in *Y* then *Y* is *vg*-regular.

Theorem 4.16: If $f:X\to Y$ is almost *vg*-closed and $A \in RO(X)$, then $f_A:(X,\tau(A))\to(Y,\sigma)$ is almost *vg*-closed.

Proof: Let F be an r-closed set in *A*. Then F = A∩E for some r-closed set E of *X* and so F is r-closed in $X \Rightarrow f(A)$ is *vg*-closed in *Y*. But $f(F) = f_A(F)$. Hence f_A is almost *vg*-closed.

Theorem 4.17: If $f:X\to Y$ is almost *vg*-closed, *X* is $rT_{½}$ and A is rg-closed set of *X* then $f_A:(X,\tau(A))\to(Y,\sigma)$ is almost *vg*-closed.

Proof: Let F be a r-closed set in *A*. Then F = A∩E for some r-closed set E of *X* and so F is r-closed in *X* $\Rightarrow f(A)$ is *vg*-closed in *Y*. But $f(F) = f_A(F)$. Hence f_A is almost *vg*-closed.

Corollary 4.9: If $f:X\to Y$ is *g*-[*rg-; sg-; gs-; βg-; rαg-; rgα-*; *r-*; *rα-*; *α-; s-; p-*; *β-; v-;* π-] closed and $A \in RO(X)$, then $f_A:(X,\tau(A))\to(Y,\sigma)$ is almost *vg*-closed.

Proof: Let F be an r-closed set in *A*. Then F = A∩E for some r-closed set E of *X* and so F is r-closed in $X \Rightarrow f(A)$ is *rαg*-closed in *Y*. But $f(F) = f_A(F)$. Hence f_A is almost *vg*-closed[since every *rαg*-closed set is *vg*-closed].

Corollary 4.10: If $f:X\to Y$ is al-*g*-[al-*rg-;* al-*sg-;* al-*gs-;* al-*βg-;* al-*rαg-;* al-*rgα-*; al-*r-*; al-*rα-*; al-*α-;* al-*s-;* al-*p-*; al-*β-;* al-*v-;* al-π-] closed and $A \in RO(X)$, then $f_A:(X,\tau(A))\to(Y,\sigma)$ is almost *vg*-closed.

Proof: Let F be an r-closed set in *A*. Then F = A∩E for some r-closed set E of *X* and so F is r-closed in $X \Rightarrow f(A)$ is *rgα*-closed in *Y*. But $f(F) = f_A(F)$. Hence f_A is almost *vg*-closed[since every *rgα*-closed set is *vg*-closed].

Theorem 4.18: If $f_i:X_i\to Y_i$ be almost *vg*-closed for i =1, 2. Let $f: X_1xX_2\to Y_1xY_2$ be defined as $f(x_1,x_2) = (f_1(x_1), f_2(x_2))$. Then $f:X_1xX_2\to Y_1xY_2$ is almost *vg*-closed.

Proof: Let $U_1xU_2 \subseteq X_1xX_2$ where U_i is r-closed in X_i for i = 1,2. Then $f(U_1xU_2) = f_1(U_1)$ x $f_2(U_2)$ is *vg*-closed set in Y_1xY_2. Hence *f* is almost *vg*-closed.

Corollary 4.11: If $f_i:X_i\to Y_i$ be al-*g*-[al-*rg-;* al-*sg-;* al-*gs-;* al-*βg-;* al-*rαg-;* al-*rgα-*; al-*r-*; al-*rα-*; al-*α-;* al-*s-;* al-*p-*; al-*β-;* al-*v-;* al-π-]closed for i =1, 2.
Let $f:X_1xX_2\to Y_1xY_2$ be defined as $f(x_1,x_2) = (f_1(x_1), f_2(x_2))$, then $f:X_1xX_2\to Y_1xY_2$ is almost *vg*-closed.

Proof: Let $U_1xU_2 \subseteq X_1xX_2$ where U_i is r-closed in X_i for i = 1,2. Then $f(U_1xU_2) = f_1(U_1)$ x $f_2(U_2)$ is *vg*-closed set in Y_1xY_2 [since every *α*-closed set is *vg*-closed]. Hence *f* is almost *vg*-closed.

Corollary 4.12: If $f_i:X_i\to Y_i$ be *g*-[*rg-; sg-; gs-; βg-; rαg-; rgα-*; *r-*; *rα-*; *α-; s-; p-*; *β-; v-;* π-]closed for i =1, 2. Let $f:X_1xX_2\to Y_1xY_2$ be defined as $f(x_1,x_2) = (f_1(x_1), f_2(x_2))$, then $f:X_1xX_2\to Y_1xY_2$ is almost *vg*-closed.

Proof: Let $U_1xU_2 \subseteq X_1xX_2$ where U_i is r-closed in X_i for i = 1,2. Then $f(U_1xU_2) = f_1(U_1)$ x $f_2(U_2)$ is *vg*-closed set in Y_1xY_2 [since every *α*-closed set is *vg*-closed]. Hence *f* is almost *vg*-closed.

Theorem 4.19: Let $h:X\to X_1xX_2$ be almost *vg*-closed. Let $f_i:X\to X_i$ be defined as $h(x) = (x_1,x_2)$ and $f_i(x) = x_i$. Then $f_i: X\to X_i$ is almost *vg*-closed for i =1, 2.

Proof: Let U_1 be r-closed in X_1, then U_1xX_2 is r-closed in $X_1x X_2$, and $h(U_1x X_2)$ is *vg*-closed in X. But $f_1(U_1) = h(U_1x X_2)$, Hence f_1 is almost *vg*-closed. Similarly we can show that f_2 is also almost *vg*-closed and thus $f_i:X\to X_i$ is almost *vg*-closed for i = 1, 2.

Corollary 4.13: Let h: $X\to X_1xX_2$ be al-*g*-[al-*rg-;* al-*sg-;* al-*gs-;* al-*βg-;* al-*rαg-;* al-*rgα-*; al-*r-*; al-*rα-*; al-*α-;* al-*s-;* al-*p-*; al-*β-;* al-*v-;* al-π-] closed.

Let f_i: $X \to X_i$ be defined as $h(x) = (x_1, x_2)$ and $f_i(x) = x_i$. Then f_i: $X \to X_i$ is almost *vg*-closed for i =1,2.

Proof: Let $U_1xU_2 \subseteq X_1xX_2$ where U_i is *r*-closed in X_i for i = 1,2. Then $f(U_1xU_2) = f_1(U_1)$ x $f_2(U_2)$ is *vg*-closed set in Y_1xY_2 [since every β-closed set is *vg*-closed]. Hence *f* is almost *vg*-closed.

Corollary 4.14: Let h:$X \to X_1xX_2$ be *g*-[*rg-; sg-; gs-; βg-; rαg-; rgα-; r-; rα-; α-; s-; p-; β-; v-;* π-] closed. Let f_i: $X \to X_i$ be defined as $h(x) = (x_1, x_2)$ and $f_i(x) = x_i$. Then f_i: $X \to X_i$ is almost *vg*-closed for i =1,2.

Proof: Let $U_1xU_2 \subseteq X_1xX_2$ where U_i is r-closed in X_i for i = 1,2. Then $f(U_1xU_2) = f_1(U_1)$ x $f_2(U_2)$ is *vg*-closed set in Y_1xY_2 [since every *s*-closed set is *vg*-closed]. Hence *f* is almost *vg*-closed.

CONCLUSION:

In this paper Author introduced the concept of *vg*-closed mappings and almost *vg*-closed mappings, studied their basic properties and the interrelationship between other closed maps.

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