Pre-Generalization Compact Space and Pre-Generalization Lindel of Space

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Abstract:

The aim of this paper is to introduce pre- Generalization Compact Space and Pre-Generalization Lindelof Space. Also, we study the properties and prove some theorems about these spaces, finally explain the relation between these concepts.

Key words :- *PG – compact , PG –lindelof , pg – open cover .*

1-Introduction:

Let X be a topological space . Recall that a subset A of X is said to be preclosed set if cl int A ⊆ A. Moreover A is said to be preopen if X –A is preclosed . The smallest preclosed set containing A is called preclosuer of A and denoted by pcl A.

In 1970, Levine [1] initialed the investigation of the so called generalized closed set. By definition ,A subset A of a topological X is called generalized closed set, if clA ⊆ U whenever A ⊆ U is open set also, A is called generalized open , if $X - A$ is generalized closed set and denoted by g-closed set and g-open set .

In [2] Maki introduce to concepts pg-closed and pg –open set in anadogaous manner

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2- Basic Definitions:

Definition 2.1. [1]

Let X be a topological space. A subset A of X is called

1- Pre-generalized closed (denoted by pg-closed), if $pclA \subseteq U$ whenever $A \subseteq U$ and U is preopen set, the complement is called pg-open.

2- Generalized preclosed (denoted by pg -closed), if pcl A ⊆ U whenever A ⊆ U and U is open, the complement is called pg-open.

Definition 2.2 .

If$G = \{A_i : i \in \lambda\}$ is a cover of X, by pg-open set in X then G is called pg-open cover of X. See[3]

Definition 2.3 .

A space (X,T) is said to be PG-compact space if every pg- open cover of X has finite sub cover. See [3], [4] and [5].

3- Main Results:

The following theorems explain the relation between compact space and PG-compact space.

Theorem 3.1.

Every PG- compact space is compact space .

Proof:

Let (X,T) be a PG-compact space , $A = \{G_i : i \in \Lambda\}$ is open cover of X then $A \subseteq \beta$ where is pg - open cover of X .Since is pg- compact space then X has finite sub cover thus A has finite sub cover , so (X,T) is compact space.

Remark 3.2.

The converse of above theorem is not necessary true , as the following example illustrates :

Example 3.3.

(R, T_c) is compact space but not PG - compact space.
To prove (R, T_c)
is not PG- compact.

Let $G = \{A_n : (-\infty, n) : n = 0,1,2,\dots\}$ be pg-open cover of R.

$$\bigcup_{n=0}^{\infty} A_n = R$$

But

$$\bigcup_{n=\{1,2,\dots k\}} A_n \subset R$$

Then (R, T_c) is not PG-compact.

In the next theorem below we introduce some characterization of PG-compact space.

Theorem 3.4.

Let (X, T) be a topological space then the following statements are equivalent .

1. *X is PG- compact.*
2. *Every pg-closed subset of X is pg- compact. Gg*

Proof : 1 ⟶ 2

Suppose A be pg-closed subset of X and $G = \{B_i : i \in \Lambda\}$ be pg-open cover A, if $W = G \cup A^c$ then W is pg- open cover of X.
Since X is PG- compact space then $\exists i_1, i_2, \dots, i_n$ such that

$$X \subseteq \left(\bigcup_{i=1}^{n} \beta_i\right) \cup A^c$$

Thus

$$A \subseteq \bigcup_{i=1}^{n} \beta_i$$

So A is pg – compact subset of X .

proof :2 ⟶ 1

Let $S = \{J\alpha : \alpha \in \Lambda\}$ is pg-open cover of X and $J a_0$ is pg-open of X so $J^c_{\alpha o}$ is pg-closed of X.
$S^* = \{J\alpha : \alpha \in \Lambda - \{a_0\}\}$ is pg-open cover of $J^c_{\alpha o}$. Since $J^c_{\alpha o}$ is pg- compact set then

$$J_{ao}^{c} \subseteq \bigcup_{i=1}^{n} J_{\alpha i} \ ,$$

So

$$X \subseteq \bigcup_{i=0}^{n} J_{\alpha i}$$

Then X is PG- compact space. ■

Definition 3.5.

A function $f: X \to Y$ is said to be PG^{**} – continuous if every (pg-open) A in Y then $f^{-1}(A)$ is (pg- open) in X.

The following theorem introduces the topological property of PG- compact space .

Theorem 3.6.

If $f: X \to Y$ to be PG^{**}- continuous from PG- compact space into Y ,then Y is PG- compact space .

Proof:

Let $G = \{W_\alpha : \alpha \in \Lambda\}$ be pg- open cover of Y.

Then $G^* = \{f^{-1}(W\alpha):\ \alpha \in \Lambda\ \}$ be pg- open cover of X.

Since X is a PG - compact space ,then $\exists \alpha_1, \alpha_2, \dots, \alpha_n$ such that

$$X \subseteq \bigcup_{i=1}^{n} f^{-1}(W_{\alpha_i})$$

Then

$$Y = f(x) \subseteq f(\bigcup_{i=1}^{n} f^{-1}(W_{\alpha_i}))$$

Then

$Y \subseteq \bigcup_{i=1}^{n} W_{\alpha_i}$,so Y is PG- compact space . ■

Definition 3.7.

Let (X, T) be a topological space we say that X is PG- Lindelof space if every pg- open cover in X has finite countable sub cover .

Theorem 3.8.

Let (X, T) be a topological space if X is PG - Lindelof space then X is PG - Lindelof space .

Proof:

Let $\{G_\alpha : \alpha \in \Lambda\ \}$ be open cover of X , then $\{G_\alpha : \alpha \in \Lambda\ \}$ be pg-open cover of X .

Since X is PG - Lindelof space then $\exists \alpha_1, \alpha_2, \dots \in \Lambda$ such that

$$X = \bigcup_{i=1}^{\infty} G_{\alpha i}$$

Then X is Lindelof space . ■

Remark 3.9.

The converse of this theorem is not need to be true .

Example 3.10.

Let (X, τ_i) be indiscrete topology where X is not countable, then (X, τ_i) is Lindelof space but not PG - Lindelof space .

We can give the relation between PG - compact and PG - Lindelof by thefollowing theorem :

Theorem 3.11.

Let (X, T) be a topological space if X is PG - compact space then X is PG - Lindelof space .

Proof:

Let $= \{G_\alpha : \alpha \in \Lambda\}$ be pg- open cover of X , Since PG - compact space then $\exists \alpha_1, \alpha_2, ..., \alpha_n$ such that

$$X = \bigcup_{n=1}^{n} G_{\alpha i}$$

Let $B = \{G_{\alpha 1}, G_{\alpha 2}, \dots, G_{\alpha n}\}$ finite sub cover of A to X and
let $C = \{G_{\alpha 1}, G_{\alpha 2}, \dots, G_{\alpha n}, \Phi\}$ such that
$C = \{W_i : i = 1,2, \dots\}$ sub cover of A to X such that $W_i = \{G_{\alpha i} : i = 1,2, \dots$ and $\Phi, i > n\}$.
Then X is PG - Lindelof space. ■

Theorem 3.12.

Let (X, T) be a PG - Lindelof space , $A \subseteq B$ be pg - closed set in X then A is pg - Lindelof set in X .

Proof :

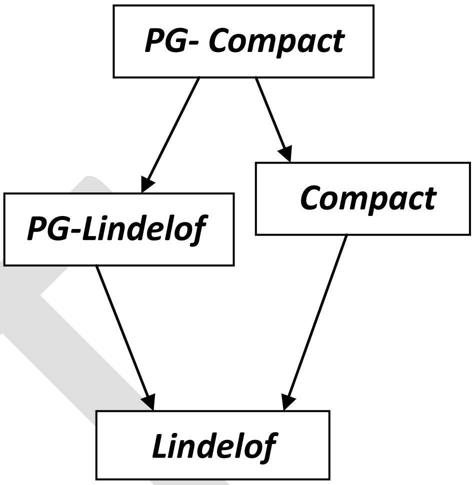
Let $A \subseteq B$ be pg - closed set and $= \{G_a : \alpha \in \Lambda\}$ be pg - open cover to A , let $E_1 = E \cup A^c$, then E_1 is pg- open cover to X .
Since X is Lindelof then $\exists \alpha_1, \alpha_2, ... \in \Lambda$ such that

$$X \subseteq \bigcup_{i=1}^{\infty} G_{\alpha i} \cup A^c \quad ,$$

$$X \subseteq \bigcup_{i=1}^{\infty} G_{\alpha i}$$

Thus A is pg - Lindelof set in X.

Therefore we have the following diagram



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