DOI : https://dx.doi.org/10.26808/rs.st.i7v5.07
International Journal of Advanced Scientific and Technical Research Issue 7 volume 5 Sep-Oct 2017
Available online on http://www.rspublication.com/ijst/index.html ISSN 2249-9954

Strongly Pure Fuzzy Ideals And Strongly Pure Fuzzy Submodules

Hatam.Y.Khalaf, Shaymaa.A.Alkareem

Department of Mathematics, College of Education, University of Baghdad

Abstract

The main aim of this paper is to extend and study the notion of (ordinary) S-pure ideal(submodule) into S-pure fuzzy ideal (submodule) and S-regular ring (module) into S-regular fuzzy ring (module) This lead us to introduced and study other notions such as S-pure fuzzy ideal (submodule) and S-regular fuzzy ring (module).

Introduction

Let I fuzzy ideal of a ring R. it is well known that I is called S-pure fuzzy ideal of R if for each $r_{\ell} \subseteq I$, there exists a prime fuzzy singleton $x_t \subseteq I$ such that $r_{\ell} = r_{\ell} x_t$, $\forall$ t, $\ell \in(0,1]$.

And a fuzzy ring R is called S-regular fuzzy if and only if for each fuzzy singleton r_{ℓ} of R , there exists a prime fuzzy singleton x_t of R such that $r_{\ell} = r_{\ell} x_t r_{\ell}$, $\forall$ t, $\ell \in(0,1]$.

In this paper, we fuzzify these concepts S-pure fuzzy ideal (submodule) and S-regular fuzzy ring (module) , moreover we generalize many properties of S-pure fuzzy ideal (submodule) and S-regular fuzzy ring (module).

This paper consists of four part. In part one, various basis properties about strongly pure fuzzy ideals are discussed . part two included strongly regular fuzzy ring and basic properties about this concept. Part three study the strongly pure fuzzy submodules. Part four is definition the strongly regular fuzzy module and study the property of strongly regular fuzzy module.

§1.Strongly Pure Fuzzy Ideals

Definition (1.1)

Let I be a fuzzy ideal of a ring R ,I is called pure fuzzy ideal if for each $x_t \subseteq I$, there exists $r_{\ell} \subseteq I$ such that $x_t = x_t r_{\ell}$, $\forall$ t, $\ell \in(0,1]$.

Definition (1.2)

Let x_t: R→[0,1] such that $x_t(p) = \begin{cases} t & \text{if } x = p \\ o & \text{otherwise} \end{cases}$ Where p is prime number of R.

Definition (1.3)

Let K be a fuzzy ideal of a ring R ,K is called strongly pure fuzzy ideal denoted by S-pure fuzzy ideal if for each $r_\ell \subseteq K$, there exists a prime fuzzy singleton $x_t \subseteq K$ such that $r_\ell = r_\ell x_t$, $\forall$ t, ℓ $\in$(0,1].

Proposition (1.4)

Let I be a fuzzy ideal of R then I is S-pure if and only if I_t is a S-pure ideal of R,$\forall$ t$\in$(0,1].

Proof:

($\Longrightarrow$)Let $r_\ell = r_\ell x_t$,$\forall \ell$,t$\in$(0,1]. To show that r=rx

$r_\ell = (rx)_\ell$ where ℓ=min{ ℓ, t} by[1]

r=rx by[4]

Then I_t is S-pure ideal of R ,$\forall$ t$\in$(0,1].

($\Longleftarrow$) let r=rx To prove $r_\ell = r_\ell x_t$ $\forall \ell$, t$\in$(0,1]

r=rx implies $r_\ell = (rx)_\ell$ by[4]

$r_\ell = r_\ell x_t$ where ℓ=min{ ℓ, t} by[1]

Therefore I is a S-pure fuzzy ideal of R.

Definition (1.5)

An fuzzy singleton a_t of R is called fuzzy idempotent if $(a_t)^2 = a_t$ $\forall$ t$\in$(0,1].

Remarks and Examples (1.6)

1-Let K be a fuzzy ideal of a ring R, if K is S-pure fuzzy ideal then K is pure fuzzy ideal.

Proof: it is clear

The converse not true by

Example: A ring Z_6 and N=$(\bar{3})$={$\bar{0}, \bar{3}$} ,K=$(\overline{2})$={$\bar{0}, \bar{2}, \bar{4}$}

Define K: $Z_6 \longrightarrow$[0,1] by $I(x) = \begin{cases} t & \text{if } x \in N \\ o & \text{otherwise} \end{cases}$ $\forall$ t$\in$(0,1]

DOI : https://dx.doi.org/10.26808/rs.st.i7v5.07
International Journal of Advanced Scientific and Technical Research Issue 7 volume 5 Sep-Oct 2017
Available online on http://www.rspublication.com/ijst/index.html ISSN 2249-9954

Define $J: Z_6 \longrightarrow [0,1]$ by $J(x) = \begin{cases} t & \text{if } x \in K \\ o & \text{otherwise} \end{cases}$ $\forall$ t∈(0,1]

It is clear that K and J is fuzzy ideal of Z_6 and J_t=K ,K_t=N

K_t is S-pure ideal of Z_6 by [5]

Thus K is S-pure fuzzy ideal of a ring Z_6 by(Proposition 3.1.4)

But J_t is not S-pure ideal of a ring Z_6 by [5]

Hence J is not S-pure fuzzy ideal of Z_6 by(Proposition 3.1.4)

But J is pure fuzzy ideal.

2- Let K be a fuzzy ideal of a ring R. If K is S-pure fuzzy ideal of R, then JK=J∩ K for each fuzzy ideal J of R.

3- Let K be a fuzzy ideal of a ring R. If K generated by prime idempotent fuzzy singleton , then K is S-pure fuzzy ideal

Proof:

Let K=(p_s) be a fuzzy ideal generated by prime fuzzy singleton p_t, ∀ t∈(0,1]

Such that p_s=p_s^2. If $x_t \subseteq K$ there exists fuzzy singleton r_ℓ of R such that x_t= r_ℓ p_s implies x_t= r_ℓ p_s=$r_\ell p_s^2$=r_ℓ p_s p_s=x_t p_s

Therefore K is S-pure fuzzy ideal of a ring R.

4-Let K be a fuzzy ideal of a ring R, if K is S-pure fuzzy ideal then K is idempotent .

Proof:

Let K be a S-pure fuzzy ideal of R. and let $r_\ell \subseteq K, \forall \ell \in (0,1]$

Then there exists a prime fuzzy $x_t \subseteq K$.

Such that $r_\ell = r_\ell x_t$ $\forall \ell$, t∈(0,1] but $r_\ell x_t \subseteq K.K$

Hence $r_\ell \subseteq K^2$. Thus K⊆ K^2 and it is clear that $K^2 \subseteq K$. implies K= K^2.

Therefore K is idempotent fuzzy ideal of R.

Definition (1.7)

Let R be a fuzzy ring, then there exists 1_t of R such that $a_t.1_t$=$(a.1)_t$ =a_t for all fuzzy singleton a_t of R . a_t is called unit fuzzy singleton.

DOI : https://dx.doi.org/10.26808/rs.st.i7v5.07
International Journal of Advanced Scientific and Technical Research Issue 7 volume 5 Sep-Oct 2017
Available online on http://www.rspublication.com/ijst/index.html ISSN 2249-9954

Definition (1.8)

Let x_t be a fuzzy singleton of R is called fuzzy irreducible if $x_t = r_\ell\, y_s$. where $r_\ell \neq 0_1 \neq y_s$ is a fuzzy singleton of R ,$\forall \ell$,s, t∈(0,1] it is non unit fuzzy singleton of R then either r_ℓ or y_s is unity of R.

Now, we interdace the concepts of fuzzy factorial ring

Definition (1.9)

let S be a non empty fuzzy subset of R, and has no fuzzy singlet unit of integral domain of R ,then R is called fuzzy factorial if every non empty fuzzy singleton of R .written uniquely form $y_r\, x_{t1} \ldots.. x_{tk}$ where y_r is unit of R and $x_{t1} \ldots.. x_{tk} \subseteq S$,∀r, t∈(0,1].

Lemma (1.10)

Let R be a factorial fuzzy ring. Then every irreducible fuzzy singleton y_r of R is fuzzy prime every , $x_t \subseteq S$ is prime fuzzy singleton and every prime fuzzy singleton of set S is the product of unit of R ,∀ t∈(0,1].

Proof:

Let y_r irreducible fuzzy singleton of R

Thus y_r is non unit and if $a_s\, b_r \subseteq (\, y_r)$

Then $a_s\, b_r = x_t\, y_r$ with $x_t \subseteq S$. we write $a_s, b_r,\ x_t$ as product of irreducible

$a_s = y_{r1} \ldots.. y_{ri}$ $b_r = q_{k1} \ldots..\ q_{km}$ $x_t = r_{\ell 1} \ldots.. r_{\ell n}$,∀ r,k,t∈(0,1].

Here , one of those first two product may be empty

$y_{r1} \ldots.. y_{ri}\, q_{k1} \ldots..\ q_{km} = r_{\ell 1} \ldots.. r_{\ell n}\, y_r$

It is mean that either $a_s \subseteq (\, y_r)$ or $b_r \subseteq (\, y_r)$

Thus $(\, y_r)$ is prime fuzzy ideal of R and it is generated by prime

Proposition (1.11)

Let K be fuzzy ideal of R . And let R be a factorial fuzzy ring, such that $y_r \neq 0_1$ non unit fuzzy singleton of R is fuzzy irreducible. Then K is S-pure fuzzy ideal ⟺ K is pure fuzzy ideal.

Proof:

Let K be a pure fuzzy ideal of R, and let $r_\ell \subseteq K$, there exists $x_t \subseteq K$, such that $r_\ell = r_\ell x_t$. since $x_t \subseteq$R, is irreducible fuzzy singleton of R

Hence x_t is prime of K by (lemma3.1.10)

Therefore K is S-pure fuzzy ideal of R.

DOI : https://dx.doi.org/10.26808/rs.st.i7v5.07
International Journal of Advanced Scientific and Technical Research Issue 7 volume 5 Sep-Oct 2017
Available online on http://www.rspublication.com/ijst/index.html ISSN 2249-9954

The conversely is clear .

Proposition (1.12)

Let K and H are two fuzzy ideal of a ring R, if K is S-pure fuzzy ideal of R then $K \cap H$ is S-pure fuzzy ideal of R.

Proof: obviously.

Proposition (1.13)

Let K and H are two fuzzy ideal of a ring R, such that $K \subseteq H$ if $K \cap H$ is S-pure fuzzy ideal of R, then K is S-pure fuzzy ideal of R.

Proof: it is clear

Corollary (1.14)

Let K and H are two S-pure fuzzy ideal of a ring R, then $K \cap H$ is S-pure fuzzy ideal of R.

Corollary (1.15)

Let K and H are two fuzzy ideal of a ring R, then K is S-pure fuzzy ideal of $R \Leftrightarrow K \cap H$ is S-pure fuzzy ideal of R.

Proposition (1.16)

Let K and H are two fuzzy ideal of a ring R, if $K \oplus H$ is S-pure fuzzy ideal of R. then either K or H is S-pure fuzzy ideal of R.

Proof:

Let $x_t \subseteq K$ and $y_t \subseteq H$, implies $x_t + y_t \subseteq K \oplus H$

Since $K \oplus H$ is S-pure fuzzy ideal of R. there exists a prime fuzzy $r_\ell \subseteq K \oplus H$

Where $r_\ell = r_\ell + 0 \subseteq K \oplus H$

Such that $x_t + y_t = (x_t + y_t)\, r_\ell = (x_t + y_t)(r_\ell + 0) = x_t r_\ell + y_t r_\ell \subseteq K \oplus H$

Since $y_t r_\ell \subseteq K \cap J$ and $K \cap H = \{0\}$

Hence $y_t r_\ell = 0$. Thus $x_t = x_t r_\ell \subseteq K$.

Therefore K is S-pure fuzzy ideal of R.

And if $r_\ell = 0 + r_\ell \subseteq K \oplus J$. then we can get H is S-pure fuzzy ideal of R.

DOI : https://dx.doi.org/10.26808/rs.st.i7v5.07
International Journal of Advanced Scientific and Technical Research Issue 7 volume 5 Sep-Oct 2017
Available online on http://www.rspublication.com/ijst/index.html ISSN 2249-9954

Corollary (1.17)

Let K and H are two fuzzy ideal of R and let R be a factorial fuzzy ring , such that K ⊕ H is S-pure fuzzy ideal of R. then K and H are S-pure fuzzy ideal of R.

"**Definition (1.18)**

The fuzzy Jacobson radical of a ring R denoted by F-J(R) is the intersection of all fuzzy maximal ideal of R.[3]"

Proposition (1.19)

Let K is S-pure fuzzy ideal of R, such that K ⊆ F-J(R), then K={0}

Proof:

Let $r_\ell \subseteq$ K, since K is S-pure fuzzy ideal of R there exists a prime $x_t \subseteq$ K, such that $r_\ell = r_\ell x_t$

Implies $r_\ell(1\text{-}x_t)$=0. And since K ⊆ F-J(R), then $x_t \subseteq$ F-J(R).

Hence r_ℓ=0, so K={0}.

§2.Strongly Regular Fuzzy Ring

Definition (2.1)

A fuzzy ring R is called regular if and only if for each fuzzy singleton x_t of R , there exists fuzzy singleton r_ℓ of R such that $x_t = x_t r_\ell x_t$, ∀ t, ℓ ∈(0,1].

Definition (2.2)

Let R be a fuzzy ring . R is called strongly regular denoted by S-regular fuzzy if and only if for each fuzzy singleton r_ℓ of R , there exists a prime fuzzy singleton x_t of R such that $r_\ell = r_\ell x_t r_\ell$, ∀ t, ℓ ∈(0,1].

Equivalent a ring R is S-regular fuzzy if for each fuzzy singleton of R is S-regular fuzzy.

Proposition (2.3)

Let R be a S-regular fuzzy ring ⇔ R_t be S-regular ring , ∀t∈(0,1].

Proof:

(⟹)Let R be a S-regular fuzzy ring To prove R_t is S-regular ring

$r_\ell = r_\ell x_t r_\ell$,∀ℓ,t∈(0,1].

implies $r_\ell = (rxr)_\ell$ where ℓ=min{ ℓ, t} by[1]

r=r x r by[4]

Then R_t is S-regular ring , ∀t∈(0,1].

DOI : https://dx.doi.org/10.26808/rs.st.i7v5.07
International Journal of Advanced Scientific and Technical Research Issue 7 volume 5 Sep-Oct 2017
Available online on http://www.rspublication.com/ijst/index.html ISSN 2249-9954

($\Leftarrow$) let R_t be S-regular ring to show that R is S-regular fuzzy ring

Let r=r x r To prove $r_\ell = r_\ell x_t r_\ell$ $\forall \ell, t\in(0,1]$

r=rxr implies $r_\ell = (rxr)_\ell$ where ℓ=min{ ℓ, t} by[4]

$r_\ell = r_\ell x_t r_\ell$ by[1]

Therefore R is S-regular fuzzy ring .

Remarks and Examples (2.4)

1- Let R: Z_4 →[0,1] define by

$$R(r) = \begin{cases} t & \text{if } r \in Z_4 \\ o & \text{otherwise} \end{cases} \qquad \forall\ t\in(0,1]$$

It is clear that $R_t = Z_4$ and Z_4 is S-regular ring [5]

Thus R is S-regular fuzzy ring. by proposition(3.2.3)

By the same mothed we can to show that if R_t= Z_6 is not S-regular fuzzy ring and we get R is not S-regular fuzzy ring .

2-Let R be a fuzzy ring , if R is S-regular fuzzy, then R is regular fuzzy ring.

Proof: it is clear

The converse not true for example

Example: Let R: Z_6 →[0,1] define by

$$R(r) = \begin{cases} t & \text{if } r \in Z_6 \\ o & \text{otherwise} \end{cases} \qquad \forall\ t\in(0,1]$$

It is clear that $R_t = Z_6$ and Z_6 is regular ring ,but Z_6 is not S-regular ring

Then R is not S-regular fuzzy ring . by proposition(3.2.3)

3- every fuzzy ideal of R is irreducible and let R be a factorial fuzzy ring, then R is S-regular fuzzy ring $\Leftrightarrow$ fuzzy ideal of R is S-pure fuzzy.

Proposition (2.5)

Let R_1 and R_2 are two fuzzy ring , if $R_1 \oplus R_2$ is S-regular fuzzy ring. Then either R_1 or R_2 is S-regular fuzzy ring.

Proof:

Let fuzzy singleton $r_{\ell 1}$ of R_1 and $r_{\ell 2}$ of R_2

Implies $r_{\ell 1} + r_{\ell 2} \subseteq R_1 \oplus R_2$. Put x_t= $r_{\ell 1} + r_{\ell 2}$

DOI : https://dx.doi.org/10.26808/rs.st.i7v5.07
International Journal of Advanced Scientific and Technical Research Issue 7 volume 5 Sep-Oct 2017
Available online on http://www.rspublication.com/ijst/index.html ISSN 2249-9954

Since $R_1 \oplus R_2$ is S-regular fuzzy ring, there exists a prime fuzzy singleton $y_t = y_t + 0 \subseteq R_1 \oplus R_2$, such that $x_t = x_t y_t x_t = (r_{\ell 1} + r_{\ell 2}) y_t (r_{\ell 1} + r_{\ell 2})$

$= r_{\ell 1} y_t r_{\ell 1} + r_{\ell 1} y_t r_{\ell 2} + r_{\ell 2} y_t r_{\ell 1} + r_{\ell 2} y_t r_{\ell 2}$

But $r_{\ell 1} y_t r_{\ell 2}, r_{\ell 2} y_t r_{\ell 1} \subseteq R_1 \cap R_2$ and $R_1 \cap R_2 = (0)$

Thus $x_t = r_{\ell 1} + r_{\ell 2} = r_{\ell 1} y_t r_{\ell 1} + r_{\ell 2} y_t r_{\ell 2}$.

Implies that $r_{\ell 1} = r_{\ell 1} y_t r_{\ell 1} \subseteq R_1$

Therefore R_1 is S-regular fuzzy ring.

And if $0 + y_t \subseteq R_1 \oplus R_2$, by same method we get $r_{\ell 2} = r_{\ell 2} y_t r_{\ell 2} \subseteq R_2$

Hence R_2 is S-regular fuzzy ring.

§3.Strongly Pure Fuzzy Submodules

Definition (3.1)

Let X be a fuzzy module of an R-module M and let A be a fuzzy sub modules of X. A is called Strongly pure fuzzy submodule denoted by S-pure fuzzy submodule . if there exists a prime fuzzy ideal P of a ring R such that $PX \cap A = PA$.

Proposition (3.2)

let B be fuzzy submodules of a fuzzy module X. Then B is S-pure fuzzy submodule of X if and only if B_t is S-pure submodules of X_t, $\forall$ $t \in (0,1]$.

Proof:

Let I be a prime ideal of ring R

Define $P: R \to [0,1]$ by $P(x) = \begin{cases} t & \text{if } x \in I \\ o & \text{otherwise} \end{cases}$ $\forall$ $t \in (0,1]$

And let N be a submodule of an R-module M.

Define $B: M \longrightarrow [0,1]$ by $B(x) = \begin{cases} t & \text{if } x \in N \\ o & \text{otherwise} \end{cases}$ $\forall$ $t \in (0,1]$ It is clear that P is a prime fuzzy ideal of R and B is fuzzy submodules of X .

Now, $B_t = N$, $P_t = I$, $X_t = M$

($\Longrightarrow$) Let B is S-pure fuzzy submodule of X . To prove B_t is S-pure submodules of X_t. $\forall$ $t \in (0,1]$.

To show that $P_t X_t \cap B_t = B_t P_t$

$P_t X_t \cap B_t = (PX)_t \cap B_t$ by[2]

$= (PX \cap B)_t$ by[6]

$=(PB)_t$ since B is S-pure

$=B_t\, P_t$ by[2]

Thus B_t is S-pure submodules of X_t. $\forall\, t\in(0,1]$.

conversely Let P be a prime fuzzy ideal of R and B be a fuzzy submodules of X T.p B is S-pure fuzzy submodule of X

$(PX \cap B)_t=(PX)_t \cap B_t \quad \forall\, t\in(0,1]$. By[6]

$=P_tX_t \cap B_t$ by[2]

but B_t is S-pure submodules of X_t.

Then $P_tX_t \cap B_t=P_tB_t$

$= (PB)_t$ by[2]

Hence $(PX \cap B)_t=(PB)_t$

$PX \cap A = PB$

Therefore B is S-pure fuzzy submodule of X .

Remarks and Examples (3.3)

1- Let $M=Z_6$ as Z-module and $N=(\bar{3})$ and $K=(\bar{2})$

Define $X: Z_6 \longrightarrow [0,1]$ by $X(x) = \begin{cases} 1 & \text{if } x \in M \\ o & \text{otherwise} \end{cases}$

Define $A: Z_6 \longrightarrow [0,1]$ by $A(x) = \begin{cases} t & \text{if } x \in N \\ o & \text{otherwise} \end{cases} \quad \forall\, t\in(0,1]$

Define $B: Z_6 \longrightarrow [0,1]$ by $B(x) = \begin{cases} t & \text{if } x \in K \\ o & \text{otherwise} \end{cases} \quad \forall\, t\in(0,1]$

It is clear that X is fuzzy module, and A, B is fuzzy submodules of X

$X_t=M$, $A_t=N$ and $B_t=K$

A_t is S-pure submodules of X_t. by[5]

Then A is S-pure fuzzy submodule of X by(proposition 3.3.2)

But B_t is not S-pure fuzzy submodule of Z_6

Therefore B is not S-pure fuzzy submodule of X. by(proposition 3.3.2)

2- Let X be a fuzzy module of an R-module M ,and let C be S-pure fuzzy submodule of X, then C is pure fuzzy submodule of X.

DOI : https://dx.doi.org/10.26808/rs.st.i7v5.07
International Journal of Advanced Scientific and Technical Research Issue 7 volume 5 Sep-Oct 2017
Available online on http://www.rspublication.com/ijst/index.html ISSN 2249-9954

Proof: it is clear

The converse not true for example

Example: Let M=Z_{12} as Z-module and N= $(\bar{3})$

Define X: M⟶[0,1] by $X(x) = \begin{cases} 1 & \text{if } x \in M \\ o & \text{otherwise} \end{cases}$

Define C: M⟶[0,1] by $A(x) = \begin{cases} t & \text{if } x \in N \\ o & \text{otherwise} \end{cases}$ $\forall$ t∈(0,1]

It is clear that X is fuzzy module, C is fuzzy submodules of X, X_t=M and C_t=N

C_t is pure submodules of X_t. by[18]

Thus C is pure fuzzy submodule of X by[4]

But C_t is not S-pure submodules of X_t . by [5]

Therefore C is not S-pure fuzzy submodule of X. by(proposition 3.3.2)

Proposition (3.4)

let A be fuzzy submodules of a fuzzy module X and let B be fuzzy submodules of A. if A is pure fuzzy submodule of X and B is S-pure fuzzy submodules of A, then B is S-pure fuzzy submodules of X.

Proof:

Let B is S-pure fuzzy submodules of A, there exists a prime fuzzy ideal P of R such that $PA \cap B = PB$

Since A is pure fuzzy submodule of X ,then $PX \cap A = PA$.

Implies that B∩A∩XP=BP, and since B⊆A, then B∩A=B

Implies $XP \cap B = PB$

Therefore B is S-pure fuzzy submodules of X.

Corollary (3.5)

let A and B are two fuzzy submodule of a fuzzy module X. if A is pure fuzzy submodule of X and A∩B is S-pure fuzzy submodules of A. then A∩B is S-pure fuzzy submodules of X.

DOI : https://dx.doi.org/10.26808/rs.st.i7v5.07
International Journal of Advanced Scientific and Technical Research Issue 7 volume 5 Sep-Oct 2017
Available online on http://www.rspublication.com/ijst/index.html ISSN 2249-9954

§4.Strongly Regular Fuzzy Module

Definition (4.1)

Let X be a fuzzy module of an R-module M, an fuzzy $x_t \subseteq X, \forall\ t \in (0,1]$ is called strongly regular fuzzy denoted by S-regular fuzzy if there exists a fuzzy module homomorphism $\theta: M \longrightarrow R$, such that $\theta(x_t)\, x_t = x_t$ where $\theta(x_t)$ is S-regular fuzzy singleton in a ring R.

Definition (4.2)

Let X be a fuzzy module of an R-module M, is called S-regular fuzzy if every $x_t \subseteq X, \forall\ t \in (0,1]$ is S-regular fuzzy

Remark (4.3)

Let X is S-regular fuzzy modules and A be a prime fuzzy submodule of X, then A is S-regular fuzzy submodule of X.

Proof:

Let A be a fuzzy submodule of X and I be a fuzzy ideal of R

To show that $IX \cap A = IA$

It is clear that $IA \subseteq IX \cap A$

To prove $IX \cap A \subseteq IA$

Let $x_t \subseteq IX \cap A$, then $x_t = \sum_{i=1}^{n} r_{\ell i}\, x_{ti}$ $\forall\ t, \ell \in (0,1]$. Where $r_{\ell i} \subseteq I$ and $x_{ti} \subseteq X$

Since X is S-regular fuzzy R-modules ,hence x_t S-regular fuzzy singleton

Thus there exists a fuzzy module homomorphism $\theta: M \longrightarrow R$, such that $x_t = \theta(x_t)\, x_t$, so $\theta(x_t) = \sum_{i=1}^{n} r_{\ell i}\, \theta(x_{ti})$ and $x_t = \theta(x_t)\, x_t = \sum_{i=1}^{n} r_{\ell i}\, \theta(x_{ti}) x_t$

And since $x_t \subseteq A$, hence $x_t = \sum_{i=1}^{n} r_{\ell i}\, \theta(x_{ti}) x_t \subseteq IA$

Thus $IX \cap A \subseteq IA$

Then $IX \cap A = IA$

Therefore A is S-regular fuzzy submodule of X.

Proposition (4.4)

Let R be a S-regular fuzzy ring $\Leftrightarrow$ R is S-regular fuzzy R-module.

Proof:

Let R be a S-regular fuzzy ring to prove R is S-regular fuzzy R-module.

DOI : https://dx.doi.org/10.26808/rs.st.i7v5.07
International Journal of Advanced Scientific and Technical Research Issue 7 volume 5 Sep-Oct 2017
Available online on http://www.rspublication.com/ijst/index.html ISSN 2249-9954

Let $x_t \subseteq R$, thus there exists a prime fuzzy singleton $p_s \subseteq R$ such that $x_t = x_t p_s x_t$, $\forall$ t, $\ell \in (0,1]$.

Now, define a function $\theta: R \longrightarrow R$, by $\theta(x_t) = x_t p_s$ for each fuzzy singleton x_t of R

Then $\theta(x_t)\, x_t = x_t p_s x_t$, so $\theta(x_t)\, x_t = x_t$

Thus R is S-regular fuzzy R-module.

Conversely let R is S-regular fuzzy R-module to prove R is S-regular fuzzy ring .

Let $x_t \subseteq R$, there exists a fuzzy module homomorphism $\theta: R \longrightarrow R$, such that $x_t = \theta(x_t)\, x_t$, where $\theta(x_t)$ is S-regular fuzzy singleton of R.

Since $\theta(x_t) = \theta(1.x_t) = \theta(1) x_t$,

Hence $x_t p_s x_t = x_t \theta(1) x_t$

Therefore R is S-regular fuzzy ring .

<u>Proposition (4.5)</u>

Let X is S-regular fuzzy module and fuzzy divisible over an fuzzy integral domain R, then every fuzzy submodule of X is fuzzy divisible.

<u>Proof:</u>

Let A be a fuzzy submodule of X , and $0 \neq r_\ell \subseteq R$

We show that $r_\ell A = A$

By remark(3.4.3) A is S-pure fuzzy submodules of X.

so $< r_\ell > A = A \cap < r_\ell > X$

We show that $r_\ell A = A \cap r_\ell X$, if $x_t \subseteq A \cap r_\ell X$, then $x_t = r_\ell s_r$

Since x_t is S-regular fuzzy of X there exists a fuzzy module homomorphism $\theta: M \longrightarrow R$, such that $x_t = \theta(x_t)\, x_t$ and hence $x_t = \theta(x_t)\, x_t = r_\ell\, \theta(s_r)\, x_t$, as $x_t \subseteq A$

This implies that $x_t \subseteq r_\ell$ A, hence $A \cap r_\ell X \subseteq r_\ell$ A,

Hence $r_\ell A = A \cap r_\ell X$.

As $r_\ell X = X$. thus $A \cap X = r_\ell A$

So $r_\ell A = A$

Therefore A is fuzzy divisible.

Proposition (4.6)

Let X be a S-regular fuzzy R-module M , then F-J(R) X=0

Proof:

Since X is S-regular fuzzy R-module,

Thus every fuzzy submodules of X is S-pure fuzzy submodules by remark(3.4.3)

Let F-J(R) X≠0, therefore there exists $x_t \subseteq$F-J(R)

So Rx_t is S-pure fuzzy submodules of X

Thus $Rx_t \cap$ F-J(R)X= F-J(R) Rx_t

Then Rx_t =F-J(R) Rx_t and by lemma(3.4.6) Rx_t=0 so x_t=0

Hence F-J(R) X=0

REFERENCES

1-Inaam M-A.H, ‘’On Fuzzy Ideals of Fuzzy Rings’’, Math. And Physics, Vol. 16, PP.4, (2001).

2-Kumar, R., S.k. Bhambir , Kumar, p.,” Fuzzy Submodules, Some Analogous and Deviation”, Fuzzy Sets and Systems, 70,125-130,(1995).

3-Malik, D. S. and Mordeson, J.N., “Fuzzy Maximal, Radical and Primary Ideals of a Ring”, Information Sciences, Vol.53,237-250,(1991).

4-Maysoun A. H., ’’F-Regular Fuzzy Modules’’, M. Sc. Thesis , University of Baghdad,(2002).

5-Nada K. A.,” Strongly Pure Ideals And Strongly Pure Submodules”, Kirkuk University Journal ,Vol.10, Issue 1 , PP. 12-28 ,(2015).

6-Zadeh L. A., ‘’Fuzzy Sets, Information and Control’’ ,Vol. 8,PP.338-353,(1965).