

ON INTUITIONISTIC FUZZY πgb AND πgb^* -SETS

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ABSTRACT

In this paper, two kinds of intuitionistic fuzzy sets called πgb and πgb^* sets in intuitionistic fuzzy topological spaces are introduced and some of their basic properties are studied. In addition the relationships between πgb and πgb^* , also the relationships between πgb and πgb^* separately with other are investigated. Furthermore the πgb and πgb^* continuous functions are introduced which resulted in obtaining some interesting properties of both.

Key words: *Intuitionistic fuzzy πgb -closed set, intuitionistic fuzzy πgb^* -closed set.*

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1. Intruduction

The concept of fuzzy set was introduced by L. A. Zadah [21]. The fuzzy concept has invaded almost all branches of mathematics. The concept of fuzzy topological space was introduced and developed by C. L. Chang [4]. Atanasov [3] was introduced the concept of intuitionistic fuzzy set, as a generalization of fuzzy set. This approach provided a wide field to the generalization of various concepts of fuzzy mathematics. In 1997 Coker[9] defined intuitionistic fuzzy topological spaces. Recently many concepts of fuzzy topological space have been extended in intuitionistic fuzzy (IF) topological spaces. D. Sreeja and C. Janaki [18] introduced the concepts of πgb -closed set. Dhanya and A. Parvath [7] introduced the concept of πgb^* sets. Amal M. Al-Dowais and AbdulGawad A. Al-Qubati [2] have studied the slightly πgb -continuous functions in intuitionistic fuzzy topological spaces. In this paper we introduce two kinds of intuitionistic fuzzy sets which are called πgb and πgb^* sets and discuss the relationship between them and several kinds of intuitionistic fuzzy sets such as intuitionistic fuzzy πgb closed set and other. Also, we study two kinds of continuity which are intuitionistic fuzzy πgb -continuous functions and intuitionistic fuzzy πgb^* -continuous functions. Lastly, our discussion focuses on the relationship between intuitionistic fuzzy πgb and πgb^* sets.

2. Preliminaries

Definition 2.1[3] Let X be a non-empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$, where the function $\mu_A(x): X \rightarrow [0, 1]$ and $\nu_A(x): X \rightarrow [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A , respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by $IFS(X)$, the set of all intuitionistic fuzzy sets in X .

Definition 2.2[3] Let A and B be IFSs of the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$ and $B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle : x \in X \}$. Then:

- (a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$.
 (b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.
 (c) $A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle : x \in X \}$.
 (d) $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle : x \in X \}$.
 (e) $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle : x \in X \}$.
 (f) $0_{\sim} = \{ \langle x, 0, 1 \rangle : x \in X \}$ and $1_{\sim} = \{ \langle x, 1, 0 \rangle : x \in X \}$.
 (g) $0_{\sim}^c = 1_{\sim}$ and $1_{\sim}^c = 0_{\sim}$.

Definition 2.3 [5] Let $\alpha, \beta \in [0, 1]$ such that $\alpha + \beta \leq 1$. An intuitionistic fuzzy point (IFP) $p_{(\alpha, \beta)}$ is intuitionistic fuzzy set defined by

$$p_{(\alpha, \beta)}(x) = \begin{cases} (\alpha, \beta) & \text{if } x = p \\ (0, 1) & \text{if otherwise} \end{cases}$$

In this case, p is called the support of $p_{(\alpha, \beta)}$ and α, β are called the value and no value of $p_{(\alpha, \beta)}$ respectively.

Clearly an intuitionistic fuzzy point can be represented by an ordered pair of fuzzy point as follows: $p_{(\alpha, \beta)} = (p_{\alpha}, p_{(1-\beta)})$

In $IFP p_{(\alpha, \beta)}$ is said to belong to an IFS $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$ denoted by $p_{(\alpha, \beta)} \in A$, if $\alpha \leq \mu_A(x)$ and $\beta \geq \nu_A(x)$.

Definition 2.4 [6] Let X and Y be two non-empty sets and $f: X \rightarrow Y$ be a function. Then:

- (a) If $B = \{ \langle y, \mu_B(y), \nu_B(y) \rangle : y \in Y \}$ is an IFS in Y , then the pre image of B under f denoted by $f^{-1}(B)$ is the IFS in X defined by
- $$f^{-1}(B) = \{ \langle x, f^{-1}(\mu_B)(x), f^{-1}(\nu_B)(x) \rangle : x \in X \}, \text{ where}$$

$$f^{-1}(\mu_B)(x) = \mu_B(f(x)) \text{ and } f^{-1}(\nu_B)(x) = \nu_B(f(x)).$$

- (b) If $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$ is an IFS in X , then the image of A under f denoted by $f(A)$ is the IFS in Y defined by

$$f(A) = \{ \langle y, f(\mu_A)(y), 1 - f(1 - \nu_A)(y) \rangle : y \in Y \}$$

where,

$$f(\mu_A)(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \mu_A(x) & \text{if } f^{-1}(y) \neq \emptyset \\ 0, & \text{if otherwise} \end{cases}$$

$$1 - f(1 - \nu_A)(y) = \begin{cases} \inf_{x \in f^{-1}(y)} \nu_A(x) & \text{if } f^{-1}(y) \neq \emptyset \\ 1, & \text{if otherwise} \end{cases}$$

Replaying fuzzy sets [21] by intuitionistic fuzzy sets in Chang definition of fuzzy topological space [4] we get the following;

Definition 2.5[6] *An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms:*

- (i) $0_{\sim}, 1_{\sim} \in \tau$
- (ii) If $G_1, G_2 \in \tau$, then $G_1 \cap G_2 \in \tau$
- (iii) If $G_{\lambda} \in \tau$ for each λ in Λ , then $\bigcup_{\lambda \in \Lambda} G_{\lambda} \in \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFT in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X . the complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X .

Definition 2.6[9] *A subset A of an intuitionistic fuzzy space X is said to be cl open if it is intuitionistic fuzzy open set and intuitionistic fuzzy closed set.*

Definition 2.7 [4] *Let (X, τ) be an IFTS and $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$ be an IFS in X . Then the intuitionistic fuzzy interior and an intuitionistic fuzzy clouser are defined by:*
 $\text{int}(A) = \bigcup \{ G : G \text{ is an IFOS in } X \text{ and } G \subseteq A \}$, $\text{cl}(A) = \bigcap \{ K : K \text{ is an IFCS in } X \text{ and } A \subseteq K \}$.

Definition 2.8 An IFS A of an IFTS (X, τ) is an:

1. Intuitionistic fuzzy regular open set (IFROS in short)[9] if $\text{int}(\text{cl}(A)) = A$. [9]
2. Intuitionistic fuzzy regular closed set (IFRCs in short)[9] if $\text{cl}(\text{int}(A)) = A$. [9]
3. Intuitionistic fuzzy π -open set (IF π OS in short)[15] if the finite union of intuitionistic fuzzy regular open sets. [15]
4. Intuitionistic fuzzy π -closed set (IF π CS in short)[15] if the finite intersection of intuitionistic fuzzy regular closed sets. [15]
5. Intuitionistic fuzzy generalized closed set (IFGCS in short)[20] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X . [20]
6. Intuitionistic fuzzy b-open set (IFbOS in short)[10] if $A \subseteq \text{cl}(\text{int}(A)) \cup \text{int}(\text{cl}(A))$. [1]
7. Intuitionistic fuzzy b-closed set (IFbCS in short)[10] if $\text{cl}(\text{int}(A)) \cup \text{int}(\text{cl}(A)) \subseteq A$. [1]
8. Intuitionistic fuzzy semi-closed set (IFSCS in short) if $\text{int}(\text{cl}(A)) \subseteq A$. [9]
9. Intuitionistic fuzzy α -closed set (IF α CS in short) if $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$. [9]
10. Intuitionistic fuzzy pre-closed set (IFPCS in short) if $\text{cl}(\text{int}(A)) \subseteq A$. [9]
11. Intuitionistic fuzzy gb-closed set (IFgbCS in short)[13] if $\text{bcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is IFOS in (X, τ) .
12. Intuitionistic fuzzy gs-closed set (IFgsCS in short) [16] if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is IFOS in (X, τ) .
13. Intuitionistic fuzzy gp-closed set (IFgpCS in short) [14] if $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is IFOS in (X, τ) .
14. Intuitionistic fuzzy α g-closed set (IF α gCS in short) [17] if $\alpha \text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is IFOS in (X, τ) .
15. Intuitionistic fuzzy π g-closed set (IF π gCS in short) [8] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IF π OS in (X, τ) .
16. Intuitionistic fuzzy π gs-closed set (IF π gsCS in short) [11] if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IF π OS in (X, τ) .
17. Intuitionistic fuzzy π g α -closed set (IF π g α CS in short) [19] if $\alpha \text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IF π OS in (X, τ) .

18. Intuitionistic fuzzy π gb-closed set (IF π gbCS in short) [12] if $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IF π OS in (X, τ) .

Definition 2.9 [13] Let (X, τ) be an IFTS and A be an IFS in X . Then the intuitionistic fuzzy b-interior and an intuitionistic fuzzy b-closure are defined by

$$\text{bint}(A) = \bigcup \{ G : G \text{ is an IFbOS in } X \text{ and } G \subseteq A \},$$

$$\text{cl}(A) = \bigcap \{ K : K \text{ is an IFbCS in } X \text{ and } A \subseteq K \}$$

Theorem 2.10 Let A be an intuitionistic fuzzy set of an IFTS (X, τ) , then :

$$(i) \text{ bcl}(A) = A \cup [\text{int}(\text{cl}(A)) \cap \text{cl}(\text{int}(A))]$$

$$\text{bint}(A) = A \cap [\text{int}(\text{cl}(A)) \cup \text{cl}(\text{int}(A))]$$

Proof.

Since $\text{bcl}(A)$ is an IFbCS, we have $\text{int}(\text{cl}(A)) \cap \text{cl}(\text{int}(A)) \subseteq \text{int}(\text{cl}(\text{bcl}(A))) \cap \text{cl}(\text{int}(\text{bcl}(A))) \subseteq \text{bcl}(A)$ and we have, $A \subseteq \text{bcl}(A)$. Then, $A \cup (\text{int}(\text{cl}(A)) \cap \text{cl}(\text{int}(A))) \subseteq \text{bcl}(A)$. $\text{---} \text{---} \text{---} \text{---} > (1)$

On the other hand, Since $\text{int}(\text{cl}(A \cup \text{int}(\text{cl}(A)))) \subseteq A \cup \text{int}(\text{cl}(A))$, and $\text{cl}(\text{int}(A \cup \text{cl}(\text{int}(A)))) \subseteq A \cup \text{cl}(\text{int}(A))$, also

$$\text{int}(\text{cl}(A \cup (\text{int}(\text{cl}(A)) \cap \text{cl}(\text{int}(A)))) \subseteq A \cup \text{int}(\text{cl}(A))$$

$$, \text{cl}(\text{int}(A \cup (\text{cl}(\text{int}(A)) \cap \text{int}(\text{cl}(A)))) \subseteq A \cup \text{cl}(\text{int}(A))$$

Then, $\text{int}(\text{cl}(A \cup (\text{int}(\text{cl}(A)) \cap \text{cl}(\text{int}(A)))) \cap \text{cl}(\text{int}(A \cup (\text{cl}(\text{int}(A)) \cap \text{int}(\text{cl}(A)))) \subseteq [A \cup \text{int}(\text{cl}(A))] \cap [A \cup \text{cl}(\text{int}(A))] = A \cup [\text{int}(\text{cl}(A)) \cap \text{cl}(\text{int}(A))]$. Hence $A \cup [\text{int}(\text{cl}(A)) \cap \text{cl}(\text{int}(A))]$ is an IFbCS and thus $\text{bcl}(A) \subseteq A \cup [\text{int}(\text{cl}(A)) \cap \text{cl}(\text{int}(A))]$ $\text{---} \text{---} \text{---} \text{---} > (2)$.

From (1) and (2) it follows that $\text{bcl}(A) = A \cup [\text{int}(\text{cl}(A)) \cap \text{cl}(\text{int}(A))]$ (ii) can be proved easily by taking complement in (i).

3. Intuitionistic fuzzy π gb-set and Intuitionistic fuzzy π gb*-set

Definition 3.1 [2] An IFS A of an IFTS (X, τ) is an :

1. Intuitionistic fuzzy π gb-open set (IF π gbOS in short) if $F \subseteq \text{bint}(A)$ whenever $F \subseteq A$ and F is an IF π CS in (X, τ) .

2. Intuitionistic fuzzy π gb-closed set (IF π gbCS in short) if $\text{bcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IF π OS in (X, τ) .

Definition 3.2 Let (X, τ) be an IFTS and $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$ be an IFS in (X, τ) . Then the intuitionistic fuzzy π gb-interior and an intuitionistic fuzzy π gb-closure of A are defined by:

$$\pi\text{gb-int}(A) = \bigcup \{ G : G \text{ is an IF}\pi\text{gbOS in } X \text{ and } G \subseteq A \},$$

$$\pi\text{gb-cl}(A) = \bigcap \{ K : K \text{ is an IF}\pi\text{gbCS in } X \text{ and } A \subseteq K \}.$$

Theorem 3.3 If A is IF π gbCS in X , then $\pi\text{gb-cl}(A) = A$.

Proof. Since A is an IF π gbCS, $\pi\text{gb-cl}(A)$ is the smallest IF π gbCS which contains A , which is nothing but A . Hence $\pi\text{gb-cl}(A) = A$. ■

Remark 3.4 If $A = \pi_{gb}\text{-cl}(A)$, then A need not be an $\text{IF}\pi_{gb}\text{CS}$.

Example 3.5 Let $X = \{a, b, c\}$ and let $\tau = \{0_-, 1_-, A, B, C\}$ is IFT on X , where $A = \{\langle a, 1, 0 \rangle, \langle b, 0, 1 \rangle, \langle c, 0, 1 \rangle\}$, $B = \{\langle a, 0, 1 \rangle, \langle b, 0.8, 0.2 \rangle, \langle c, 0, 1 \rangle\}$ and $C = \{\langle a, 1, 0 \rangle, \langle b, 0.8, 0.2 \rangle, \langle c, 0, 1 \rangle\}$. Clearly $\pi_{gb}\text{-cl}(C) = C$ but C is not $\text{IF}\pi_{gb}\text{CS}$.

Theorem 3.6 If A is $\text{IF}\pi_{gb}\text{OS}$ in X , then $\pi_{gb}\text{-int}(A) = A$.

Proof. Similar to the (theorem 3.3)

Remark 3.7 If $A = \pi_{gb}\text{-int}(A)$, then A need not be an $\text{IF}\pi_{gb}\text{OS}$.

Example 3.8 Let $X = \{a, b, c\}$ and let $\tau = \{0_-, 1_-, A, B, C\}$ is IFT on X , where $A = \{\langle a, 1, 0 \rangle, \langle b, 0, 1 \rangle, \langle c, 0, 1 \rangle\}$, $B = \{\langle a, 0, 1 \rangle, \langle b, 0.8, 0.2 \rangle, \langle c, 0, 1 \rangle\}$ and $C = \{\langle a, 1, 0 \rangle, \langle b, 0.8, 0.2 \rangle, \langle c, 0, 1 \rangle\}$. let $D = \{\langle a, 0, 1 \rangle, \langle b, 0.2, 0.8 \rangle, \langle c, 1, 0 \rangle\}$, clearly $\pi_{gb}\text{-int}(D) = D$ but D is not $\text{IF}\pi_{gb}\text{OS}$.

Remark 3.9 The union of two $\text{IF}\pi_{gb}\text{-closed}$ sets is generally not a $\text{IF}\pi_{gb}\text{-closed}$ set and the intersection of two $\text{IF}\pi_{gb}\text{-open}$ sets is generally not an $\text{IF}\pi_{gb}\text{-open}$ set

Example 3.10 Let $X = \{a, b\}$ and let $\tau = \{0_-, 1_-, A, B, C\}$ is IFT on X , where $A = \{\langle a, 1, 0 \rangle, \langle b, 0, 1 \rangle\}$, $B = \{\langle a, 0, 1 \rangle, \langle b, 0.9, 0.1 \rangle\}$ and $C = \{\langle a, 1, 0 \rangle, \langle b, 0.9, 0.1 \rangle\}$. Then the IFSs A^c, B^c are $\text{IF}\pi_{gb}\text{OSs}$ but $A^c \cap B^c = C^c$ is not an $\text{IF}\pi_{gb}\text{OS}$ of X , since $C^c \subseteq C^c$ and $C^c \not\subseteq \text{int}(C^c) = 0_-$. And the IFSs A, B are $\text{IF}\pi_{gb}\text{CSs}$ but $A \cup B = C$ is not an $\text{IF}\pi_{gb}\text{CS}$ of X , since $C \subseteq C$ and $\text{bcl}(C) = 1_- \not\subseteq C$.

Theorem 3.11 Every intuitionistic fuzzy closed set is an intuitionistic fuzzy $\pi_{gb}\text{-closed}$ set.

Proof. Let A be an intuitionistic fuzzy closed set of (X, τ) such that $A \subseteq U$ and U is an $\text{IF}\pi$ -open set in (X, τ) . Since $\text{int}(\text{cl}(A)) \cap \text{cl}(\text{int}(A)) \subseteq \text{int}(\text{cl}(A)) = \text{cl}(A) = A$, $A \cup [\text{int}(\text{cl}(A)) \cap \text{cl}(\text{int}(A))] \subseteq A \cup A = A$, then $\text{bcl}(A) \subseteq A \subseteq U$. Hence A is an intuitionistic fuzzy $\pi_{gb}\text{-closed}$ set. ■

The converse of above theorem is not true in general, as we will see in the following example.

Example 3.12 Let $X = \{a, b\}$ and let $\tau = \{0_-, 1_-, U\}$ is an IFT on X , where $U = \{\langle a, 0.5, 0.4 \rangle, \langle b, 0.6, 0.4 \rangle\}$. Let $A = \{\langle a, 0.3, 0.6 \rangle, \langle b, 0.2, 0.8 \rangle\}$, then A is an $\text{IF}\pi_{gb}\text{CS}$ but it is not IFCS . Since $\text{cl}(A) = U^c \neq A$.

Theorem 3.13 Every intuitionistic fuzzy semi-closed set is an intuitionistic fuzzy $\pi_{gb}\text{-closed}$ set.

Proof. Let A be an intuitionistic fuzzy semi-closed set of (X, τ) such that $A \subseteq U$ and U is an $\text{IF}\pi$ -open set in (X, τ) . Since

$$\text{int}(\text{cl}(A)) \cap \text{cl}(\text{int}(A)) \subseteq \text{int}(\text{cl}(A))$$

$$\Rightarrow \text{int}(\text{cl}(A)) \cap \text{cl}(\text{int}(A)) \subseteq A$$

$$\text{and } A \cup (\text{int}(\text{cl}(A)) \cap \text{cl}(\text{int}(A))) \subseteq A \cup A = A$$

This implies $\text{bcl}(A) \subseteq A \subseteq U$. Hence A is an intuitionistic fuzzy $\pi_{gb}\text{-closed}$ set.

The converse of above theorem is not true in general, as we will see in the following example.

Example 3.14 Let $X = \{a, b\}$ and let $\tau = \{0, 1, U\}$ is an IFT on X , where $U = \{\langle a, 0.8, 0.2 \rangle, \langle b, 0.9, 0.1 \rangle\}$. Let $A = \{\langle a, 0.4, 0.6 \rangle, \langle b, 0.5, 0.5 \rangle\}$, then A is an IF π gbCS but it is not IFSCS. Since $\text{int}(\text{cl}(A)) = 1 \not\subseteq A$.

Theorem 3.15 Every intuitionistic fuzzy α -closed set is an intuitionistic fuzzy π gb-closed set.

Proof. Let A be an intuitionistic fuzzy α -closed set of (X, τ) such that $A \subseteq U$ and U is an IF π -open set in (X, τ) . Since $A \subseteq \text{cl}(A)$, $\text{cl}(\text{int}(A)) \subseteq \text{cl}(\text{int}(\text{cl}(A))) \Rightarrow \text{cl}(\text{int}(A)) \subseteq A$ and $\text{cl}(\text{int}(A)) \cap \text{int}(\text{cl}(A)) \subseteq A \cap \text{int}(\text{cl}(A))$
 $\text{cl}(\text{int}(A)) \cap \text{int}(\text{cl}(A)) \subseteq A \cap \text{cl}(\text{int}(\text{cl}(A))) \Rightarrow \text{cl}(\text{int}(A)) \cap \text{int}(\text{cl}(A)) \subseteq A \cap A$ and
 $\text{cl}(\text{int}(A)) \cap \text{int}(\text{cl}(A)) \subseteq A$
 then $A \cup (\text{cl}(\text{int}(A)) \cap \text{int}(\text{cl}(A))) \subseteq A \cup A = A$
 This implies $\text{bcl}(A) \subseteq A \subseteq U$. Hence A is an intuitionistic fuzzy π gb-closed set. ■

The converse of above theorem is not true in general, as we will see in the following example.

Example 3.16 Let $X = \{a, b, c\}$ and let $\tau = \{0, 1, A, B, C\}$ is an IFT on X , where $A = \{\langle a, 1, 0 \rangle, \langle b, 0, 1 \rangle, \langle c, 0, 1 \rangle\}$, $B = \{\langle a, 0, 1 \rangle, \langle b, 0.8, 0.2 \rangle, \langle c, 0.3, 0.7 \rangle\}$ and $C = \{\langle a, 1, 0 \rangle, \langle b, 0.8, 0.2 \rangle, \langle c, 0.3, 0.7 \rangle\}$. Then B is an IF π gbCS but it is not IF α CS. Since $\text{cl}(\text{int}(\text{cl}(B))) = A^c \not\subseteq B$.

Theorem 3.17 Every intuitionistic fuzzy pre-closed set is an intuitionistic fuzzy π gb-closed set.

Proof. Let A be an intuitionistic fuzzy pre-closed set of (X, τ) such that $A \subseteq U$ and U is an IF π -open set in (X, τ) . Since $\text{int}(\text{cl}(A)) \cap \text{cl}(\text{int}(A)) \subseteq \text{cl}(\text{int}(A))$ and $\text{int}(\text{cl}(A)) \cap \text{cl}(\text{int}(A)) \subseteq A$
 then $A \cup (\text{int}(\text{cl}(A)) \cap \text{cl}(\text{int}(A))) \subseteq A \cup A = A$ This implies $\text{bcl}(A) \subseteq A \subseteq U$. Hence A is an intuitionistic fuzzy π gb-closed set. ■

The converse of above theorem is not true in general, as we will see in the following example.

Example 3.18 Let $X = \{a, b, c\}$ and let $\tau = \{0, 1, A, B, C\}$ is an IFT on X , where $A = \{\langle a, 1, 0 \rangle, \langle b, 0, 1 \rangle, \langle c, 0, 1 \rangle\}$, $B = \{\langle a, 0, 1 \rangle, \langle b, 0.5, 0.4 \rangle, \langle c, 0.4, 0.6 \rangle\}$ and $C = \{\langle a, 1, 0 \rangle, \langle b, 0.5, 0.4 \rangle, \langle c, 0.4, 0.6 \rangle\}$. Then B is an IF π gbCS but it is not IFPCS. Since $\text{cl}(\text{int}(B)) = A^c \not\subseteq B$.

Theorem 3.19 Every intuitionistic fuzzy b-closed set is an intuitionistic fuzzy π gb-closed set.

Proof. Let A be an intuitionistic fuzzy b-closed set of (X, τ) such that $A \subseteq U$ and U is an IF π -open set in (X, τ) . Since $\text{bcl}(A) = A \subseteq U$. Hence A is an intuitionistic fuzzy π gb-closed set. ■

The converse of above theorem is not true in general, as we will see in the following example.

Example 3.20 Let $X = \{a, b\}$ and let $\tau = \{0_-, 1_-, U\}$ is an IFT on X , where $U = \{ \langle a, 0.8, 0.2 \rangle, \langle b, 0.9, 0.1 \rangle \}$. Let $A = \{ \langle a, 0.9, 0.1 \rangle, \langle b, 1, 0 \rangle \}$, then A is an IF π gbCS but it is not IFbCS, since $\text{int}(\text{cl}(A)) \cap \text{cl}(\text{int}(A)) = 1_- \not\subseteq A$.

Theorem 3.21 Every intuitionistic fuzzy g-closed set is an intuitionistic fuzzy π gb-closed set.

Proof. Let A be an intuitionistic fuzzy g-closed set of (X, τ) such that $A \subseteq U$ and U is an IF π -open set in (X, τ) , since every IF π -open set is an intuitionistic fuzzy open set and $\text{cl}(A) \subseteq U$. Since

$$\text{int}(\text{cl}(A)) \cap \text{cl}(\text{int}(A)) \subseteq \text{int}(\text{cl}(A)) \subseteq \text{cl}(\text{cl}(A)) = \text{cl}(A), \quad A \cup (\text{int}(\text{cl}(A)) \cap \text{cl}(\text{int}(A))) \subseteq A \cup \text{cl}(A) = \text{cl}(A)$$

This implies $\text{bcl}(A) \subseteq \text{cl}(A) \subseteq U$. Hence A is an intuitionistic fuzzy π gb-closed set. ■

The converse of above theorem is not true in general, as we will see in the following example.

Example 3.22 Let $X = \{a, b\}$ and let $\tau = \{0_-, 1_-, U\}$ is an IFT on X , where $U = \{ \langle a, 0.7, 0.3 \rangle, \langle b, 0.6, 0.4 \rangle \}$. Let $A = \{ \langle a, 0.2, 0.8 \rangle, \langle b, 0.4, 0.6 \rangle \}$, then A is an IF π gbCS but it is not IFgCS. Since $\text{cl}(A) = U^c \not\subseteq U$.

Theorem 3.23 Every intuitionistic fuzzy α g-closed set is an intuitionistic fuzzy π gb-closed set.

Proof. Let A be an intuitionistic fuzzy α g-closed set of (X, τ) such that $A \subseteq U$ and U is an IF π -open set in (X, τ) . Since every IF π -open set is an intuitionistic fuzzy open set, $\alpha\text{cl}(A) \subseteq U$. Since

$$\begin{aligned} A &\subseteq \text{cl}(A) \\ , \quad \text{cl}(\text{int}(A)) &\subseteq \text{cl}(\text{int}(\text{cl}(A))) \\ , \quad \text{cl}(\text{int}(A)) \cap \text{int}(\text{cl}(A)) &\subseteq \text{cl}(\text{int}(\text{cl}(A))) \cap \text{int}(\text{cl}(A)) \\ \Rightarrow \quad \text{cl}(\text{int}(A)) \cap \text{int}(\text{cl}(A)) &\subseteq \text{cl}(\text{int}(\text{cl}(A))) \cap \text{cl}(\text{int}(\text{cl}(A))) \\ \Rightarrow \quad \text{cl}(\text{int}(A)) \cap \text{int}(\text{cl}(A)) &\subseteq \text{cl}(\text{int}(\text{cl}(A))) \\ \text{then } A \cup (\text{cl}(\text{int}(A)) \cap \text{int}(\text{cl}(A))) &\subseteq A \cup \text{cl}(\text{int}(\text{cl}(A))) \end{aligned}$$

This implies $\text{bcl}(A) \subseteq \alpha\text{cl}(A) \subseteq U$. Hence A is an intuitionistic fuzzy π gb-closed set. ■

The converse of above theorem is not true in general, as we will see in the following example.

Example 3.24 Let $X = \{a, b\}$ and let $\tau = \{0_-, 1_-, U\}$ is an IFT on X , where $U = \{ \langle a, 0.3, 0.4 \rangle, \langle b, 0.3, 0.4 \rangle \}$. Let $A = \{ \langle a, 0.1, 0.5 \rangle, \langle b, 0.1, 0.6 \rangle \}$, then A is an IF π gbCS but it is not IF α gCS, since $\alpha\text{cl}(A) = U^c \not\subseteq U$.

Theorem 3.25 Every intuitionistic fuzzy gs-closed set is an intuitionistic fuzzy π gb-closed set.

Proof. Let A be an intuitionistic fuzzy gs-closed set of (X, τ) such that $A \subseteq U$ and U is an IF π -open set in (X, τ) . Since every IF π -open set is an intuitionistic fuzzy open set, $\text{scl}(A) \subseteq U$. Since

$$\begin{aligned} \text{int}(\text{cl}(A)) \cap \text{cl}(\text{int}(A)) &\subseteq \text{int}(\text{cl}(A)) \\ \text{then } A \cup (\text{int}(\text{cl}(A)) \cap \text{cl}(\text{int}(A))) &\subseteq A \cup \text{int}(\text{cl}(A)) \end{aligned}$$

This implies $\text{bcl}(A) \subseteq \text{scl}(A) \subseteq U$. Hence A is an intuitionistic fuzzy π gb-closed set. ■

The converse of above theorem is not true in general, as we will see in the following example.

Example 3.26 Let $X = \{a, b\}$ and let $\tau = \{0_-, 1_-, U\}$ is an IFT on X , where $U = \{\langle a, 0.7, 0.2 \rangle, \langle b, 0.7, 0.3 \rangle\}$. Let $A = \{\langle a, 0.5, 0.5 \rangle, \langle b, 0.5, 0.5 \rangle\}$, then A is an $IF\pi gbCS$ but it is not $IFgsCS$, since $scl(A) = 1_- \not\subset U$.

Theorem 3.27 Every intuitionistic fuzzy πg -closed set is an intuitionistic fuzzy πgb -closed set.

Proof. Let A be an intuitionistic fuzzy πg -closed set of (X, τ) such that $A \subseteq U$ and U is an $IF\pi$ -open set in (X, τ) . Since every $IF\pi$ -open set is an intuitionistic fuzzy open set, $pcl(A) \subseteq U$. Since

$$\text{int}(\text{cl}(A)) \cap \text{cl}(\text{int}(A)) \subseteq \text{cl}(\text{int}(A)), A \cup (\text{int}(\text{cl}(A)) \cap \text{cl}(\text{int}(A))) \subseteq A \cup \text{cl}(\text{int}(A))$$

This implies $bcl(A) \subseteq pcl(A) \subseteq U$. Hence A is an intuitionistic fuzzy πgb -closed set. ■

The converse of above theorem is not true in general, as we will see in the following example.

Example 3.28 Let $X = \{a, b\}$ and let $\tau = \{0_-, 1_-, U\}$ is an IFT on X , where $U = \{\langle a, 0.5, 0.5 \rangle, \langle b, 0.2, 0.6 \rangle\}$. Let $A = U$, then A is an $IF\pi gbCS$ but it is not $IFgpCS$, since $pcl(A) = U^c \not\subset U$.

Theorem 3.29 Every intuitionistic fuzzy gb -closed set is an intuitionistic fuzzy πgb -closed set.

Proof. Let A be an intuitionistic fuzzy gb -closed set of (X, τ) such that $A \subseteq U$ and U is an $IF\pi$ -open set in (X, τ) . Since every $IF\pi$ -open set is an intuitionistic fuzzy open set, $bcl(A) \subseteq U$. Hence A is an intuitionistic fuzzy πgb -closed set. ■

The converse of above theorem is not true in general, as we will see in the following example.

Example 3.30 Let $X = \{a, b\}$ and let $\tau = \{0_-, 1_-, U\}$ is an IFT on X , where $U = \{\langle a, 0.6, 0.3 \rangle, \langle b, 0.8, 0.2 \rangle\}$. Let $A = U$, then A is an $IF\pi gbCS$ but it is not $IFgbCS$, since $bcl(A) = 1_- \not\subset U$.

Theorem 3.31 Every intuitionistic fuzzy πg -closed set is an intuitionistic fuzzy πgb -closed set.

Proof. Let A be an intuitionistic fuzzy πg -closed set of (X, τ) such that $A \subseteq U$ and U is an $IF\pi$ -open set in (X, τ) . Then $\text{cl}(A) \subseteq U$ and $asbcl(A) \subseteq \text{cl}(A) \subseteq U$. Hence A is an intuitionistic fuzzy πgb -closed set. ■

The converse of above theorem is not true in general, as we will see in the following example.

Example 3.32 Let $X = \{a, b\}$ and let $\tau = \{0_-, 1_-, U\}$ is an IFT on X , where $U = \{\langle a, 0.1, 0.9 \rangle, \langle b, 0.3, 0.7 \rangle\}$. Let $A = \{\langle a, 0, 1 \rangle, \langle b, 0.2, 0.7 \rangle\}$, then A is an $IF\pi gbCS$ but it is not $IF\pi gCS$, since $\text{cl}(A) = U^c \not\subset U$.

Theorem 3.33 Every intuitionistic fuzzy $\pi g\alpha$ -closed set is an intuitionistic fuzzy πgb -closed set.

set.

Proof. Let A be an intuitionistic fuzzy $\pi\alpha$ -closed set of (X, τ) such that $A \subseteq U$ and U is an $IF\pi$ -open set in (X, τ) . Then $\alpha cl(A) \subseteq U$, and $asbcl(A) \subseteq \alpha cl(A) \subseteq U$. Hence A is an intuitionistic fuzzy πgb -closed set. ■

The converse of above theorem is not true in general, as we will see in the following example.

Example 3.34 Let $X = \{a, b\}$ and let $\tau = \{0, 1, U\}$ is an IFT on X , where $U = \{ \langle a, 0.3, 0.6 \rangle, \langle b, 0.5, 0.5 \rangle \}$. Let $A = \{ \langle a, 0.2, 0.8 \rangle, \langle b, 0.4, 0.6 \rangle \}$, then A is an $IF\pi gbCS$ but it is not $IF\pi\alpha CS$, since $\alpha cl(A) = U^c \not\subseteq U$.

Theorem 3.35 Every intuitionistic fuzzy πgp -closed set is an intuitionistic fuzzy πgb -closed set.

Proof. Let A be an intuitionistic fuzzy πgp -closed set of (X, τ) such that $A \subseteq U$ and U is an $IF\pi$ -open set in (X, τ) . Then $pcl(A) \subseteq U$, and $asbcl(A) \subseteq pcl(A) \subseteq U$. Hence A is an intuitionistic fuzzy πgb -closed set. ■

The converse of above theorem is not true in general, as we will see in the following example.

Example 3.36 Let $X = \{a, b\}$ and let $\tau = \{0, 1, U_1, U_2, U_3\}$ is an IFT on X , where $U_1 = \{ \langle a, 0.1, 1 \rangle, \langle b, 0.7, 0.3 \rangle \}$, $U_2 = \{ \langle a, 0.8, 0.2 \rangle, \langle b, 0.1, 1 \rangle \}$, $U_3 = \{ \langle a, 0.8, 0.2 \rangle, \langle b, 0.7, 0.3 \rangle \}$. Let $A = U_2$, then A is an $IF\pi gbCS$ but it is not $IF\pi gpCS$, since $pcl(A) = U^c \not\subseteq U_3$.

Theorem 3.37 Every intuitionistic fuzzy πgs -closed set is an intuitionistic fuzzy πgb -closed set.

Proof Let A be an intuitionistic fuzzy πgs -closed set of (X, τ) such that $A \subseteq U$ and U is an $IF\pi$ -open set in (X, τ) . Then $scl(A) \subseteq U$, and $asbcl(A) \subseteq scl(A) \subseteq U$. Hence A is an intuitionistic fuzzy πgb -closed set. ■

The converse of above theorem is not true in general, as we will see in the following example.

Example 3.38 Let $X = \{a, b\}$ and let $\tau = \{0, 1, U_1, U_2, U_3, U_4, U_5\}$ is an IFT on X , where $U_1 = \{ \langle a, 0.1, 0.4 \rangle, \langle b, 0.3, 0.2 \rangle \}$, $U_2 = \{ \langle a, 0.3, 0.2 \rangle, \langle b, 0.2, 0.3 \rangle \}$, $U_3 = \{ \langle a, 0.1, 0.4 \rangle, \langle b, 0.2, 0.3 \rangle \}$, $U_4 = \{ \langle a, 0.3, 0.2 \rangle, \langle b, 0.3, 0.2 \rangle \}$, $U_5 = \{ \langle a, 0.4, 0.2 \rangle, \langle b, 0.3, 0.2 \rangle \}$. Let $A = \{ \langle a, 0.3, 0.4 \rangle, \langle b, 0.3, 0.4 \rangle \}$, then A is an $IF\pi gbCS$ but it is not $IF\pi gsCS$, since $scl(A) = U_5^c \not\subseteq U_4$.

Definition 3.39 An IFS A of an IFTS (X, τ) is an :

1. Intuitionistic fuzzy πgb^+ -open set ($IF\pi gb^+OS$ in short) if $F \subseteq cl(bint(A))$ whenever $F \subseteq A$ and F is an $IF\pi CS$ in (X, τ) .
2. Intuitionistic fuzzy πgb^+ -closed set ($IF\pi gb^+CS$ in short) if $int(bcl(A)) \subseteq U$ whenever $A \subseteq U$ and U is an $IF\pi OS$ in (X, τ) .

Remark 3.40 The union of finite $IF\pi gb^+$ -closed sets is generally not an $IF\pi gb^+$ -closed set and the intersection of finite $IF\pi gb^+$ -open sets is generally not an $IF\pi gb^+$ -open set.

Example 3.41 Let $X = \{a, b\}$ and let $\tau = \{0, 1, A, B, C\}$ is IFT on X , where $A = \{ \langle a, 1, 0 \rangle, \langle b, 0, 1 \rangle \}$, $B = \{ \langle a, 0, 1 \rangle, \langle b, 0.9, 0.1 \rangle \}$ and $C = \{ \langle a, 1, 0 \rangle, \langle b, 0.9, 0.1 \rangle \}$. Then the IFSs A^c, B^c are $IF\pi gb^+OS$ but $A^c \cap B^c = C^c$ is not an

IF π gb' O So fX, since $C^c \subseteq C^c$ and $C^c \not\subseteq \text{cl}(\text{bint}(C^c)) = 0_\sim$.

And the IFSs A,B are IF π gb' CSs but $A \cup B = C$ is not an IF π gb' CS of X, since $C \subseteq C$ and $\text{int}(\text{bcl}(C)) = 1_\sim \not\subseteq C$.

Theorem 3.42

1. Every intuitionistic fuzzy closed set is an intuitionistic fuzzy π gb*-closed set.
2. Every intuitionistic fuzzy semi-closed set is an intuitionistic fuzzy π gb* - closed set.
3. Every intuitionistic fuzzy pre-closed set is an intuitionistic fuzzy π gb* - closed set.
4. Every intuitionistic fuzzy α -closed set is an intuitionistic fuzzy π gb* -closed set.
5. Every intuitionistic fuzzy b- closed set is an intuitionistic fuzzy π gb* -closed set.
6. Every intuitionistic fuzzy g- closed set is an intuitionistic fuzzy π gb* -closed set.
7. Every intuitionistic fuzzy gs- closed set is an intuitionistic fuzzy π gb* -closed set.
8. Every intuitionistic fuzzy gp- closed set is an intuitionistic fuzzy π gb* -closed set.
9. Every intuitionistic fuzzy $g\alpha$ -closed set is an intuitionistic fuzzy π gb* - closed set.
10. Every intuitionistic fuzzy gb- closed set is an intuitionistic fuzzy π gb* -closed set.
11. Every intuitionistic fuzzy π g- closed set is an intuitionistic fuzzy π gb* -closed set.
12. Every intuitionistic fuzzy π gs-closed set is an intuitionistic fuzzy π gb* - closed set.
13. Every intuitionistic fuzzy $\pi g\alpha$ -closed set is an intuitionistic fuzzy π gb* - closed set.
14. Every intuitionistic fuzzy π gp-closed set is an intuitionistic fuzzy π gb* - closed set.

Proof. It is similar to that of (theorems (3.11), (3.13), (3.15), (3.17), (3.19), (3.21), (3.23), (3.25), (3.27), (3.29), (3.31), (3.33), (3.35) and (3.37)). ■

4. Intuitionistic Fuzzy π gb Continuous Functions and Intuitionistic Fuzzy π gb* Continuous Functions

Definition 4.1 [2] Let f be a function from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be an intuitionistic fuzzy π gb-continuous if $f^{-1}(F)$ is an intuitionistic fuzzy π gb-closed in (X, τ) for every intuitionistic fuzzy closed set F of (Y, σ) .

Theorem 4.2 A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is intuitionistic fuzzy π gb- continuous if and only if the inverse image of every intuitionistic fuzzy open set of (Y, σ) is intuitionistic fuzzy π gb-open set in (X, τ) .

Proof. Necessity: Let A be an IFOS in (Y, σ) . Then A^c is an IFCS in (Y, σ) . By hypothesis, $f^{-1}(A^c)$ is an IF π gbCS in (X, τ) . Then $f^{-1}(A^c) = (f^{-1}(A))^c$, implies $f^{-1}(A)$ is an IF π gbOS in (X, τ) .

Sufficiency: Let A be an IFCS in (Y, σ) . Then A^c is an IFOS in (Y, σ) . By hypothesis, $f^{-1}(A^c)$ is an IF π gbOS in (X, τ) . But $f^{-1}(A^c) = (f^{-1}(A))^c$, which in turn implies $f^{-1}(A)$ is an IF π gbCS in (X, τ) . Hence f is intuitionistic fuzzy π gb-continuous. ■

Theorem 4.3 Let $f: (X, \tau) \rightarrow (Y, \sigma)$ from a IFTS (X, τ) to another IFTS (Y, σ) is IF π gb-continuous. Then the following statements hold.

1. $f(\pi\text{gb-cl}(A)) \subseteq \text{cl}(f(A))$, for every intuitionistic fuzzy set A in (X, τ) .
2. $\pi\text{gb-cl}(f^{-1}(B)) \subseteq f^{-1}(\text{cl}(B))$, for every intuitionistic fuzzy set B in (Y, σ) .

Proof.

1. Let A be any set in (X, τ) . Then $\text{cl}(f(A))$ is an IFCS in (Y, σ) . Since f is an IF π gb-continuous, $f^{-1}(\text{cl}(f(A)))$ is an IF π gbCS in (X, τ) . Since $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(\text{cl}(f(A)))$ and $f^{-1}(\text{cl}(f(A)))$ is an IF π gbCS, implies $\pi\text{gb-cl}(A) \subseteq \pi\text{gb-cl}(f^{-1}(\text{cl}(f(A)))) = f^{-1}(\text{cl}(f(A)))$. Hence

$$f(\pi_{gb}\text{-cl}(A)) \subseteq \text{cl}(f(A)).$$

2. Replacing A by $f^{-1}(B)$ in (i), we get $f(\pi_{gb}\text{-cl}(f^{-1}(B))) \subseteq \text{cl}(f(f^{-1}(B))) \subseteq \text{cl}(B)$ $f(\pi_{gb}\text{-cl}(f^{-1}(B))) \subseteq \text{cl}(B)$ Hence $\pi_{gb}\text{-cl}(f^{-1}(B)) \subseteq f^{-1}(\text{cl}(B))$. ■

Definition 4.4 A bijection function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called an intuitionistic fuzzy π_{gb} homeomorphism (IF π_{gb} home in short) if f and f^{-1} are IF π_{gb} -continuous functions.

Example 4.5 Let $X = \{a, b\}$, $Y = \{u, v\}$ and $U_1 = \{ \langle a, 0.3, 0.6 \rangle, \langle b, 0.3, 0.7 \rangle \}$, $U_2 = \{ \langle u, 0.4, 0.4 \rangle, \langle v, 0.8, 0.2 \rangle \}$, then $\tau = \{0_-, 1_-, U_1\}$ And $\sigma = \{0_-, 1_-, U_2\}$ are IFT's on X and Y , respectively. Define a bisection function $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = v$ and $f(b) = u$. Then f is an IF π_{gb} -continuous function and f^{-1} is also an IF π_{gb} -continuous function. Therefore f is an intuitionistic fuzzy π_{gb} homeomorphism.

Definition 4.6 Let f be a function from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be an intuitionistic fuzzy π_{gb}^* -continuous if $f^{-1}(F)$ is an intuitionistic fuzzy π_{gb}^* -closed in (X, τ) for every intuitionistic fuzzy closed set F of (Y, σ) .

Theorem 4.7 Let $f: (X, \tau) \rightarrow (Y, \sigma)$ from a IFTS (X, τ) to another IF (Y, σ) is IF π_{gb}^* -continuous. Then the following statements hold.

- (i) $f(\pi_{gb}^*\text{-cl}(A)) \subseteq \text{cl}(f(A))$, for every intuitionistic fuzzy set A in (X, τ) .
- (ii) $\pi_{gb}^*\text{-cl}(f^{-1}(B)) \subseteq f^{-1}(\text{cl}(B))$, for every intuitionistic fuzzy set B in (Y, σ) .

Proof. It is similar to that of (theorem 4.3). ■

Definition 4.8 A bijection function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called an intuitionistic fuzzy π_{gb}^* homeomorphism (IF π_{gb}^* home in short) if f and f^{-1} are IF π_{gb}^* -continuous functions.

Theorem 4.9

1. Every intuitionistic fuzzy continuous function is an intuitionistic fuzzy π_{gb} -continuous.
2. Every intuitionistic fuzzy semi-continuous function is an intuitionistic fuzzy π_{gb} -continuous.
3. Every intuitionistic fuzzy pre-continuous function is an intuitionistic fuzzy π_{gb} -continuous.
4. Every intuitionistic fuzzy α -continuous function is an intuitionistic fuzzy π_{gb} -continuous.
5. Every intuitionistic fuzzy β -continuous function is an intuitionistic fuzzy π_{gb} -continuous.
6. Every intuitionistic fuzzy g -continuous function is an intuitionistic fuzzy π_{gb} -continuous.
7. Every intuitionistic fuzzy gs -continuous function is an intuitionistic fuzzy π_{gb} -continuous.
8. Every intuitionistic fuzzy gp -continuous function is an intuitionistic fuzzy π_{gb} -continuous.
9. Every intuitionistic fuzzy $g\alpha$ -continuous function is an intuitionistic fuzzy π_{gb} -continuous.
10. Every intuitionistic fuzzy gb -continuous function is an intuitionistic fuzzy π_{gb} -continuous.

πgb - continuous.

11. Every intuitionistic fuzzy πg -continuous function is an intuitionistic fuzzy πgb - continuous.
12. Every intuitionistic fuzzy πgs -continuous function is an intuitionistic fuzzy πgb - continuous.
13. Every intuitionistic fuzzy $\pi g\alpha$ -continuous function is an intuitionistic fuzzy πgb - continuous.
14. Every intuitionistic fuzzy πgp -continuous function is an intuitionistic fuzzy πgb - continuous.

Proof.

1. Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be an intuitionistic fuzzy continuous function. Let V be an intuitionistic fuzzy closed set in (Y,σ) . Since f is an intuitionistic fuzzy continuous $f^{-1}(V)$ is an intuitionistic fuzzy closed set in (X,τ) . As every intuitionistic fuzzy closed set is an intuitionistic fuzzy πgb -closed set, $f^{-1}(V)$ is an intuitionistic fuzzy πgb -closed set. Hence f is an intuitionistic fuzzy πgb -continuous.

2. It is similar to that of above. ■

The converse of above theorem is not true in general, as we will see in the following example.

Example 4.10 Let $X=\{a, b\}$, $Y=\{u, v\}$ and $U_1 = \{ \langle a, 0.1, 0.8 \rangle, \langle b, 0.5, 0.5 \rangle \}$,

$U_2 = \{ \langle u, 0.6, 0.4 \rangle, \langle v, 0.9, 0.1 \rangle \}$, then $\tau = \{0_-, 1_-, U_1\}$

And $\sigma = \{0_-, 1_-, U_2\}$ are IFT's on X and Y respectively. Define A function $f:(X,\tau)\rightarrow(Y,\sigma)$ by $f(a)=v$ and $f(b)=u$. Therefore f is an intuitionistic fuzzy πgb -continuous but not an intuitionistic fuzzy continuous function, since

$$f^{-1}(U_2^c) = \{ \langle a, 0.1, 0.9 \rangle, \langle b, 0.4, 0.6 \rangle \}, cl(f^{-1}(U_2^c)) = U_2^c \neq f^{-1}(U_2^c)$$

Theorem 4.11

1. Every intuitionistic fuzzy continuous function is an intuitionistic Fuzzy πgb^* -continuous.
2. Every intuitionistic fuzzy semi-continuous function is an intuitionistic fuzzy πgb^* -continuous.
3. Every intuitionistic fuzzy pre-continuous function is an intuitionistic fuzzy πgb^* -continuous.
4. Every intuitionistic fuzzy α -continuous function is an intuitionistic fuzzy πgb^* -continuous.
5. Every intuitionistic fuzzy b -continuous function is an intuitionistic fuzzy πgb^* -continuous.
6. Every intuitionistic fuzzy g -continuous function is an intuitionistic fuzzy πgb^* -continuous.
7. Every intuitionistic fuzzy $g\alpha$ -continuous function is an intuitionistic fuzzy πgb^* -continuous.
8. Every intuitionistic fuzzy gs -continuous function is an intuitionistic fuzzy πgb^* -continuous.
9. Every intuitionistic fuzzy gp -continuous function is an intuitionistic fuzzy πgb^* -continuous.
10. Every intuitionistic fuzzy gb -continuous function is an intuitionistic fuzzy πgb^* -continuous.
11. Every intuitionistic fuzzy πg -continuous function is an intuitionistic fuzzy πgb^* -continuous.
12. Every intuitionistic fuzzy πgs -continuous function is an intuitionistic fuzzy πgb^* -continuous.

continuous.

13. Every intuitionistic fuzzy π gb-continuous function is an intuitionistic fuzzy π gb^{*}-continuous.
14. Every intuitionistic fuzzy π ga-continuous function is an intuitionistic fuzzy π gb^{*}-continuous.

Proof. It is similar to that of (theorem 4.9). ■

5 Relation Between IF π gb-Sets and IF π gb^{*}-Sets

Theorem 5.1 Every intuitionistic fuzzy π gb-closed set is an intuitionistic fuzzy π gb^{*}-closed set.

Proof. Let A be an intuitionistic fuzzy π gb-closed set of (X, τ) such that $A \subseteq U$ and U is an IF π -open set in (X, τ) . Since every IF π -open set is an IF open set, $\text{bcl}(A) \subseteq U$. Thus $\text{int}(\text{bcl}(A)) \subseteq \text{int}(U) = U$. Hence A is an intuitionistic fuzzy π gb^{*}-closed set. ■

The converse of above theorem is not true in general, as we will see in the following example

Example 5.2 Let $X = \{a, b\}$ and let $\tau = \{0_\sim, 1_\sim, U_1, U_2, U_3\}$ is an IFT on X , where $U_1 = \{\langle a, 0.1 \rangle, \langle b, 0.7, 0.3 \rangle\}$, $U_2 = \{\langle a, 0.8, 0.2 \rangle, \langle b, 0.1 \rangle\}$, $U_3 = \{\langle a, 0.8, 0.2 \rangle, \langle b, 0.7, 0.3 \rangle\}$. Let $A = \{\langle a, 0.7, 0.3 \rangle, \langle b, 0.7, 0.3 \rangle\}$, then A is an IF π gb^{*}CS but it is not IF π gbCS, since $\text{bcl}(A) = \{\langle a, 0.7, 0.3 \rangle, \langle b, 1, 0 \rangle\} \not\subseteq U_3$.

Theorem 5.3 Every intuitionistic fuzzy π gb-continuous function is an intuitionistic fuzzy π gb^{*}-continuous.

Proof. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an intuitionistic fuzzy π gb-continuous function. Let V be an intuitionistic fuzzy closed set in (Y, σ) . Since f is an intuitionistic fuzzy π gb-continuous $f^{-1}(V)$ is an intuitionistic fuzzy π gb-closed set in (X, τ) . As every intuitionistic fuzzy π gb-closed set is an intuitionistic fuzzy π gb^{*}-closed set, $f^{-1}(V)$ is an intuitionistic fuzzy π gb^{*}-closed set. Hence f is an intuitionistic fuzzy π gb^{*}-continuous. ■

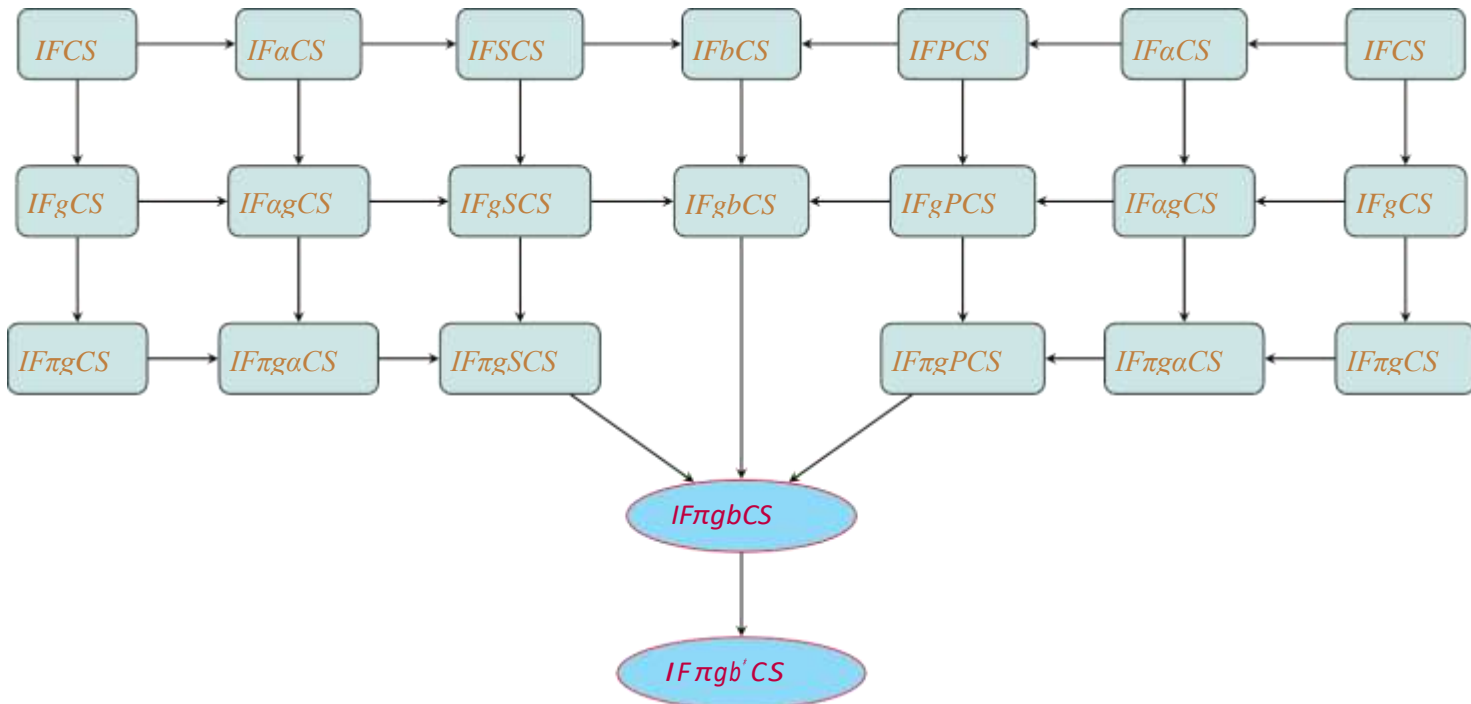
The converse of above theorem is not true.

Example 5.4 Let $X = \{a, b\}$, $Y = \{u, v\}$ and $U_1 = \{\langle a, 0.1 \rangle, \langle b, 0.6, 0.4 \rangle\}$, $U_2 = \{\langle a, 0.7, 0.3 \rangle, \langle v, 0, 0.1 \rangle\}$, $U_3 = \{\langle a, 0.7, 0.3 \rangle, \langle b, 0.6, 0.4 \rangle\}$, $U_4 = \{\langle u, 0.4, 0.6 \rangle, \langle v, 0.4, 0.6 \rangle\}$, then $\tau = \{0_\sim, 1_\sim, U_1, U_2, U_3\}$ and $\sigma = \{0_\sim, 1_\sim, U_4\}$ are IFT's on X and Y respectively. Define a function $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Therefore f is an intuitionistic fuzzy π gb^{*}-continuous but not an intuitionistic fuzzy π gb-continuous function.

Theorem 5.5 Every intuitionistic fuzzy π gb homeomorphism is an intuitionistic fuzzy π gb^{*} homeomorphism.

Proof. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an IF π gb homeomorphism. Then f and f^{-1} are IF π gb-continuous functions. This implies f and f^{-1} are IF π gb^{*}-continuous functions. Therefore, f is an intuitionistic fuzzy π gb^{*} homeomorphism. ■

The converse of above theorem is not true.



Example 5.6 Let $X = \{a, b\}$, $Y = \{u, v\}$ and $U_1 = \{ \langle a, 0.1 \rangle, \langle b, 0.6, 0.4 \rangle \}$, $U_2 = \{ \langle a, 0.8, 0.2 \rangle, \langle b, 0, 1 \rangle \}$, $U_3 = \{ \langle a, 0.8, 0.2 \rangle, \langle b, 0.6, 0.4 \rangle \}$, $U_4 = \{ \langle u, 0.4, 0.6 \rangle, \langle v, 0.4, 0.6 \rangle \}$, then $\tau = \{0_\sim, 1_\sim, U_1, U_2, U_3\}$ and $\sigma = \{0_\sim, 1_\sim, U_4\}$ are IFT's on X and Y , respectively. Define a bijection function $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an intuitionistic fuzzy πgb^* homeomorphism but not an intuitionistic fuzzy πgb homeomorphism, since f is not an $IF\pi gb$ -continuous function. Hence f is not an $IF\pi gb$ homeomorphism.

Conclusion

As an extension of this paper, can be study more properties of intuitionistic fuzzy πgb and πgb^* -sets with other intuitionistic fuzzy sets.

References

- [1] D. Adnadjevic, "On b-Open Sets", Matematički Vesnik, 48 (1996), pp. 59- 64.
- [2] Amal M. Al-Dowai and Abdul Gawad A. Al-Qubati, "ON Intuitionistic Fuzzy Slightly πgb -Continuous Functions", IJARJSET, Vol.4(2017), Issue, pp. 84 - 88.
- [3] K. T. Atanassov, "Intuitionistic fuzzy sets", Fuzzy Sets and Systems, 20(1986), pp.87-96.
- [4] C. L. Chang, "Fuzzy topological space", J. Math. Anal. Appl., 24 (1968), pp.182 -190.
- [5] D. Coker, M. Demirci, "On intuitionistic fuzzy points", Notes IFS, 1,2 (1995), pp. 79-84.
- [6] D. Coker and A. Es. Hyder, "An introduction to intuitionistic fuzzy topological spaces ", Fuzzy Sets and Systems, 88(1997), pp. 81-89.
- [7] R. Dhanya and A.Parvathi, "On πgb^* -closed sets in topological spaces", IJARJSET, Vol. 3 (2014), Issue 5 pp 2319-8753.
- [8] J. Dontche and T. Noiri, "Quasi Normal Spaces and πg -closed sets", Acta math. Hungar., 89(3) (2000), pp. 211-219.
- [9] H. Gurcay, A. Haydar and D. Coker, "On fuzzy continuity in intuitionistic fuzzy topological spaces", jour. of fuzzy math, 5 (1997), pp.365-378.
- [10] I. M. Hanafy, "Intuitionistic fuzzy γ continuity ", Canad. Math Bull, 52(2009), pp. 1-11.
- [11] S. Maragathavalli and K. Ramesh, "Intuitionistic fuzzy π -generalized semi closed sets ",

Advances in Theoretical and Applied Mathematics, 1 (2012), pp. 33-42.

[12] J. H. Park, "On π gp-closed sets in Topological Spaces ", Indian J.Pure Appl. Math., (2004).

[13] P. Rajarajeswari and R. Krishna Moorthy, "On intuitionistic fuzzy generalized b closed sets", International journal of Computer Applications, 63(2013), pp. 41-46.

[14] P. Rajarajeswari and L. Senthil Kumar, "Generalized pre-closed sets in intuitionistic fuzzy topological spaces ", Apple. Math. Sci. (Ruse), 6(94) (2012), pp. 4691-4700.

[15] M.S.Sarsak,N.Rajesh," π -Generalized Semi-Preclosed Sets",Int.Math- ematical Forum, 5 (2010), pp.573-578.

[16] R. Santhi and K. Sakthivel, "Intuitionistic fuzzy generalized semi continuous mappings", Advances in Theoretical and Applied Mathematics,5(2009), pp. 73-82.

[17] K. Sakthivel, "Intuitionistic fuzzy Alpha generalized continuous mappings and intuitionistic fuzzy Alpha generalized irresolute mappings ", Applied Mathematical Sciences, Vol. 4, No. 37, (2010), pp. 1831-1842.

[18] D. Sreeja and C.Janaki,"On π gb-closed sets in topological spaces",Int. International Journal of Mathematical Archieve, 2(8) (2011),pp. 1314-1320.

[19] N. Seenivasagan, O. Ravi and S. SatheeshKanna, " π ga Closed Sets in intuitionisticfuzzytopologicalspaces",J.ofAdvancedResearchinScientific Computing, 6 (2014),pp. 1-15.

[20] S. S. Thakur and Rekha Chaturvedi, "Regular generalized closed sets in intuitionistic fuzzy topological spaces ", Universitatea Din Bacau Studii Si Cercertar Stiintifice, 6 (2006), pp. 257-272.

[21] L.A. Zadeh, "Fuzzy Sets", Inform. And control, 8 (1965), pp.338 – 353