Image Compression using Fractional Fourier Transform

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ABSTRACT

The Fourier transform can be used in the field of image processing, communications and data compression applications. The Discrete Fractional Fourier Transform (DFrFT) is a generalization of the Discrete Fourier Transform and is used for compression of high resolution images. With the extra degree of freedom provided by the DFrFT and its fractional order "a", decompressed image can be obtained. The performance parameters such as Peak Signal-to-Noise Ratio (PSNR), mean square error (MSE) and Compression Ratio (CR) are determined for the image. The given image is subdivided and DFrFT is applied for the subdivided image to form transformed coefficients and IDFrFT is applied for reconstruction of original images. Discrete Fractional Fourier Transform is computed by eigen decomposition method. The significant improvement is observed using DFrFT as compared to the image compression based on Discrete Cosine Transform (DCT).

KEYWORDS
Image Compression, DFrFT, PSNR, MSE, CR.

I INTRODUCTION

Image compression is a technique of reducing dimensions of the image, to a level that can be used. A common characteristic of images is that the neighboring pixels are correlated to each other and contains same information. So it is required to find less correlated representation of the image. Compression can be achieved by two types namely redundancy and irrelevancy reduction. Redundancies reduction is nothing but removing of duplication from the image. Irrelevancy reduction neglects the parts of the signal that will not be noticed by the signal receiver, namely the Human Visual System. There are three types of redundancies in order to compress image size. They are listed as coding redundancy is represented by using higher number of bits per pixel than it is actually needed. Second one is inter pixel redundancy in which neighboring pixels have similar values. The last one is psycho visual redundancy that refers; some information in the image is irrelevant so it is ignored by the human vision system.

Image compression techniques aim at reducing the number of bits required to represent an image by using advantage of these redundancies. Advantages of image compression are, it not only reduces storage requirements but also overall execution time. It also reduces the probability of transmission error because of fewer bits are transmitted and also provides a level of security against illegal monitoring.

The image compression techniques are classified into two categories as lossless technique and lossy technique. In lossless compression technique, the original image can be perfectly recovered from the compressed image. It is also known as entropy coding since it use statistics techniques to eliminate/minimize redundancy. In lossy compression, the reconstructed image contains degradation relative to the original. Lossy schemes provide much higher compression ratios than lossless method.

The rest of the paper is organized as follows: Section II gives an overview of the DfrFT. Section III details the algorithm. Section IV gives the simulation results and analysis. Finally section V gives the concluding remarks.

II DISCRETE FRACTIONAL FOURIER TRANSFORM

The FrFT is a generalization of the ordinary Fourier Transform with fractional order parameter "a" and is similar to the Fourier Transform, when fractional order α is equal to π/2. The FrFT consists of time– frequency representation. In all the time–frequency representations, one normally uses a plane with two orthogonal axes corresponding to time and frequency. The FRFT is defined with the help of the transformation kernel K_α as

$$k_\alpha(t,u)=\begin{cases}\delta(t-u) & \text{if } \alpha \text{ is a multiple of } 2\pi\\ \delta(t+u) & \text{if } \alpha+\pi \text{ is a multiple of } 2\pi\\ \sqrt{\frac{1-j\cot\alpha}{2\pi}}\,e^{j\left(\left(u^2+t^2\right)/2\right)\cot\alpha-jut\,\mathrm{cosec}(\alpha)} & \text{if } \alpha \text{ is not a multiple of } \pi\end{cases}$$

The 2D forward and inverse DFrFT are computed from above 2D transformation kernel as:

$$X_{(\alpha,\beta)}(m,n)=\sum_{p=0}^{M-1}\sum_{q=0}^{N-1}x(p,q)R_{(\alpha,\beta)}(p,q,m,n) \quad (1)$$

$$x(p,q)=\sum_{m=0}^{M-1}\sum_{N=0}^{N-1}X_{(\alpha,\beta)}(m,n)R_{(-\alpha,-\beta)}(p,q,m,n) \quad (2)$$

In two-dimensional DFrFT we have to consider two angles of rotation α=aπ/2 and β=bπ/2 and If one of these angles is zero, the 2D transformation kernel reduces to the 1D transformation kernel.

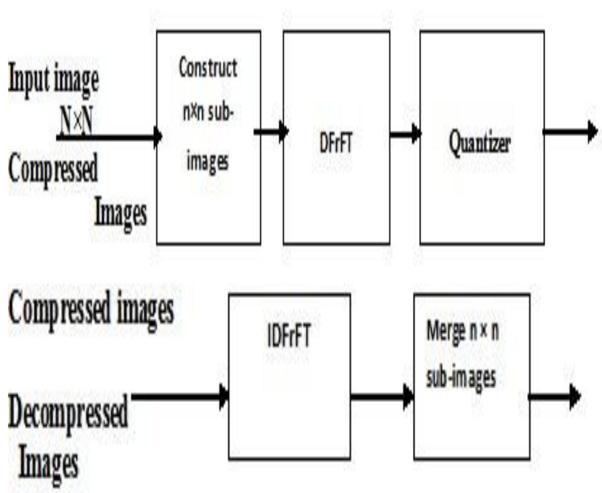

Fig 1: Image Compression Model (a) Encoder; (b) Decoder

In image compression using FrFT, a compression model encoder performs three operations i.e. Sub image decomposition, Transformation and Quantization. The decoder implements the inverse sequence of steps of encoder. Because quantization results in irreversible information loss, the decoder does not contain inverse quantizer block hence it is a lossy compression method. The process detailed in figure 1.

Sub-image Decomposition:

An image is first partitioned into non-overlapped nXn sub images. The most popular sub image sizes are 8×8, 16×16. In present implementation sub image size chosen is 8×8. As sub image size increases, error decreases but computational complexity increases. Compression techniques that operate on block of pixels i.e. sub images are often described as block coding techniques. The transform of a block of pixels may suffer from discontinuity effects resulting in the presence of blocking artifacts in the image after it has been decompressed.

Transformation:

In this step, a 2D-FRFT is applied to each block to convert the gray levels of pixels in the spatial domain into coefficients in the frequency domain. By using FRFT a large amount of information is pack into smallest number of transform coefficients, hence small amount of compression is achieved at this step. At decoder inverse FRFT is applied.

Quantization:

The final step in compression process is quantizes the transformed coefficients according to cutoff selected and variation of ‘a’. By adjusting the cutoff of the transform coefficients, a compromise can be made between image quality and compression factor. With the FRFT by varying its free parameter ‘a’, high compression ratio can be achieved even for the same cutoff. The quantized coefficients are then rearranged in a zigzag scan order to form compressed image which can be stored or transmitted. At decoder, simply inverse process of encoder is performed by using inverse 2D-FRFT. The non-overlapped sub images merged to get decompressed image. As we know it is lossy compression technique so the decompressed image is not exactly same as that of original image.

Due to the advent of computers and enhanced computational complexities the Discrete Fractional Fourier Transform (DFrFT) came into existence. The DFRFT has been considered to be the combination of four parts:
1) The original signal;2) Its DFT;3) Circular flipped of signal;4) Circular flipped of its DFT.

These DFRFTs uses the DFT Hermite eigenvectors as their eigenvectors and have similar an eigen decomposition form as the continuous FRFT. The computation of DFrFT is based upon the eigen decomposition of the DFT kernel matrix. The kernel matrix of DFT has only four distinct eigen values [l, - j, -1, j]. The DFRFT is developed based on the eigen decomposition, and its transform kernel is written as

$$S = \begin{bmatrix} 2 & 1 & 0 & 0 & \cdots & 1 \\ 1 & 2\cos\omega & 1 & 0 & \cdots & 0 \\ 0 & 1 & 2\cos 2\omega & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & & \\ 1 & 0 & 0 & 0 & \cdots & 2\cos((N-1)\omega) \end{bmatrix}$$

III ALGORITHM

1. Read the Image.
2. Convert the colour image into gray scale image.
3. Resize the original image into a required dimension that is to 256X256
5. Divide the original image into 16 sub-blocks by partitioning the image into 4X4.
6. Applying the 2D-DFrFT to each sub-blocks in order to convert the gray levels of pixels in the spatial domain into coefficients in the frequency domain.
7. The coefficients are normalized by different scales according to the cutoff selected in order to get the compressed image.
8. At decoder simply inverse process of encoding by using inverse 2D-DFrFT is performed.
9. After getting the decompressed image the performance parameter like PSNR, MSE and CR are calculated using appropriate formulas.

IV SIMULATION RESULTS AND ANALYSIS

The numerical simulation for images is performed using MATLAB code to examine the validity of image compression technique. Figure 2 represents the original image whereas figure 5 shows the decompressed image.

Fig. 2: Original Image

Fig. 3: Sub Divided Image

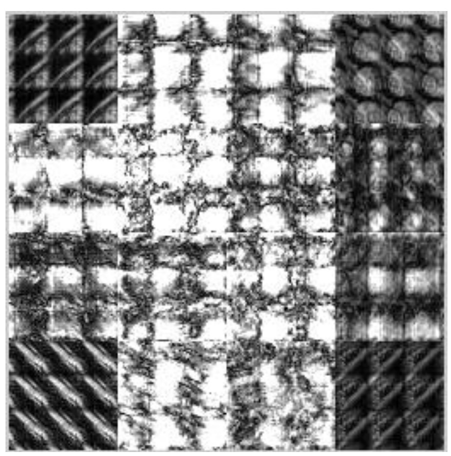

Fig. 4: Encoded Image

Fig. 5: Reconstructed Image

The Original satellite image is of size 65Kb (256x256) is shown in figure 3 which is taken for analysis and is partitioned into 4x4 sub-images. Applying DFrFT method to the sub-image we got image as shown in figure 4.

Figure 5 shows the reconstructed image of size 12.7Kb (256x256)

Table.1 Variation of PSNR and MSE with varying Fractional Order keeping CR Constant

Fractional Order(a)	PSNR at 50% CR	MSE at 50% CR
0	27.72	109.79
0.1	31.26	48.61
0.2	31.76	43.30
0.3	32.35	47.84
0.4	32.95	32.92
0.5	33.39	29.75
0.6	32.11	39.99
0.7	32.33	37.96
0.8	32.87	33.52
0.9	35.32	19.08
1	45.92	1.66

PERFORMANCE PARAMETER

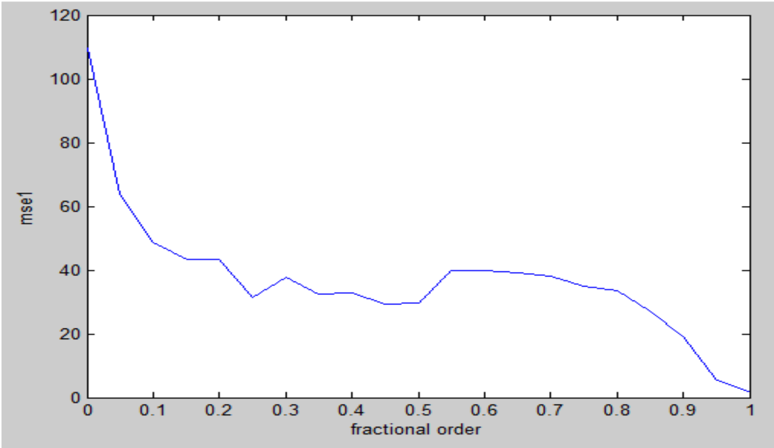

Fig.6: MSE versus Fractional Order

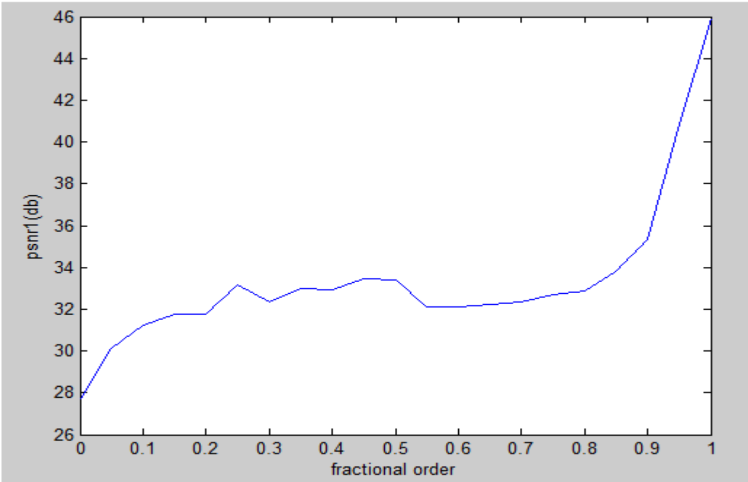

Fig.7: PSNR versus Fractional Order

V CONCLUSION

It is concluded that, good image compression is achieved by using DFrFT for the image. It is seen that, by using the DFrFT better PSNR and min MSE and same amount of CR are gained, which results better image compression. It is also observed that with increase in CR the quality of image decreases.

REFERENCES

[1] Nair,G.M, (2008). Role of Communications Satellites in National Development, *IETE Technical Review*, Vol. 25, pp. 3-8.
[2] Min-Hung Yeh and Soo-Chang Pei, (March 2003). A Method for the Discrete Fractional Fourier Transform Computation, *IEEE Transactions on Signal Processing*, Vol. 51, No. 3.
[3] Pei.S.C. AndYeh.M.H,(1996). Discrete Fractional Fourier transform, *IEEE International Symposium on Circuits and Systems*, Vol. 2, pp. 536 – 539.
[4] Papiya Chakraborty, (September-2012). A Survey Analysis for Lossy Image Compression using Discrete Cosine Transform, *International Journal of Scientific Engineering Research*, Vol 3, Issue 9.
[5] Dickinson,B. W. and Steiglitz,K. (1982). Eigenvectors and Functions of the Discrete Fourier Transform, *IEEE Transaction Acoustic., Speech, and Signal Processing*, Vol. 3, pp. 25-31.