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ANALYSIS OF KRYLOV-BOGOLIUBOV METHOD IN APPROXIMATING SOLUTION OF NONLINEAR OSCILLATORY DIFFERENTIAL EQUATIONS

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ABSTRACT

There has been extensive research on how to find solutions of non-linear oscillatory differential equations. This has been triggered by the fact that it is difficult to get an analytical solution of static nonlinear differential equations. The methods of approximation have remained as the only way to find solutions of these equations. Several methods of approximation like numerical method, perturbation method, and Krylov-Bogoliubov methods have been formulated. Out of these methods numerical method has been found to be more accurate. However, it may not be a reliable method for approximating the solution of non-linear differential equations. This is because numerical methods can still encounter problems such as diverging solution and tracking wrong solutions. In addition computer manipulations on numerical methods do not provide much insight to the scientists running them. Hence, other methods of approximation like Krylov-Bogoliubov methods offer an alternative approach to solving non-linear differential equations. Solving these equations using these methods helps in understanding physical problems better, and may also help improve future procedures and designs used in solving problems of oscillatory motion. The aim of this study is to analyze Krylov-Bogoliubov methods in solving non-linear oscillatory ordinary differential equations. Krylov-bogoliubov method is found to be suitable for approximating solution of non-linear oscillatory differential equations.

Key words: krylov-bogoliubov, numerical method, nonlinear oscillatory differential equations

INTRODUCTION

The use of Krylov-Bogoliubov approximation allows approximate solution to be determined for problems which cannot be solved by traditional analytical methods (Berezansky, 2013). Various static element computer programs are currently in use for the analysis of complex nonlinear problems. This involves selection of a representative static element model, the analysis of model and the interpretation of the results (Klaus *et al*, 1979). In engineering practice selection of an appropriate static element model and the corresponding interpretation of the results are crucial. However a reliable and accurate response prediction of the model is essential so that the analysis can be used with confidence. Unfortunately, considering the present nonlinear analysis procedures, the accurate analysis of a static element model can present great difficulties (Talus, 2011). The cost can be high, but a more serious factor is that considerable knowledge and judgment by the user may be required to assure a stable and accurate solution. Hence, there is a great need for solution algorithm with increased accuracy and stability properties (Renfrey, 1980).

The important contribution of Krylov-Bogoliubov approximation is that they developed a general averaging approach and proved that the solution of the averaged system approximates the exact dynamics (Krylov, 1937). Hence this study analyses the accuracy of this method in reference to numerical method which have been found to be more approximate. The absolute error deviation of this method is used in the analysis.

MATHEMATICAL FORMULATION AND SIMULATION

Krylov-Bogoliubov and numerical analysis among other methods of approximations are important in finding the solution of nonlinear ordinary differential equations. Two equations were solved with the above methods and their solution was used to generate data that helped in plotting the solutions of the two methods in the same plane. MATLAB software version 12 was used to generate the data. An analysis of the methods was derived from the drawn graphs.
The study involved solving two nonlinear oscillatory differential equations. The equations are;

Van der pol equation

$$\frac{d^2y}{dx^2} + y + \varepsilon(y^2 - 1)\frac{dy}{dx} = 0$$

Duffing equation

$$\frac{d^2y}{dx^2} + y + \varepsilon\alpha y^3 = 0$$

The graphs were compared against numerical method whose solution was obtained by utilizing MATLAB's built fifth order Runge-kutta method with degree four interpolant. The graphs drawn for the methods of approximation and their absolute error were used in the analysis and discussion of the study.

RESULTS AND DISCUSSION

Solution of duffing equation

$$y = a_0 \sin\left[(\omega x + \frac{3\mu a_0^2}{8\omega})x + \phi_0\right]$$

Solution of van der pol equation

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$$y = a_0 \sin\left[\omega x + (\frac{\mu}{8\omega})x + \phi_0 \right]$$

GENERATED DATA

Table 1: Krylov-Bogoliubov and Numerical values for the duffing equation ($\phi_0 = 3, \mu = 0.3, \omega = 1$)

X	Y Krylov-Bogoliubov	Y Numerical	Absolute error (Krylov-Bogoliubov vs Numerical)
0	0.1411	1	0.8589
3.926991	0.6026	-0.6515	1.3273
7.853982	-0.9908	-0.1443	0.8542
11.78097	0.7945	0.8421	0.2510
15.70796	-0.1295	-0.9573	1.2058
19.63495	-0.6120	0.4067	1.3132
23.56194	0.9923	0.4211	0.4804
27.48894	-0.7873	-0.9618	0.7244
31.41593	0.1178	0.8334	1.4337
35.34292	0.6212	-0.1286	1.1277
39.26991	-0.9937	-0.6634	0.0039
43.1969	0.7800	0.9998	1.1522
47.12389	-0.1060	-0.6393	1.5000
51.05088	-0.6304	-0.1599	0.7797
54.97787	0.9950	0.8506	0.5191

Table 2: Krylov-Bogoliubov and numerical values for van der pol equation

($\phi_0 = 3, \mu = 0.3, \omega = 1$)

X	Y Krylov-Bogoliubov	Y Numerical	Absolute error (Krylov-Bogoliubov vs Numerical)
0	0.1411	1	0.8589
3.926991	0.6757	-0.6515	1.2541
7.853982	-0.9985	-0.1443	0.8465
11.78097	0.5911	0.8421	0.0476
15.70796	0.2485	-0.9573	0.8279
19.63495	-0.9064	0.4067	1.0187
23.56194	0.9016	0.4211	0.5711
27.48894	-0.2375	-0.9618	0.1746
31.41593	-0.6002	0.8334	0.7157

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35.34292	0.9991	-0.1286	0.7499
39.26991	-0.6673	-0.6634	0.3302
43.1969	-0.1523	0.9998	0.2199
47.12389	0.8606	-0.6393	0.5333
51.05088	-0.9396	-0.1599	0.4705
54.97787	0.3315	0.8506	0.1444

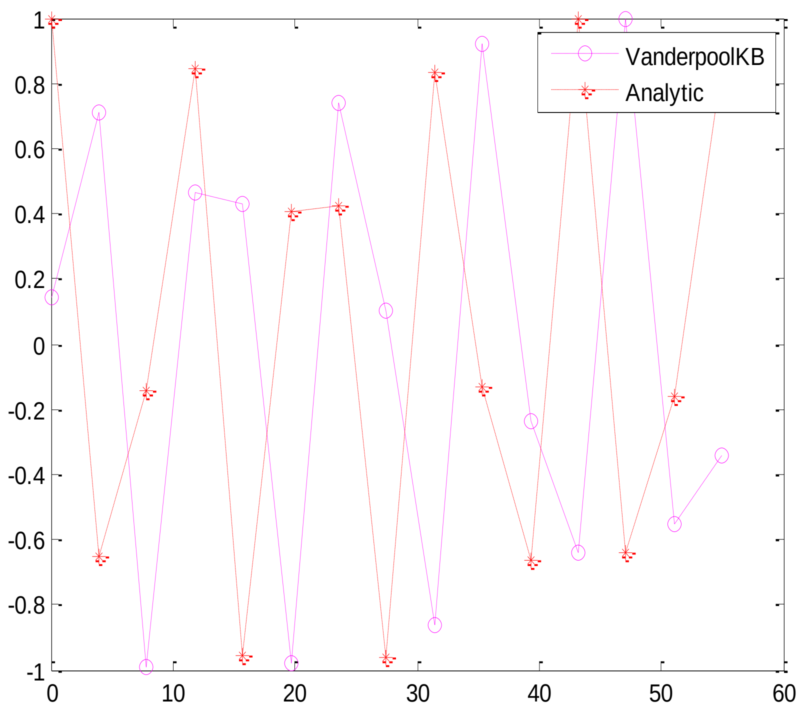

Figure 1: GRAPH OF KRYLOV-BOGOLIUBOV METHOD AGAINST NUMERICAL METHOD IN SOLVING VAN DER POL EQUATION.

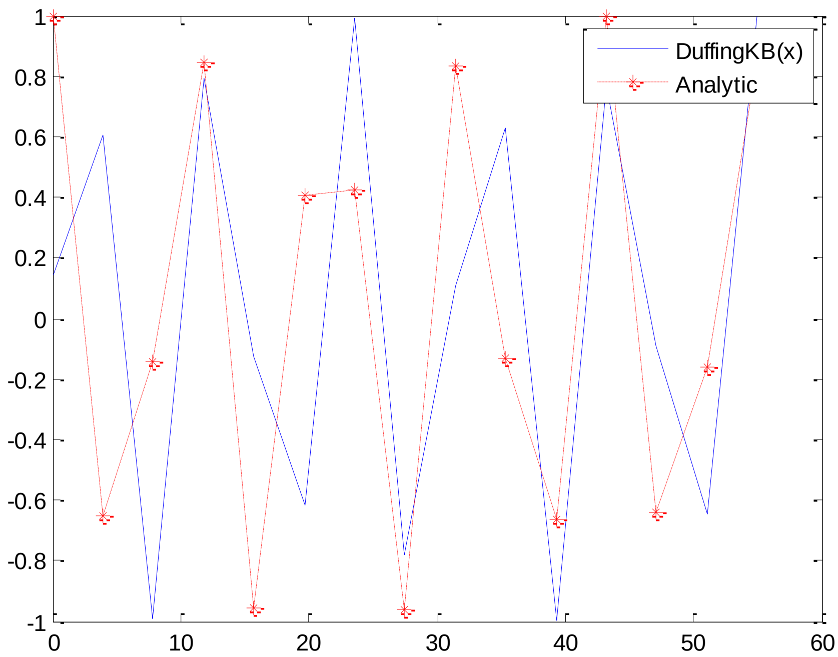

Figure 2: GRAPH OF KRYLOV –BOGOLIUBOV METHOD AGAINST NUMERICAL METHOD IN SOLVING THE DUFFING EQUATION.

DISSCUSION AND CONCLUSION

Considering the absolute error deviation of krylov-bogoliubov method against numerical, it is evident that krylov-bogoliubov method is a better analytical method of approximating the solution. The deviation of the method for the two equations is more the same. This indicates that krylov –bogoliubov is approximate for approximating the solution of non-linear oscillatory differential equations. It is also clear from the graphs of the two equations that krylov- bogoliubov method tracks the solution of the said equation. The deviation from the numerical method is not very significant to have a great effect on the approximated solution. Despite, it is evident that the method requires some improvement to reduce deviation from the numerical method. The method of krylov-bogoliubov is found to a better method of approximating the solution of non-linear differential equations due the fact it is analytical and more practical as compared to numerical method that relies on use of computer software. This computer software may be expensive to purchase especially in tough economic times. Krylov-bogoliubov method is also preffered since it is analytical and procedure followed may be improved in future. Numerical method relies on use of computer, hence difficult to improve the procedure.

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