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Some Types of Contra αrps –Continuous Functions

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Abstract
The purpose of this paper is to define anew kinds of contra – continuous functions which is called contraαrps –continuous functions. As well as , we will introduce other types of contra αrps– continuousfunctions which are ($\alpha rps^{\#}$ – continuous functions and $\alpha rps^{\#\#}$ –continuous functions) in topological spaces .Also, we will give some of its fundamental properties and discuss the relationships among these functions.
Keywords:-
αrps –closed sets , αrps–open sets,αrps–continuous functions ,contra- continuous.

1- Introduction :-

The notion of contra – continuous functions have been introduced and investigated by Dontchv J. [2] . Recently anew weaker from of this class of function called contra g-continuous functions is introduced And investigated by Caldas.M , Jafari .S ,Noiri .T ,Simeos.T[4] .In 2014, Dunya M. Hamed [3]introduced and studiedαrps–closed sets and also introduce the notion (αrps–continuous , αrps– irresolute and strongly αrps –continuous) functions .

In this paper , we introduce and investigate some types of Contra – continuous which are (contra αrps– continuous functions ,$\alpha rps^{\#}$ –continuous functions and $\alpha rps^{\#\#}$ –continuous functions) .Further , we discuss some properties of these functions.

Throughout this paper (X ,τ) , (Y,ρ)and (Z ,δ) (or simply X ,Y and Z) represent non-empty topological spaces . For a sub set K of a space .cl (K) , int(K) andK^c denoted the closure of K, the interior of K and the complement of K respectively.

2.Preliminaries:-

Definition (2-1) :- A subset K of a space X is said to

1- Semi-preopen[1]if $K \subseteq cl(int(cl(K))$and semi-preclosed if$int(cl(int(K))\subseteq K$.

2- regular open [13] if K = int(cl(K)) and regular closed if K = cl(int(K)) .

3- regular α-open [14] if there is a regular open set L such that L⊆ K ⊆ αcl(L) .

The semi-pre closureof sub set K of X is the intersection of all semi-pre closed sets containing Kand denoted byspcl(K)).

Definition (2-2) : A sub set K of a space is said to be :

1- generalized closed set (briefly , g- closed) [7] if cl(K) ⊆ L whenever K ⊆L and L is an open set in .

2- regular generalized closed set (briefly , rg- closed) [10] if cl(K) ⊆ L whenever K⊆ L and L is a regular open set .

3- regular pre-semi closed (briefly , rps-closed)[8] if spcl(K) ⊆ L whenever K⊆ L and L is an rg- open set in .

4- αrps-closed set [3] if αcl(K) ⊆L whenever K⊆L and L is a rps-open set in

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The complement of g-closed (resp. rg-closed, rps – closed and $\alpha rps -$ closed) sets is called a g-open (resp. rg-open, rps- open and $\alpha rps -$ open)sets ,

The class of all g-closed [resp. αrps–closed] subsets of is denoted by GC(X ,τ)[resp. αRPSC(X ,τ)] .

Remark (2-3),[3]:

Every closed (resp. open) set in is αrps– closed(resp. αrps–open)set.

Definition (2-4),[7] :

A topological space is said to be $T_{1/2}$ - space if every g- closed sets is closed.

Proposition (2-5) ,[3] :

A topological space X is said to be $T^*_{1/2}$ −spaceif every $\alpha rps -$ closed (resp. αrps −open) sets is closed (resp. open)set.

Definition (2-7): A function f : (X, τ) → (Y, σ) is said to be:

1. continuous function [6] if the inverse image of each open(closed) set in Y is an open(closed) set in X .
2. αrps −continuous [3]if $f^{-1}(K)$is αrps− closed (resp. αrps- open)set in X , for every closed (resp . open) set K in Y.
3. αrps −irresolute continuous[3] if f^{-1} (K) is an αrps −closed(resp. αrps − open) set in X , for every αrps − closed (resp. αrps −open) set K in Y .
4. strongly αrps −continuous[3] if f^{-1} (K) is closed(resp.open) set in X , for every αrps −closed(resp. αrps −open) set K in Y .
5. Contra-continuous [2] if $f^{-1}(K)$ is closed set in X , for every open set K in Y .
6. Contra g -continuous[4] if $f^{-1}(K)$ is g- closed set in X , for every open set K in Y .

3-Main Results :

In this section ,we introduce and define new kinds of contra -continuous functions namely contra αrps −continuous functions and other types of contra αrps–continuous.

Definition (3-1):

A function f: (X ,τ) ⟶ (Y , σ) is called **contra αrps- continuous function** if the inverse image of each open set in Y is αrps −closed set in X .

Example(3-2):-Let X= {u, v, w} with the topology τ ={X,ϕ,{u},{v, w}} , we define f : (X , τ) → (X, τ)by f(u)=u , f(v)=v and f(w)=w ,it is observe that f is contra αrps-continuous function.

Proposition(3-3):

A function f: (X ,τ) ⟶ (Y , σ) is contra αrps − function iff $f^{-1}(K)$ is αrps −open set in X , for every closed set K in Y .

Proof :

Suppose that f is contra αrps −continuousfunction and let K be a closed set in Y Then, $K^c$is an open set in Y, since f is contra αrps −continuous . Thus, $f^{-1}(K^c)$ is αrps −closed set in X . But $f^{-1}(K^c)$ =$-f^{-1}$ (K) =$(f^{-1}(K))^c$.Hence, f^{-1} (K) is an αrps −open set in X .Conversely, let K be an open set in Y. Then $K^c$is closed set in .By assumption $f^{-1}(K^c)$ is αrps −open set in X . But $f^{-1}(K^c)$ =$-f^{-1}$ (K) =$(f^{-1}(K))^c$.Hence, f^{-1} (K) is a closed set in X .Therefore , f is contra αrps −continuousfunction .

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Proposition(3-4):-
f: (X ,τ) ⟶ (Y , σ) is contra-continuous function ,then f is contraαrps −continuous.
Proof :-It clear from the definition of contra-continuousand fact that every closed set is αrps −closed .
The converse of above proposition need not be true as seen from the following example .
Example(3-5):-Let X=Y= {u, v, w} with the topologies τ = {X , ϕ , {u}}and ρ={Y, ϕ,{u}, {u, w} } , let f : (X , τ) → (Y,ρ) define by f(u)=v , f(v)=u and f(w)=w ,clearly f is contra αrps −continuous , but f is not contra-continuous , since for the open set K = {u} in Y , but $f^{-1}(K) = f^{-1}(\{u\}) = \{v\}$ is not closed set in X .
The following proposition give the condition in order to make proposition(3-4) true:

Proposition (3-6):
iff : (X , τ) → (Y,ρ) is contra αrps−continuousand is $T^*_{1/2}$ −space, then f is contra continuous.
Proof :-
Let K be an open set in Y .Thus ,$f^{-1}(K)$ isαrps–closed set in X .By hypotheses is$T^*_{1/2}$ −space, then by proposition(2-5)we get ,$f^{-1}(K)$ is closed set in X Therefore , f is contra continuous function .

Remark (3-7):
The concept of contra g-continuous function is independent to concept contra αrps −continuous function . As shows in the following examples:
Example (3-8):
(i) Let X=Y= {u ,v, w} with the topologies τ = { X, ϕ ,{v}, {u,w}}and ρ={Y,ϕ,{v}}. Let f : (X , τ) → (Y,ρ) be function define by f(u)=v , f(v)=u and f(w)=w ,clearly f is contra g-continuous function but is not contraαrps −continuous , since for the open set K = {v} in Y , but $f^{-1}(K) = f^{-1}(\{v\}) = \{u\}$ is notαrps− closed set in X .
(ii) Let X=Y= {u ,v, w} with the topologies τ = {X , ϕ ,{u}, {u ,w}}and ρ={Y,ϕ,{u} ,{v},{u ,v}}. Let f : (X , τ) → (Y,ρ) by f(u)=w , f(v)=u and f(w)=v ,clearly f is contra αrps-continuous function but is not contra g-continuous , since for the open set K = {v} in Y , but $f^{-1}(K) = f^{-1}(\{v\}) = \{w\}$ is not g- closed set in X .
The following propositions give the condition to make Remark(3-7)true**:**

Proposition (3-9):
Letf : (X, τ) → (Y,ρ) be a contra g-continuous function and (X , τ) is $T_{1/2}$ – space , then f is contraαrps −continuous .
Proof:
Let K be an open set in Y .Thus ,$f^{-1}(K)$ isg –closed set in X .By hypotheses X is$T_{1/2}$ –space, then by proposition(2-6)step-i- we get $f^{-1}(K)$ is αrps −closed in XTherefore , f is contra αrps −continuous function .
Similarly, we proof the following proposition :

Proposition (3-10):
Letf : (X , τ) → (Y,ρ) be contra αrps −continuous function and (X , τ) is $T^*_{1/2}$ −space, then f is contrag–continuous .

Now , we will define other types of contraαrps-continuous functionsnamely (contra $\alpha rps^{\#}$ −continuous function andcontra $\alpha rps^{\#\#}$-continuous function) .

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Definition (3-11):
A function f: (X ,τ) ⟶ (Y , σ) is called **contra $\mathfrak{\alpha rps}^{\#}$ −continuousfunction**if the inverse image of each $\mathfrak{\alpha rps}$-open set G in Y is $\mathfrak{\alpha rps}$ −closed set in X .

Example(3-12):-
Let X =Y= {u ,v ,w} with the topologiesτ ={X,ϕ,{u},{v},{v ,w}} and ρ={Y,,ϕ,{u}}, define f : (X , τ) → (Y,ρ) by f(u)=v , f(v)=w and f(w)=u ,it is observe that f is contra $\mathfrak{\alpha rps}^{\#}$ −continuous function.

Proposition(3-13):
A function f: (X ,τ) ⟶ (Y , σ)is contra $\mathfrak{\alpha rps}^{\#}$ −continuous function if and only if f^{-1}(G) is αrps- open set in X , for every αrps-closed set G in Y .
Proof : The proof is similar to the proof of proposition(3-3).

Proposition(3-14):-
If f: (X ,τ) ⟶ (Y , σ) is contra$\mathfrak{\alpha rps}^{\#}$ −continuousfunction then f is contra$\mathfrak{\alpha rps}$ −continuous
Proof :
Let K be an open set in Y , since [every open set is$\mathfrak{\alpha rps}$-open set], then K is $\mathfrak{\alpha rps}$ −open set in Y.Thus,f^{-1} (K) is an$\mathfrak{\alpha rps}$-closed set in X. Therefore , f iscontra$\mathfrak{\alpha rps}$ −continuousfunction .

the converse of proposition (3-14) need not be true in general the following example to show :

Example (3-15): Let X=Y= {u, v, w} with the topologies τ = { X, ϕ ,{u}, {u ,w}}and ρ={Y, ϕ, {u}} ,and define f : (X , τ) → (Y,ρ) by f(u)=v , f(v)=u and f(w)=w clearly, f is contra$\mathfrak{\alpha rps}$ −continuous function , but f is not contra$\mathfrak{\alpha rps}^{\#}$ −continuous, since for the$\mathfrak{\alpha rps}$-open set K={u ,v} in Y, but f^{-1}(K) = f^{-1} ({u, v})={u, v} is not$\mathfrak{\alpha rps}$ −closed set in X.
The following proposition give the condition to make proposition(3-14) true:

Proposition (3-16):
iff : (X , τ) → (Y,ρ) is contra $\mathfrak{\alpha rps}$ −continuousandY is $T^{*}{}_{1/2}$ −space, then f is contra $\mathfrak{\alpha rps}^{\#}$ −continuous.
Proof :
Let K be an$\mathfrak{\alpha rps}$-open set in Y . By hypotheses Y is$T^{*}{}_{1/2}$ −space, then by proposition(2-5) we get, K is an open set in Y. Also , since f is contra$\mathfrak{\alpha rps}$ −continuous , hence f^{-1}(K) is$\mathfrak{\alpha rps}$-closed set in X .Therefore , f is contra $\mathfrak{\alpha rps}^{\#}$ −continuous function .

Remark(3-17):
The concept of contra continuousis independent to concept contra $\mathfrak{\alpha rps}^{\#}$ −continuous function . As shows in the following examples:
Example (3-18):-
(i) Let X=Y={u, v, w} with the topologiesτ={X,ϕ,{u}}and ρ={Y,ϕ,{u},{v},{u, v} }. Let f : (X, τ) → (Y,ρ) be function define by f(u)=w , f(v)=u and f(w)=u ,clearly f is contra $\mathfrak{\alpha rps}^{\#}$ −continuousbut f is not contracontinuous, since for the open set K = {u} in Y , but f^{-1}(K) =f^{-1}({u}) = {v} is notclosed set in X.
(ii) Let X =Y={u, v, w} with the topologiesτ ={X, ϕ , {u},{v, w}}and ρ={Y ,ϕ,{u}}. Let f : (X , τ) → (Y,ρ) be function define by f(u)=u , f(v)=v and f(w)=w ,clearly f is contra continuous but f is notcontra $\mathfrak{\alpha rps}^{\#}$ −continuous, since for the$\mathfrak{\alpha rps}$- open set K = {1,2} in Y , but f^{-1}(K) =f^{-1} ({1,2}) = {1,2} is not$\mathfrak{\alpha rps}$-closed set in X .
The following proposition give the condition to make Remark(3-17)is true:

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Proposition (3-19):
iff : (X , τ) → (Y,ρ) is contra -continuousandY is $T^*{}_{1/2}$ −space, then f is contra αrps$^{\#}$ −continuous.

Proof :
Let K be anαrps-open set in Y . By hypotheses Y is$T^*{}_{1/2}$ −space, then by proposition(2-5) we get, K is an open set in Y. Hence, f^{-1}(K) isclosed set in X and by Remark(2-3) we get , then f^{-1}(K) isαrps–closed set in X .Therefore , f is contra αrps$^{\#}$ −continuous function .
Similarly , we proof the following proposition .

Proposition (3-20):
iff : (X , τ) → (Y,ρ) is contra αrps$^{\#}$ −continuousand is $T^*{}_{1/2}$ −space, then f is contra −continuous.

Definition (3-21):
A function f: (X ,τ) ⟶ (Y , σ) is called **contra αrps$^{\#\#}$ −continuousfunction** if the inverse image of each αrps-open set G in Y isclosed set in X .
Example(3-22):-
Let ={u, v, w} with the topologiesτ ={X,ϕ,{v},{u, w}} and f : (X , τ) → (X, τ) by f(u)=u , f(v)=v and f(w)=w ,it is observe that f is contra αrps$^{\#\#}$ −continuous function.

Proposition(3-23):
A function f: (X,τ) ⟶ (Y , σ) is contra αrps$^{\#\#}$ −continuous function if and only if f^{-1}(G) is open set in X , for every αrps-closed set G in Y .
Proof : The proof is similar to the proof of proposition(3-3).

Proposition(3-24):-If f: (X ,τ) ⟶ (Y , σ) is contra αrps$^{\#\#}$ −continuous function , then f is
- **(i)** Contra - continuous .
- **(ii)** Contra αrps −continuous.
- **(iii)** Contra αrps$^{\#}$ −continuous function

Proof(i) : Let K be an open set in Y , since [every open set isαrps-open set], then K is αrps-open set in Y. Also, since f is contraαrps$^{\#\#}$ −continuous function. Thus,f^{-1} (K) is closed set in X . Therefore , f iscontra-continuousfunction .
The proof of step –ii- and –iii- are similar to step-i- .
The converse of proposition (3-24) need not be true in general the following examples to show :
Example (3-25):-Let X=Y= {u, v, w} with the topologies τ = {X , ϕ ,{u}, {v, w}}and ρ={Y, ϕ, {u}} , and define f : (X , τ) → (Y,ρ) by f(u)=u , f(v)=w and f(w)=v ,clearly f is contra continuous (resp. contra**αrps** −continuous) function , but f is not contra**αrps**$^{\#\#}$ −continuous function , since for the **αrps**-open set K={u, w} in Y, but f^{-1}(K) = f^{-1} ({u, w})={u, v} is not closed(resp. **αrps**- closed) set in X.
Example (3-26):-Let X =Y= {u, v, w} with the topologies τ = {X , ϕ ,{u}, {u, w}}and ρ={Y, ϕ, {u}}.Let f : (X, τ) → (Y,ρ) define by f(u)=v , f(v)=w and f(w)=u ,clearly f iscontraαrps$^{\#\#}$ −continuous , but f is notcontra continuous (resp. contraαrps −continuous) function ,since for the open (resp.αrps- open) set K={u} in Y, but f^{-1}(K) = f^{-1} ({u})={w} is not closed set in X.
The following proposition give the condition to make proposition(3-24) are true:

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Proposition (3-27):A functionf : (, τ) → (Y,ρ) is contra **αrps**$^{\#\#}$ −continuousif

- **(i)** f is contra –continuous and Y is $T^*_{1/2}$ −space.
- **(ii)** f is contraαrps −continuous and Y is $T^*_{1/2}$ −space.
- **(iii)** f is contra αrps$^{\#}$ −continuous and X is $T^*_{1/2}$ −space.

Proof (i): Let K be an αrps-open set in Y . By hypotheses Y is$T^*_{1/2}$ −space, then by proposition(2-5) we get, K is an open set in Y. Also , since f is contra-continuous.Hence,f^{-1}(K) isclosed set in X .Therefore , f is contra αrps$^{\#\#}$ −continuous function .

The proof of step –ii-and –iii-are similar to step-i- .

Remark (3-28):

The composition of two contra αrps −continuous functions need not becontra αrps −continuous function , the following example to show that:

Example (3-29):

Let X=Y= Z={u, v, w} with the topologies τ = {X , ϕ ,{u},{v}, {u, v}},ρ={Y,ϕ,{u} {u, w}} and μ={Z ,ϕ,{u}}, where αRPSC(X ,τ)={X,ϕ,{w},{u, w},{v, w}} andαRPSC(Y,ρ)={Y,ϕ,{v},{w},{v, w}}.Define f : (X , τ) → (Y,ρ) by f(u)=v , f(v)=w and f(w)=u and g: (Y,ρ) → (Z,μ) by g(u)=v ,g(v)=u and g(w)=w clearly f and g arecontraαrps −continuous functions , but g∘f :(X , τ) → (Z,μ)is not contraαrps −continuous , since for the open set K = {u} in Z ,$(gof)^{-1}$(K)= f^{-1}(g^{-1}(K))=f^{-1}(g^{-1}({u}))=f^{-1} ({v}) = {u} is notαrps- closed set in X .

Proposition (3-30):- Let f : (X , τ) → (Y,ρ) and g: (Y,ρ) → (Z,μ) be any functions Then , g∘f :(X , τ) → (Z,μ) is contraαrps −continuous function if

- **(i)** f is contraαrps −continuous function and g is continuous function .
- **(ii)** f is contraαrps −continuous function and g is stronglyαrps −continuous function .

Proof (i):

Let K be an open set in Z , since g is continuous function .Thusg^{-1}(K) is an open set in Y. Also, since f iscontraαrps −continuous function, thenf^{-1}(g^{-1}(G))=$(gof)^{-1}$(G) isαrps-closed set in X.Therefore, g∘f :(X , τ)→(Z,μ) is contraαrps −continuous function.

The proof of step –ii- is similar to step-i- .

Proposition (3-31): Let f : (X , τ) → (Y,ρ) and g: (Y,ρ) → (Z,μ) be any functions . Then , g∘f :(X , τ) → (Z,μ) is contraαrps −continuous function if

- **(i)** f isαrps −irresolute continuousfunction and g is contraαrps −continuous function .
- **(ii)** f is strongly αrps −continuousfunction and g iscontraαrp −continuous function .

Proof(i) :-

Let K be an open set in Z , since g is contra αrps–continuous. Thus , g^{-1}(K) is αrps–closed set in Y.Also, since f isαrps −irresolute continuous function , then f^{-1}(g^{-1}(K)) =$(gof)^{-1}$(K) isαrps-closed set in X .Therefore, g∘f : (X , τ)→(Z, μ) is contraαrps −continuous function

The proof of step –ii- is similar to step-i- .

Similarly, we proof of the following proposition.

Proposition (3-32):- Iff : (X , τ) → (Y,ρ) and g: (Y,ρ) → (Z,μ) are any functions then , g∘f :(X , τ) → (Z,μ) is contra αrps$^{\#}$ −continuousfunction if

- **(i)** f isαrps −irresolute continuousfunction and g is contraαrps$^{\#}$ −continuous.
- **(ii)** f is strongly αrps −continuous function and g is contraαrps$^{\#}$ −continuous .

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Proposition (3-33):-
Iff: $(X,\tau) \rightarrow (Y,\rho)$ is a contra$\alpha rps^{\#}$ −ccontinuous function and g: $(Y,\rho) \rightarrow (Z,\mu)$ isαrps −continuous, then g∘f:$(X,\tau) \rightarrow (Z,\mu)$ is contraαrps − continuous function.
Proof: -Let K be an open set in Z .Thus,$g^{-1}(K)$ isαrps-open set in Y. Also, since f iscontra $\alpha rps^{\#}$ −irresolute continuous, then$f^{-1}(g^{-1}(K))$ isαrps-closed set in X. But $f^{-1}(g^{-1}(K))=(gof)^{-1}(K)$.Therefore, g∘f :(X, τ)→(Z,μ) is contraαrps −continuous function .

Proposition (3-34):-
Iff : $(X , \tau) \rightarrow (Y,\rho)$ is a contra$\alpha rps^{\#}$ - continuous function and g: $(Y,\rho) \rightarrow (Z,\mu)$ is any function , then g∘f :$(X , \tau) \rightarrow (Z,\mu)$ is contra$\alpha rps^{\#}$ − continuous function if

- **(i)** g is αrps −irresolutecontinuous function .
- **(ii)** g is strongly αrps −continuous function .

Proof (i): Let K be anαrps– open set in Z , since g isαrps −irresolutecontinuous function . Thus ,$g^{-1}(K)$ isαrps–open set in Y. Also, since f iscontra$\alpha rps^{\#}$ −continuous function ,then $f^{-1}(g^{-1}(K))=(gof)^{-1}(K)is\alpha rps$-closed set in X.Therefore, g∘f :(, τ)→(Z,μ) is contra$\alpha rps^{\#}$ − continuous function
The proof of step –ii- is similar to step-i- .

Proposition (3-35):- Iff : $(X , \tau) \rightarrow (Y,\rho)$ is any function and g: $(Y,\rho) \rightarrow (Z,\mu)$ is contra $\alpha rps^{\#\#}$ −continuous function, then g∘f :$(X , \tau) \rightarrow (Z,\mu)$ is contra$\alpha rps^{\#\#}$ − continuous function if f is

- **(i)** continuous function .
- **(ii)** stronglyαrps −continuous function .

Proof (i):
Let K be an αrps– open set in Z . Thus ,$g^{-1}(K)$ is closed set in Y. Also, since f iscontinuous function , then $f^{-1}(g^{-1}(K))=(gof)^{-1}(K)$is closed set in X.Therefore,gof:(X , τ)→(Z,μ) is contra$\alpha rps^{\#\#}$ − continuous function
The proof of step –ii- is similar to step-i- .
Similarly , we proof the following propositions .

Proposition (3-36):- Iff : $(X, \tau) \rightarrow (Y,\rho)$ is any function and g: $(Y,\rho) \rightarrow (Z,\mu)$ is contra $\alpha rps^{\#\#}$ −continuous function, then gof:$(X, \tau) \rightarrow (Z,\mu)$ is contra$\alpha rps^{\#\#}$ − continuous function if f is

- **(i)** αrps –continuous function .
- **(ii)** αrps – irresolutecontinuous function .

Proposition(3-37):-Let f : $(X , \tau) \rightarrow (Y,\rho)$ and g: $(Y,\rho) \rightarrow (Z, \mu)$ be any functions , then

- **(i)** gof: $(X, \tau) \rightarrow (Z, \mu)$ is contra $\alpha rps^{\#\#}$ − continuous function iff is a contra $\alpha rps^{\#\#}$ − continuous function and g is αrps −irresolutecontinuous.
- **(ii)** gof: $(X, \tau) \rightarrow (Z, \mu)$ is contra $\alpha rps^{\#\#}$ − continuous function if f is a contra $\alpha rps^{\#\#}$ − continuous function and g strongly αrps −continuous
- **(iii)** gof: $(X , \tau) \rightarrow (Z, \mu)$ is contra**αrps** − continuous function if f is a contra $\alpha rps^{\#\#}$ − continuous function and g isαrps −continuous.

Remark (3-38):-
Here in the following diagram illustrates the relation between thecontraαrps −continuous function kinds.

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$$\begin{tikzcd} & \text{Contra –continuous} \arrow[d] \\ \text{Contra}\alpha rps^{\#\#} - \text{continuous function} \arrow[ur] \arrow[r] \arrow[dr] & \text{Contra } \alpha rps - \text{continuous} \\ & \text{Contra } \alpha rps^{\#} - \text{continuous} \arrow[u] \end{tikzcd}$$

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