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Magnetic Moment of Heavy Baryons

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Abstract:
The baryon magnetic moment is a fundamental observable as its masswhich encodes information of the underlying quark-gluon structure and dynamics. Assuminga conventional correlated perturbative chiral quark model (CPχQM) we suggest that the charmedheavy baryons are a bound state of two light diquarks and a single heavy charm antiquark, the spatially wave function of these diquarks has a P- wave and an S-wave in angular momentum in the first and second version of our model respectively, as the result of these considerations we construct the orbital - flavor - spin symmetry of contribution of quarks. Then we calculate their magnetic moments in our model.

Key words: Heavy Baryon, Magnetic Moment, Quark Model,Quark Symmetry

Introduction

The magnetic moment is an intrinsic observable of particles which may encode important information of its quark-gluon structure and will help us deepen our understanding of the underlying dynamics.The heavybaryon masses and magnetic moments in several typical models have been calculated [1]. Now we calculate masses and magnetic moments for them using our diquark model. Previously we have used our vector diquark model for calculating the mass and stability / magnetic moment of Theta + pentaquark state.

Theoretically, the study of heavy baryons has always been interesting and these baryons play an important role in our understanding of QCD at the hadronic scale. There are many theoretical treatments of heavy baryons [2, 3, 4] including quark models, QCD sum rules, Lattice QCD [5, 6], the Relativistic quark-diquark approximation. Non-relativistic QCD, NRQCD which has been able to explain the mass spectrum of light baryons which is an effective field theory obtained from QCD by integrating out modes of an energy of the order of the heavy- quark masses for describing baryons made of one or more heavy quarks[7, 8].The heavy – quarklight diquark HQLD sector of NRQCD lagrangian is a heavy quark effective theory HQET. In this effective field theory framework, EFT of heavy baryons where the typical gluon momenta are small compared with the heavy quark mass $m_{Q,}$QCD dynamics of light diquark is independent of the flavor and spin of heavy quark. For the heavy flavors, this new symmetry called heavy quark symmetry, HQS. In fact, in this limit of heavy quark mass, low energy QCD dynamics remains non-perturbative but using HQS one can separate the light quark and gluon dynamics from that of heavy one by

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systematically expanding the QCD lagrangian in powers of 1/mQ and imposing HQS effects [9, 10, and 11]. According to these effects in heavy baryons, the light degrees of freedom quantum numbers are well-defined up to corrections in the inverse of the m_Q.Consequently, the heavy quark momentum is close to the kinetic momentum resulting from the hadrons motion. Thus the kinetic energy of the internal motion of the heavy baryon system is close to the kinetic energy of the relative motion of the heavy quark and light diquark up to corrections of the m_L / m_Q where L, denotes a light quark. This is one of the bases for treating the light quark subsystem as a diquark in our calculations. The quark-diquark picture of a heavybaryon is the nice approximation used to describe the baryon properties [12]. In this picture, we reduce the task of treating a three-body system to a two body system which is a successful task especially where we approximate the heavy quark massthe m_Q to be infinity with respect to mass scale in the process, and hence enormously reduces the complexity of theoretical analysis. The paper is organized as follows. In the next section we introduce HQS effects for heavy baryons and calculate their mass spectrum using this symmetry limit.Finally, section 3 devoted to conclusions and results.

HQS limit

Theoretically, the full QCD Lagrangian for a heavy quark (c, b ort) is given by

$$L_Q = Q\,(i\gamma_\mu D^\mu - m_Q)\,Q, \qquad (1)$$

Where $D^\mu \equiv \partial^\mu - ig_s T^a A^{a\mu}$ with $T^a = \lambda^a/2$.Thus the heavy quark interacts with the light degrees of freedom by exchanging gluons with the momenta of order Λ_{QCD} which is much smaller than its mass m_Q. In the HQS limit with low energy situations, where the typical gluon momenta are small compared with the heavy quark mass ($_{MQ}$), QCD dynamics becomes independent of the heavy degrees of freedom, especially for the flavor and spin of the heavy quark. This means that the hyperfine interaction that involves the heavy quark is suppressed by the mass of the heavy quark. As a consequence, one-gluon exchange HF interaction should depend on the interacting light diquark pair, independently of the baryon the pair belongs to. In fact, the QCD hyperfine interaction andthe QED electromagnetic hyperfine interaction between i and j quarks are proportional to$1/m_i m_j$, where m_i, mj are their masses. These interactions contribute to the systematic uncertainty of the experimental results and can be ignored in HQS limit, where one of the quarks is heavy [14]. Indeed we characterize the heavy baryon mass by two widely separated scales: the large heavy quark mass, (m_Q), and the low momentum transfer between the heavy and the light quarks of the diquark, which is of order Λ_{QCD}. In this system, the light diquark circle around the nearly static heavy quark and the system behave as the QCD analog of the familiar hydrogen bounded by electromagnetic force. In HQS limit, where $m_Q \to \infty$ a good quantum number is the angular momentum of the light degrees of freedom. Thus, heavy quark baryons belong to either SU (3) antisymmetric 3_F or symmetric 6_F representations fig.1. The spin of the light diquark is 0 for 3_F, while it is 1 for 6_F For the spin of the ground state heavy baryons we have1/2 for 3F, representing the Λh and Ξh heavy baryons, while it can be both ½ or3/2for

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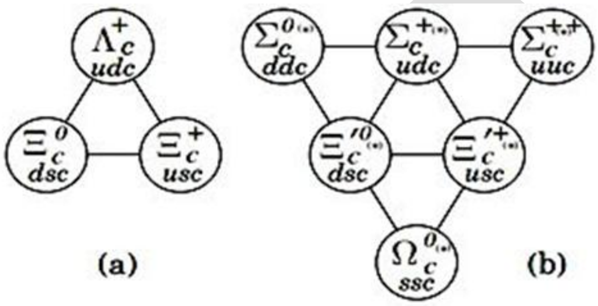
6_F, representing Σ^h, Σ^{h*}, Ξ'^h, Ξ'^{h*}, Ω^h and Ω^{h*}, where the star and h indicates spin 3/2 c b quarks respectively. The mass difference between states belonging to different representations 3_F and 6_F, do contain the dynamics of the light scalar and vector diquark subsystem respectively. But the mass splitting between states belonging to same representation is caused by the chromomagnetic interaction at the order1/ mQ and can be ignored in HQS limit. Thus baryons containing a single heavy quark should fall into almost degenerate multiplets.For example, there is no mass difference between Q^* and Q heavy baryons for 6_F. Generally, these states have the same parity as the light component.

Fig 1:SU (3) multiplets of charmed baryons, (a) 3F antisymmetric and (b) 6F symmetric Representations.



The members of the two multiplets of singly charmed baryons have flavor wave functions

Σ^{c++} = uuc, $\Sigma c^+ = 1/\sqrt{2}$ (ud + du) c, Σc0 = ddc (2)
$\Xi'^{c+} = 1/\sqrt{2}$ (us + su) c $\Xi'^{c0} = 1/\sqrt{2}$ (ds + sd) c
Ω^c = ssc,
For the sextet and
$\Lambda^{c+} = 1/\sqrt{2}$ (ud - du) c,$\Xi^{c+} = 1/\sqrt{2}$ (us - su) c (3)
$\Xi^{c0} = 1/\sqrt{2}$ (ds - sd) c

For the antitriplet whichare similarto theset of flavorwave functions for baryons containing b quark.

Table 1:The s-wave heavy baryons and their quantum numbers.

state	ΛQ	ΣQ	Σ^{*Q}	Ξ_Q	Ξ'^Q	Ξ^{*Q}	Ω_Q	Ω^{*Q}
J^P	1/2 +	1/2+	3/2+	1/2 +	1/2+	3/2+	1/2 +	3/2+
Jl	0	1	1	0	1	1	1	1

Table 2, shows the experimental masses of the Ground-state charmed and bottom

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baryons [15].

Table 2: Ground - state charmed baryons and their SU (3) multiplets
Lattice estimates (†) have been taken from (Ref [16]).

Heavy baryon	Mass(GeV)	SU(3)multiplet
Λc+ Λb+	2.285 -5.624	3bar
Σ*c++,+,0 Σ*b++,+.0	2.455-5.808	6
Ω*c0 Ω*b0	2.698-5.990†	6
Ξc+ Ξb+	2.468-5.793	3bar
Ξc0 Ξb0	2.471-5.760†	3bar
Ξ*´c+ Ξ*´b+	2.576-5.900†	6
Ξ*´c0 Ξ*´b0	2.578-5.900†	6

In the limit of HQS, where the heavy quark mass $m_Q \rightarrow \infty$, all states in the 6F representation would be degenerate and this is true for all states in the 3_F representation. In this limit, without the $m_Q \rightarrow \infty$ approximation there is a mass splitting between states belonging to each representation due to differences between the masses of the light diquark sectors of the heavy baryons. We calculated the light diquark masses by adding the two quarks mass and their binding hyperfine HF energy. Table .3.

Table 3: Quark and diquark masses and quantum numbers.

Quark mass (MeV)	**mc ms m_l m_b**
Diquark mass (MeV)	1650 460 360 4275 ll ls lc
Scalar Vector Quantum numbers	420 580 1840 673 680 1840 Flavor Color Spin Orbital
Scalar Vector	3bar 3bar 0 0 6 3bar 1 0

Now we evaluate the masses of the ground state heavy baryons in the framework of the HQS limit. Thus we can use the mass formula

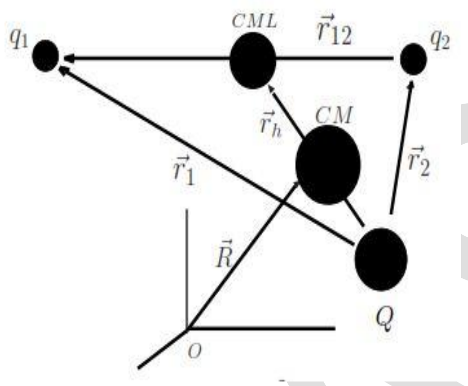
$$M = m_D + m_Q + E_L + Er \qquad (4)$$

Here, m_D is the light diquark mass, m_Q the heavy baryon mass, E_L the orbital and Er the radial exciting energies between heavy quark and light diquark respectively. According to table 3 two quarks having a closer mass have more tightly bound which is indicated by the spin-spin interaction, thus the mass splitting

$$(ud) - [ud] > (us) - [us] > (uc) - [uc] \simeq 0 \qquad (5)$$

Is expected where [ ], (), denotes scalar and vector diquarks respectively. We have accommodated the ground state, $J^P = 1/2^+$ heavy charmed and bottom baryons. These states have no orbital angular momentum, $E_L = 0$ and the mass splitting between them is indicated by radial exciting energy, Er of each ground state heavy baryon. By using this exciting energy we have evaluated the average distance between heavy quark and the center of mass of the light diquark for each heavy baryon state. We set the Jacobi coordinates for a heavy quark -light diquark description.fig.2.

Fig 2: Q2q rest frame



For the coordinates we consider the following relations

$R = m_{q1}x_{q1} + m_{q2}x_{q2} + m_Q x_h / m_{q1} + m_{q2} + m$

$r_{12} = x_q$(6)

$r_h = (m_{q1}x_{q1} + m_{q2}x_{q2} / m_{q1} + m_{q2}) - x_h$

Where x_{q1}, x_{q2}, and x_h represent the positions with respect to a certain reference frame and r_{12} and r_h are the Jacobian coordinates. Thus we would have the heavy baryon Kinetic energy

$$T(q^2Q) \simeq \nabla^2_{rh} / 2\mu \qquad (7)$$

Where∇^2 denotes the Laplacian and μ are the heavy quark-light diquark reduced mass. By using the Baryon wave function

$$\Psi_B = N[Y_{00}(rh) \exp(-a^2 r_h{}^2 / 2)](8)$$

We would have the Kinetic energy

$$Er = \langle T \rangle \psi \simeq 3a^2 / 4\mu \qquad (9)$$

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And for the relative distance between heavy quark and light diquark we have

$$r_0 = \langle r_h \rangle = \sqrt{5} / 2a^2 \quad (10)$$

We have calculated the radial kinetic energy, Er of each ground state heavy baryon listed in table 2, using their parameters, m_D m_Q and Er = 0. Also by using of Eq8-9, we obtained the average distance, r_0 between the heavy quark and the center of mass of light diquark. Table 4.

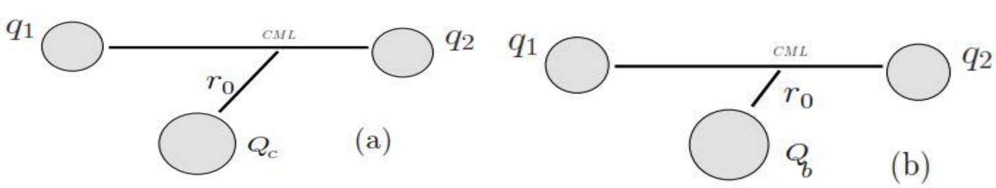
The results with QCD sum rule[16]and lattice QCD calculation [17] have suggested a clear dominance of the collinear-type configurations (the heavy quark is close to the center of mass of the light diquark).This result seems to support our calculations based on HQS limit of HQLD picture of heavy baryons. In Ref. [18], the authors studied the baryon properties using Isgur-Wise function and found the heavy quark is far from the light diquark which is against the HQS approximation of HQLD.

Table 4: Ground - state charmed and bottom baryons and their radial kinetic energy and the relative distance between heavy quark and light diquark center of mass, Experimental masses have been taken from (Ref [17]) and Lattice estimates (†) have been taken from (Ref [16]).

Heavy baryon	Mass(GeV)	E_r(MeV)	r_0(MeV)$^{-1}$
Λc+ Λb+	2.285-5.624	215-929	0.00509-0.00229
Σc++*,+,0Σ*b++,+,0	2.455-5.808	132-860	0.00535-0.00192
Ω*c0 Ω*b0	2.698-5.990	368-103	0.00320-0.00174
Ξc+ Ξb+ Ξc0	2.468-5.793	238-938	0.00425-0.00179
Ξb0 Ξ´*c+ Ξ´*b+	2.471-5.760	241-905	0.00423-0.00200
Ξ´*c0 Ξ´*b0	2.576-5.900	246-945	0.00392-0.00183
	2.578-5.900	248-945	0.00391-0.00183

The average size ofa scalar anda vector diquark is0.0045MeV $^{-1}$ and 0.0205 (0.0235) MeV $^{-1}$ respectively. According to Table 4 one sees that the average distance of the heavy quark to the center of mass of the light diquark, $r^0$is smaller than the average size of the light diquark.The picture that emerges from this analysis is the one depicted in Fig.3, where the heavy quark is too close to the center of mass of the light diquark, which is in agreement with the findings of Ref [18].

Fig 3:schematic picture of a Ground-state spin 1/2 heavy baryon with a charmed heavy quark (a), and a bottom heavy quark (b).



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These findings based on HQS limit of HQLD approximation shows a dominance of collinear-type configuration, which confirms the results of QCD sum rules[16] and lattice calculations [17].We have obtained the average distance, r^{0*} between the heavy quark and the center of mass of light diquark for charmed and bottomed baryons with spin 3/2 Table .5.One sees that this average distance for the spin 3/2 state heavy baryons is smaller than the spin 1/2 states. This distance splitting between states belonging to same representation is caused by the chromomagnetic interaction and usually can be ignored in HQS limited with $M_Q \to \infty$ approximation. The picture is depicted in Fig.4.

Fig 4:schematic picture of a Ground-state spin 3/2heavy baryon with a charmed heavy quark (a) and a bottom heavy quark (b).

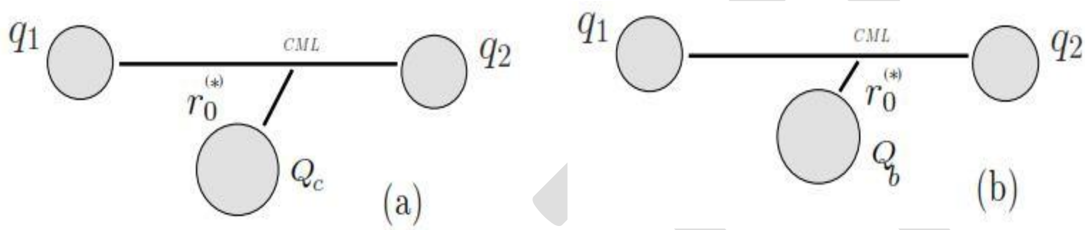

Table 5: Charmed and bottomedbaryons withspin 3/2, theirmasses (Ref [16-17] their SU (3) multiplets and the relative distance r_0*, between heavy quark and light diquark center of mass.

Heavy baryon	Mass (GeV)	SU(3) multiplet	r_0^*(MeV $^{-1}$)
$\Sigma_c^{++,+,0*}$ $\Sigma_b^{++,+,0*}$	2.518-5.833	6	0.00477-0.00190
Ω_{c*}^0 Ω_{b*}^0	2.768-6.000	6	0.00297-0.00162
Ξ'^{+0*}_c Ξ'^{+0*}_b	2.646-5.900	6	0.00350-0.00180

Magnetic Moment Calculation

For the magnetic moment of a particle we have:

$$\vec{\mu} = g\vec{S}$$

Where μ is the magnetic moment, g is gyromagnetic ratio and $S$is the spin operator, this leads to, $\mu_z = gS_z$. For the quarks we have:

$$g_q = g_s\mu_q = 2\mu_q = 2\frac{Q_q}{2m_q} = \frac{Q_q}{m_q}$$

In which$\mu_q$is quark magneton, and Q_q, $m_q$are quarks charge and mass respectively.
If the particle has angular momentum, themagnetic moment would be:

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$$\vec{\mu} = g\vec{S} + g_l \vec{l}$$

We conclude that for the heavy baryon we have:

$$\mu_z = \left\langle \psi_{fs} \,|\, \sum_i g_i S_z^i + g_{l_i} l_z^i \,|\, \psi_{fs} \right\rangle$$

In which Ψ_{fs} is the flavor- spin wave function of the heavy baryon.The contribution of the second term would be zero. Table 6 shows our magnetic moment calculations using flavor wave functions Eq (2, 3) and similar triplet and singlet spin wave functions for the scalar and vector light diquarks in our model.The spin of the light diquark is 0 for 3_F, while it is1 for 6_F.For the spin of the ground state heavy baryons we have1/2 for 3F and 1/2, 3/2 for 6_F.

Table 6: Magnetic moments of heavy baryons (Q=b, c)

Heavy baryon	**Wave Function**	**Magnetic Moment**
Λc^+ Λb^+	$1/\sqrt{2}$ (ud - du) cχ_A ,$1/\sqrt{2}$ (ud - du) b χ_A	$e_Q/2M_Q$
$\Sigma^* c^{++,+,0}$ $\Sigma^* b^{++,+,0}$	uuc (b) $\chi_{S\,\chi+}$, $1/\sqrt{2}$ (ud + du) c or (b) $\chi_{S\chi+}$, ddc or (b) $\chi_{S\,\chi+}$	$-e_Q/6M_Q + 2e_u/3M_u$, $-e_Q/6M_Q + e_u/3M_u + e_d/3M_d$, $-e_Q/6M_Q + 2e_d/3M_d$
$\Omega^* c^0$ $\Omega^* b^0$	ssc$\chi_{S\chi+}$, ssb$\chi_{S\chi+}$	$-e_Q/6M_Q + 2e_s/3M_s$
Ξc^+ Ξb^+	$1/\sqrt{2}$ (us - su) c$\chi_{A\,\chi+}$, $1/\sqrt{2}$ (us - su) b $\chi_{A\,\chi+}$	$e_Q/2M_Q$
Ξc^0 Ξb^0	$1/\sqrt{2}$ (ds - sd) c $\chi_{A\,\chi+}$, $1/\sqrt{2}$ (ds - sd) b$\chi_{A\,\chi+}$	$e_Q/2M_Q$
$\Xi'^* c^+$ $\Xi'^* b^+$	$1/\sqrt{2}$ (us + su) c $\chi_{S\,\chi+}$,$1/\sqrt{2}$ (us + su) b $\chi_{S\,\chi+}$	$-e_Q/6M_Q + e_u/3M_u + e_s/3M_s$
$\Xi'^* c^0$ $\Xi'^* b^0$	$1/\sqrt{2}$ (ds + sd) c $\chi_{S\,\chi+}$, $1/\sqrt{2}$ (ds + sd) b$\chi_{S\chi+}$	$-e_Q/6M_Q + e_d/3M_d + e_s/3M_s$

CONCLUSION

We calculated the masses and magnetic moments of charmed and bottomed heavy baryons for 3_F and 6_F multiplets with single heavy b or c quark in the framework of heavy quark symmetry limit using our distance configuration approach in perturbative chiral quark model and compared the results with the existing predictions in the literature. Our results on masses and magnetic moments of heavy baryons are in good agreement with many results listed in the literature [19] and may be checked via different non-perturbativeapproaches. Alsochecking our results by future experiments can be useful for understanding the internal structure as well as the geometric shape of these baryons.

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